Question [1]: (35 marks) [ILOs: a1]

a) Determine the Fourier sine and cosine series of the function: 10

$$f(x)=x^2 \quad , \quad 0<x<\pi$$

b) Expand $f(t)=t^2$, $on[-\pi,\pi]$ in a complex Fourier series. 10

c) Show that the set $\{\cos(nx)\}$, $n \in N$, is orthogonal but not orthonormal on the interval $[0,2\pi]$ 5

d) A refinery produces both gasoline and fuel oil, and sells gasoline for \$1 per gallon and fuel oil for \$0.90 per gallon. The refinery can produce at most 600,000 gallons a day, but must produce at least two gallons of fuel oil for every gallon of gasoline. At least 150,000 gallons of fuel oil must be produce each day to meet current demands. How must of each type of fuel should be produced to maximize daily profits? 10

Question [2]: (30 marks) [ILOs: b7]

a) Using the methods of separation of variables (M.S.V), solve the following heat equation: 10

$$8u_{xx}=u_t \quad , \quad 0<x<5, \; u(0,t)=0 \quad , \quad u(5,t)=0$$
$$u(x,0)=2\sin(\pi x)-4\sin(2\pi x)+\sin(5\pi x)$$

b) Use D'Alembert's method to solve the partial differential equation system: 10

$$u_{tt}=\tfrac{1}{\pi^2}u_{xx}, \quad t\geq 0, where: u(x,0)=\cos(\pi x), \quad u_t(x,0)=0, \quad -\infty<x<\infty$$

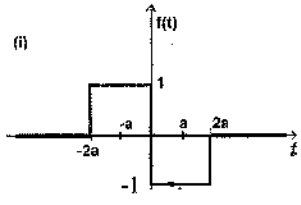
c) Solve the P.D.E. by Euler solution : 10

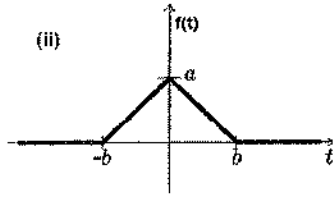
$16u_{xx}-8u_{xy}+u_{yy}=0$, and find a particular solution which satisfies:

$$u(x,0)=e^{3x}, \qquad u(0,y)=4y^2+e^{12y}$$

Question [3]: (35 marks) [ILOs: c7]

a) Find Fourier transform of : 15





b) Find Fourier transform of $f(t)=e^{-a^2t^2}$, where a is a costant, and $\int_0^\infty e^{-u}du=\frac{\sqrt{\pi}}{2}$ 10

c) Find the directional derivative of f at p in the direction of $\vec{a}$ in the following: 5

$$f(x,y,z)=xyz, \quad p(-1,1,3), \quad \vec{a}=[1,-2,2]$$

d) Compute the curl of the following vector field $\vec{F}(x,y,z)=[e^y\cos x, e^y\sin x, 0]$ and show that vector $\vec{F}$ is conservative or not. 5

Good luck >>>>==========<<<< Dr. Manal El-Sayed

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