Kafrelsheikh University
Faculty of Engineering
Dept. of Mech. Engineering
Year: Third Year
Subject: Theory of vibration

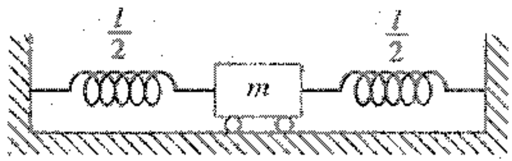
2nd Semester
Final Examination
Date: May 26th, 2019
Time allowed: 3 hours
Full Mark: 100

Question #1: (12 Marks): Give brief answers to the following:

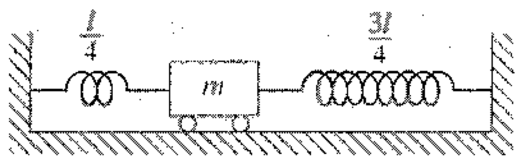
1. Why is damping considered only in the neighborhood of resonance in most cases?
2. **(ILO a.1.1)** What happens to the response of an undamped system at resonance?
3. If a vehicle vibrates badly while moving on a uniformly bumpy road, will a change in the speed improve the condition?
4. Will the force transmitted to the base of a spring-mounted machine decrease with the addition of damping?
5. **(ILO a.1.1)** How many distinct natural frequencies can exist for an n-degree-of-freedom system?
6. **(ILO a.1.3)** If the displacement of a machine is described as $x(t) = 0.15 \sin 4t + 2.0 \cos 4t$, where x is in inches and t is in seconds, find the expressions for the velocity and acceleration of the machine. Also find the amplitudes of displacement, velocity, and acceleration of the machine.

Question # 2 (12 Marks) (ILO b.2.1):

A helical spring of stiffness k and length l is cut into two halves and a mass m is connected to the two halves as shown in Fig. (a). The natural time period of this system is found to be 0.5 s. If an identical spring is cut so that one part is one-fourth and the other part is three-fourths of the original length, and the mass m is connected to the two parts as shown in Fig. (b), what would be the natural period of the system?



(a)



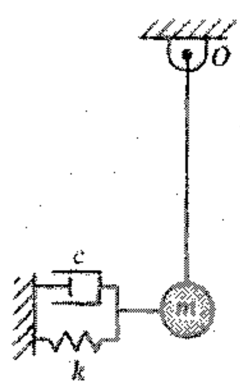
(b)

Question # 3 (10 Marks) (ILO c.1.2):

A vibrometer has a natural frequency of 4 Hz and a damping factor ξ of 0.7. This instrument is used to measure the displacement of a machine vibrating at 9 Hz. It was assumed that the amplitude measured by the vibrometer is 0.090 m. Determine the exact value of the amplitude of the machine.

Question # 4 (16 Marks) (ILO b.5.1, c.1.1):

Using Lagrange equation, obtain the differential equation of the free vibration of the pendulum shown in the figure. Then, assuming small oscillations, determine the circular frequency, the critical damping coefficient, and the damping factor of this system. Assume that the rod is massless.

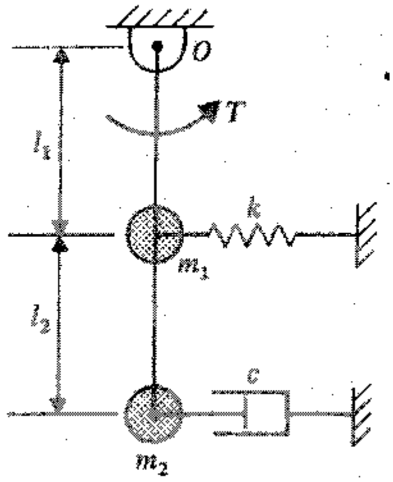


Question # 5 (20 Marks) (ILO b.2.2):

A damped single degree offreedom mass-spring system has a mass $m = 4$ kg, a spring stiffness $k = 2400$ N/m, and a damping coefficient c = 15 N· s/m. The mass is subjected to a harmonic force which has an amplitude $F_o = 16$ N and a frequency $\omega_f = 15$ rad/s. The initial conditions are $x_o = 4$ cm, and $\dot{x}_o = 0$. Determine the displacement, velocity, and acceleration of the mass after time $t = 0.5$ s.

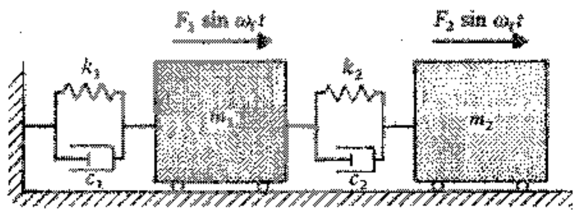
Question # 6 (20 Marks) (ILO a.5.3):

Assuming small angular oscillations, derive the differential equation of motion of the system shown in the figure. If $m_1 = m_2 = 0.4$ kg, $l_1 = l_2 = 0.6$ m, $k = 15$ kN/m, $c = 60$ N· s/m, and $T = 10\, sin\, 10t$ N· m, determine the steady-state solution as a function of time.



Question # 7 (20 Marks) (ILO b.5.2)

Derive the differential equations of motion of the two degree of freedom system shown in the figure. Then, using modal analysis, find the steady-state response of the two masses. $m_1 = 10$ kg, $m_2 = 1$ kg, $c_1 = 6$ N.s/m, $c_2 = 1$ N.s/m, $k_1 = 30$ N/m, $k_2 = 5$ N/m, $F_1 = 10$ N, $F_2 = 0$ N, $\omega_f = 5$ rad/s.



Theory of Vibration – Formulae Sheet

Forced vibration:

$X_o = \frac{F_o}{k}$, $r = \frac{\omega_f}{\omega}$

Forced undamped vibration:

$x_p = X_o \beta \sin \omega_f t$, $\beta = \frac{1}{1-r^2}$

$x(t) = X sin(\omega t + \varphi) + X_o \beta \sin \omega_f t$

At resonance ($r = 1$):

$$x_p = -\frac{\omega X_o t}{2} \cos \omega t$$

$$x(t) = X \sin(\omega t + \varphi) - \frac{\omega X_o t}{2} \cos \omega t$$

Forced vibration of damped system:

$x_p = X_o \beta \sin(\omega_f t - \psi)$, $\psi = tan^{-1}\left(\frac{2r\xi}{1-r^2}\right)$

$\beta = \frac{1}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$, $\beta_{max.} = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Force transmission:

$F_t = F_o \beta_t \sin(\omega_f t - \bar{\psi})$

Transmissibility: $\beta_t = \beta\sqrt{1 + (2r\xi)^2}$

$\bar{\psi} = \psi - \psi_t$, $\psi_t = \tan^{-1}(2r\xi)$

Work:

$W_e = \pi F_o X_o \beta \sin \psi$, $W_d = \pi c X_o^2 \beta^2 \omega_f$

$W_s = 0$

Rotating unbalance:

$$x_p(t) = \left(\frac{me}{M}\right) \beta_r \sin(\omega_f t - \psi)$$

$$\beta_r = \frac{r^2}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

$F_t = (me\omega^2)\beta_t \sin(\omega_f t - \bar{\psi})$

$\beta_t = \beta_r \sqrt{1 + (2r\xi)^2}$

Base motion:

$x_p(t) = Y_o \beta_b \sin(\omega_f t - \psi + \psi_b)$

$\beta_b = \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$, $\psi_b = \tan^{-1}(2r\xi)$

$F_t = Y_o k \beta_b r^2 \sin(\omega_f t - \psi + \psi_b)$

Relative motion:

$z = Y_o \beta_r \sin(\omega_f t - \psi)$

$$dB = 20Log(\frac{x_1}{x_2})$$

$$t_{p_1} = \frac{\pi / 2 - \phi}{\omega_n}$$

$$E = \frac{1}{2}kX^2$$

$$K_t = \frac{GJ}{L}$$

$\xi = \frac{c}{C_c}$, $C_c = 2m\omega$

$X_i = \sqrt{1-\xi^2} X e^{-\xi\omega t_i}$

logarithmic Decrement:

$$\ln \frac{X_i}{X_{i+n}} = n\xi\omega\tau_d = n\delta$$

$$\xi = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

Energy Loss:

$$\frac{\Delta U}{U_i} = 1 - e^{-2\delta}$$

IMPACT DYNAMICS:

$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2$

$v_2' - v_1' = e(v_1 - v_2)$