



The following questions measure ILOs a1, b1, b7, c1 and c7

Answer the following questions:

الامتحان عبارة عن ٦٠ نقطة اختياري يجاب عنهم في ورقة الاختيار من متعدد ويجب تظليل دائره واحده فقط هكذا ● وذلك عند تمام التأكد من الاجابة.

In the following $k = 0, \pm 1, \pm 2, \pm 3, \dots$

All points are equal marks

1. The function $f(z) = |z|$ is:

- A) analytic for all z B) not analytic C) analytic for $z=1$ D) non of these

2. The following is true except

- A) all analytic functions are harmonic ones B) all harmonic funtions are analytic ones
 C) for harmonic functions $u_{xx} + u_{yy} = 0$ D) entire function is an analytic one for all z

3. All the following functions are entire except:

- A) e^z B) $\cosh z$ C) z^3 D) $\tan z$

4. Which of the following is false?

- A) $|e^z| = e^x$ B) $\arg(e^z) = y + 2k\pi$ C) $\overline{e^z} = e^{\bar{z}}$ D) $e^z = e^x (\sin y + i \cos y)$

5. the function $f(z) = z^3$ has zero of order :

- A) 1 B) 2 C) 3 D) 4

6. In Laurent series of the function $\frac{e^{2z}}{(z-1)^3}$ about $z = 1$, the coefficient of $\frac{1}{(z-1)^3}$ is:

- A) e^2 B) e C) $2e^2$ D) $2e$

7. All the following are true for an analytic function at z_0 except

- A) $u_x = v_y, \varphi u_y = -v_x$ B) $ru_r = v_\theta, \varphi rv_r = u_\theta$ C) $f(z)$ is differentiable at z_0 D) $f(z)$ is not contiuous at z_0

8. The function $f(z) = \frac{z+1}{z^2+1}$ is:

- A) continuous for all z . B) differentiable for all z . C) continuous for all $z \neq \pm 1$. D) non of these.

9. If a function is differentiable at z_0 then

- A) it is continuous at z_0 B) it is not necessary to be continuous at z_0
 C) the limit at z_0 does not exist D) non of these

10. $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$, is

- A) exists and equal 1 B) exists and equal 0 C) does not exist. D) non of these

11. The value of $\text{Arg}(z) + \text{Arg}(\bar{z})$ is:

- A) 0 B) π C) $\pi/2$ D) non of these



12. The real part of $(1+i)^n$ is:

- A) $2^{n/2} \cos \frac{n\pi}{4}$ B) $2^n \cos n\pi$ C) $2^{n/2} \cos n\pi$ D) non of these

13. The value of $\sqrt{3+4i}$ is:

- A) $\sqrt{3+i}$ B) $2+i$ C) $2-i$ D) $\sqrt{3}+2i$

14. The value of $\left(\frac{\cos \theta + i \sin \theta}{\cos \theta - i \sin \theta}\right)^4$ is

- A) 0 B) 1 C) -1 D) non of these

15. If $\left|\frac{z-5i}{z+5i}\right|=1$, then $z = x + iy$ lie on:

- A) the real axis B) the line $x = 5$ C) the line $y=5$ D) circle

16. The following statements are true except

- A) $x = r \cos \theta$ B) $y = r \sin \theta$ C) $|z| = \sqrt{x^2 + y^2}$ D) $|z| = z\bar{z}$

17. The following statements are true except :

- A) $|z_1 + z_2| \leq |z_1| + |z_2|$ B) $|z_1 - z_2| \leq |z_1| - |z_2|$ C) $|z_1 z_2| = |z_1| |z_2|$ D) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

18. The principle value of $z = \ln 2$ is :

- A) $\ln 2 + \pi/2$ B) $\ln 2 - i\pi/2$ C) $\ln 2 + i\pi/2$ D) $\ln 2 - \pi/2$

19. The value of $(2i)^i$ is :

- A) $e^{\frac{\pi}{2} - 2k\pi} (\cos \ln 2 + i \sin \ln 2)$ B) $e^{\frac{\pi}{2} + 2k\pi} (\cos \ln 2 + i \sin \ln 2)$
 C) $e^{2k\pi} (\cos \ln 2 + i \sin \ln 2)$ D) $e^{\frac{\pi}{2}} (\cos \ln 2 - i \sin \ln 2)$

20. The n^{th} root of z is :

- A) $\sqrt[n]{r} \left(\cos \frac{\theta + 2k\pi}{n} - i \sin \frac{\theta + 2k\pi}{n}\right)$ B) $\left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n}\right)$
 C) $\left(\cos \frac{\theta + 2k\pi}{n} - i \sin \frac{\theta + 2k\pi}{n}\right)$ D) $\sqrt[n]{r} \left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n}\right)$

21. e^z is periodic function of period:

- A) 2π B) $2i\pi$ C) π D) $i\pi$

22. $\int_C \tan z dz, C : |z| = 2$ is:

- A) $2i\pi$ B) $4i\pi$ C) $-4i\pi$ D) 0



23. The residue of $\frac{z^2}{z^2+a^2}$ at $z = ia$ is:
 A) $ia/2$ B) $a/2$ C) ia D) a
24. $\int_C \frac{z^2 - z + 1}{z + 3} dz, C : |z| = 2$ is:
 A) 13 B) -13 C) 7 D) non of these
25. The function $\frac{\cot \pi z}{(z-a)^2}$ has :
 A) pole of order at $z=a$ only B) simple pole at $z=k$.
 C) simple pole at $z=a$ only D) Non of these
26. The function $x^3 - 3xy^2$ is :
 A) harmonic function B) nonharmonic function
 C) rational function D) non of these
27. The complex conjugate function of $u = \sin x \cosh y$ is :
 A) $\sin x \sinh y$ B) $\cos x \sinh y$
 C) $\cos x \cosh y$ D) $\cos x \sin y$
28. If a, b, c and d are real numbers, then $\frac{a+ib}{c+id}$ will be also real number if
 A) $bc = ad$ B) $cd = ab$ C) $bc = -ad$ D) $cd = -ab$
29. The conjugate of $(1+i)^2$ is :
 A) $(1-i)^{-2}$ B) $(1+i)^{-1}$ C) $-2i$ D) $2i$
30. The polar form of $(1-i)$ is :
 A) $\sqrt{2}e^{i\pi/4}$ B) $\sqrt{2}e^{-i\pi/4}$ C) $e^{i\pi/4}$ D) $e^{-i\pi/4}$
31. The derivative of the logarithmic function is :
 A) z B) $1/z$ C) e^z D) 1
32. The equation $\cos z = 5$ has:
 A) no real or complex solution
 B) an infinite number of complex solutions
 C) exactly two distinct solutions
 D) non of these
33. The value of $\oint_C \cos z dz, C : |z| = 1$, is
 A) $i\pi$ B) $4i\pi$ C) $2i\pi$ D) 0



34. The residue of $e^{1/z}$ at $z=0$ is :
 A) 0 B) 1 C) -1 D) i
35. The analytical function $f(z) = \frac{z-1}{z^2+1}$ has singularities at
 A) 1, -1 B) 1, -i C) i, 1 D) i, -i
36. The value of $\oint_C \frac{\cos \pi z}{(z+5)(z-3)} dz$, $C: |z|=1$, is
 A) 0 B) $2\pi i$ C) $-\pi i/8$ D) $\pi i/8$
37. The value of $\oint_C \frac{\sin z}{z} dz$, $C: |z|=1$, is
 A) 0 B) $2\pi i$ C) $-2\pi i$ D) ∞
38. The value of $\oint_C \frac{1}{z^2+1} dz$, $C: \left|z - \frac{i}{2}\right|=1$, is
 A) $-2\pi i$ B) $2\pi i$ C) π D) $\tan^{-1} z$
39. The function $w = \ln(x^2 + y^2) + i \tan^{-1}\left(\frac{y}{x}\right)$ is not analytic at the point
 A) (1,1) B) (0,1) C) (1,0) D) (0,0)
40. The value of i^i , is
 A) $e^{-\pi/2}$ B) $e^{\pi/2}$ C) $\pi/2$ D) 1
41. $\cos \phi$ can be represented as
 A) $\frac{e^{i\phi} - e^{-i\phi}}{2i}$ B) $\frac{e^{i\phi} + e^{-i\phi}}{2}$ C) $\frac{e^\phi + e^{i\phi}}{2}$ D) $\frac{e^\phi + e^\phi}{2}$
42. For the function $w(z) = ze^{2z}$, $u(x, y) =$
 A) $e^{2x}(x \cos 2y - y \sin 2y)$ B) $e^{2x}(x \cos 2y + y \sin 2y)$
 C) $e^{2x}(y \cos 2y - x \sin 2x)$ D) $e^{2x}(x \cos 2y - y \sin 2x)$
43. $\cos iz =$
 A) $\cosh z$ B) $-\cosh z$ C) $\sinh z$ D) $-\sinh z$
44. $|\cosh z|^2 =$
 A) $\sinh^2 y + \cos^2 x$ B) $\sinh^2 x - \cos^2 y$ C) $\sinh^2 x + \cos^2 y$ D) $\sinh^2 y - \cos^2 x$
45. $\frac{d}{dz} \operatorname{sech}^{-1} z =$
 A) $\frac{1}{z\sqrt{1-z^2}}$ B) $\frac{-1}{z\sqrt{1-z^2}}$ C) $\frac{-1}{z\sqrt{1+z^2}}$ D) $\frac{1}{z\sqrt{1+z^2}}$



46. $\frac{d}{dz} \cos^{-1} z =$

- A) $\frac{1}{\sqrt{z^2-1}}$ B) $\frac{1}{\sqrt{1-z^2}}$ C) $\frac{-1}{\sqrt{z^2-1}}$ D) $\frac{-1}{\sqrt{1-z^2}}$

47. The integration $\int_{(0,0)}^{(2,4)} z dz$, along the straight line joining (0,0) to (2,4), equals:

- A) $-6-8i$ B) $6+8i$ C) $6-8i$ D) $-6+8i$

48. For any closed contour C, which of the following is true:

- A) If $f(z)$ is analytic, then $\oint_C f(z) dz = 0$.
 B) If $\oint_C f(z) dz = 0$, then $f(z)$ must be analytic.
 C) If $f(z)$ is analytic, then $\oint_C f(z) dz = 2\pi i \sum \text{Res.}$
 D) non of these

49. The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, $C : |z|=2$, is

- A) $\frac{8\pi}{3} e^{-2}$ B) $\frac{8\pi i}{3} e^2$ C) $\frac{8\pi i}{3} e^{-2}$ D) $\frac{8\pi}{3} e^2$

50. The Laurent series of $\frac{\sin z}{z^5}$ at $z=0$, the coefficient of the term $\frac{1}{z}$ is

- A) $-\frac{1}{3!}$ B) 1 C) -1 D) 0

51. To evaluate $\int_{-\infty}^{\infty} \frac{p(x)}{q(x)} dx$:

- A) the degree of $q(x)$ equals the degree of $p(x)$.
 B) the degree of $p(x)$ must be greater than the degree of $q(x)$ at least by 2.
 C) the degree of $q(x)$ must be greater than the degree of $p(x)$ at least by 2.
 D) non of these

52. To evaluate $\int_0^{\infty} \frac{1}{(x^2+4)^2} dx$ we use the contour:

- A) semicircle in the lower half plane. B) unit circle.
 C) semicircle in the upper half plane D) non of these.



53. To evaluate $\int_0^{\infty} \frac{1}{(x^2 + 4)^2} dx$ we use that

- A) $z = e^{ix}$. B) $x = z$. C) $x = \cos z$ D) non of these.

54. To evaluate $\int_0^{\infty} \frac{1}{(x^2 + 4)^2} dx$ we calculate the residue at

- A) $-2i$ B) $2i, -2i$. C) $2i$ D) non of these.

55. To value of $\int_0^{\infty} \frac{1}{(x^2 + 4)^2} dx$ is

- A) $\pi/16$ B) $-\pi/32$ C) $\pi/32$ D) $-\pi/16$.

56. To evaluate $\int_0^{2\pi} \frac{d\theta}{5 + \sin \theta}$:

- A) semicircle in the upper half plane .
 B) semicircle in the lower half plane.
 C) unit circle.
 D) non of these.

57. To evaluate $\int_0^{2\pi} \frac{d\theta}{5 + \sin \theta}$ we use the substitution

- A) $\theta = z$. B) $z = e^{i\theta}$. C) $z = \sin \theta$ D) non of these.

58. In the previous problem, after using suitable substitution, the value of $d\theta$ is

- A) $d\theta = dz$. B) $d\theta = z dz$. C) $d\theta = dz / iz$ D) non of these.

59. In the previous problem, after using suitable substitution, we calculate the residue at the point

- A) $(-5 - 2\sqrt{6})i$. B) $-5 + 2\sqrt{6}$ C) $(-5 + 2\sqrt{6})i$ D) non of these.

60. The value of $\int_0^{2\pi} \frac{d\theta}{5 + \sin \theta}$ is

- A) $\pi\sqrt{6}$. B) $\pi/\sqrt{6}$. C) $\pi/2\sqrt{6}$ D) non of these.

With my best wishes
 Dr. Samah El-Kholy