



Answer the following questions:

**Question 1: [30 Marks]**

a) Let  $z, \omega \in \mathbb{C}$ , show that;

i)  $\overline{z \pm \omega} = \overline{z} \pm \overline{\omega}$ ,      ii)  $\overline{\left(\frac{1}{z}\right)} = \frac{1}{\overline{z}}$

b) Let  $z_1 = -4 + 4i$  and  $z_2 = 3i$  Find

(i)  $\text{Arg}\left(\frac{z_1}{z_2}\right)$ ,      (ii)  $|z_1 z_2|$ ,      (iii)  $\text{Im}\left(\frac{1}{z_1}\right)$

c) By writing  $z = x + iy$  Find all solution of the following equation:  $z^2 = 3 + 4i$

d) Use De Moivre's theorem to express  $\sin 3\theta$  and  $\cos 3\theta$

**Question 2: [30 Marks]**

a) Discuss the continuity of the following function:  $f(z) = \begin{cases} z^2 & , z \neq -i \\ 0 & , z = -i \end{cases}$

b) Prove that  $\frac{d}{dz} z^n = n z^{(n-1)}$

c) Verify that  $u = 3x^2y + 2x^2 - y^3 - 2y^2$  is harmonic, then find a conjugate harmonic  $v$  of  $u$ .

d) Evaluate the following integral:  $\int_{1+i}^{2+4i} z dz$

i) along the parabola  $x=t, y=t^2, 1 < t < 2$

ii) along the straight line  $1+i$  to  $2+i$  and from  $2+i$  to  $2+4i$

**Question 3: [30 Marks]**

a) Evaluate the following integrals around the contour  $C : |z| = 2$

i)  $\oint_C \sin z dz$ ,      ii)  $\oint_C \frac{6e^z}{(z-1)} dz$ ,      iii)  $\oint_C \frac{\cos z}{(z+4)z^2} dz$

b) Find Maclaurine series, of  $f(z) = \frac{1}{1-z^2}$

and hence find Maclaurine series of  $\coth^{-1} z$

c) Determine the order of the poles for the following function and find the

residue at each pole:  $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$

d) Compute the integral:  $I = \int_0^{2\pi} \frac{\cos 2\theta}{5 - 4 \cos \theta} d\theta$