نموذج أجابة

ثالثة ك

ترم أول

2015-2016

فاينال رياضيات 2015

Question [1]

a) A , B are two independent events:

$L.H.S = P(A^c \cap B^c) = P((A \cup B)^c)$ (Demorgan law)

$= 1 - P(A \cup B)$ But $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A \cap B)$

They are independent events : $P(A \cap B) = P(A) . P(B)$

$\therefore L.H.S = 1 - P(A) - P(B) + P(A) P(B)$

$= (1 - P(A))(1 - P(B))$

$= P(A^c) . P(B^c)$

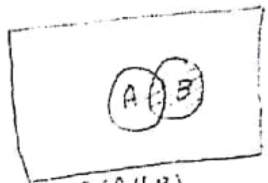
b) $P(A) = \frac{3}{8}$

$P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

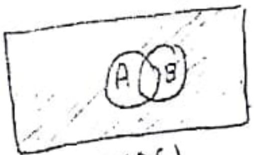
$= \frac{3}{8} + \frac{1}{2} - \frac{1}{4} = \frac{5}{8}$



$P(A \cup B)$

ii) $P(A^c) = 1 - P(A)$

$= 1 - \frac{3}{8} = \frac{5}{8}$



$P(A^c)$

iii) $P(A^c \cup B^c) = P((A \cap B)^c)$ De Morgan law

$= 1 - P(A \cap B)$

$= 1 - \frac{1}{4} = \frac{3}{4}$

$P(A^c \cup B^c)$

c) Expected value μ , variance σ^2 , standard deviation σ

x_i	$f(x_i)$	x_i^2	$x_i f(x_i)$	$x_i^2 f(x_i)$
-1	0,3	1	-0,3	0,3
0	0,1	0	0	0
1	0,1	1	0,1	0,1
2	0,3	4	0,6	1,2
3	0,2	9	0,6	1,8
Σ			1	3,4

$E(x) = \sum_{i=-1}^{3} x_i f(x_i) = 1 = \mu$

$E(x^2) = \sum_{i=-1}^{3} x_i^2 f(x_i) = 3,4$

variance $= E(x^2) - \mu^2 = 3,4 - 1 = 2,4 = \sigma^2$

standard deviation $= \sigma = +\sqrt{\sigma^2} = 1,5491$

Scanned by CamScanner

Scanned by CamScanner

Question $\boxed{2}$

a) probability Function :

هي دالة مجالها فضاء الحوادث S ومداها هو R بحيث

$P: \Omega(S) \to R$

دالة الاحتمال لأي حدث دائماً محصوره بين 0 و 1

b) 1) $p(A) = 1 - p(A^c)$

$R.H.S = 1 - p(A^c)$, $A^c = S - A$, $p(S) = 1$

$R.H.S = 1 - p(S - A) = 1 - p(S) + p(A)$

$\therefore R.H.S = 1 - 1 + p(A) = p(A)$ *

2) $p(A \cap B) = p(A) + p(B) - p(A \cup B)$]

$R.H.S = p(A) + p(B) - [p(B) + p(A \setminus B)]$ $\to p(A \setminus B) = p(A) - p(A \cap B)$

$= p(A) - p(A \setminus B)$

$= p(A) - [p(A) - p(A \cap B)]$

$= p(A \cap B) = L.H.S$ *

3) $p(A - B) = p(A) - p(A \cap B)$

$L.H.S = p(A - B) = p(A \cup B) - p(B)$

$= p(A) + p(B) - p(A \cap B) - p(B)$

$= p(A) - p(A \cap B)$

c) $S = \{a, b, c, d, e, F\}$

$p(a) = \frac{1}{16}$, $p(b) = \frac{1}{16}$, $p(c) = \frac{1}{8}$, $p(d) = \frac{3}{16}$

$p(e) = \frac{1}{4}$, $p(F) = \frac{5}{16}$

$A = \{a, c, e\} \to p(A) = p(a) + p(c) + p(e) = \frac{1}{16} + \frac{1}{8} + \frac{1}{4} = \frac{7}{16}$

$B = \{c, d, e, F\} \to p(B) = p(c) + p(d) + p(e) + p(F)$
$= \frac{1}{8} + \frac{3}{16} + \frac{1}{4} + \frac{5}{16} = \frac{7}{8}$

$C = \{b, c, F\} \to p(C) = p(b) + p(c) + p(F) = \frac{1}{2}$

i) $p(A|B) = \frac{p(A \cap B)}{p(B)}$, $p(A \cap B) = p\{c, e\} = p\{c\} + p\{e\}$
$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

$p(A|B) = \frac{3/8}{7/8} = \frac{3}{7}$

ii) $p(A^c|C) = \frac{p(A^c \cap C)}{p(C)}$, $p(A^c \cap C) = p(C \setminus A)$
$= p(C) - p(A \cap C)$

$p(A \cap C) = p\{\{c\}\} = \frac{1}{8}$ $\to p(C) - p(A \cap C) = p\{b, F\} = 3/8$

$\therefore p(A^c|C) = \frac{3/8}{1/2} = \frac{3}{4}$

$P(C|A^c) = \frac{P(A^c \cap C)}{P(A^c)}$, $P(A^c \cap C) = \frac{3}{8}$

$P(A^c) = 1 - P(A) = 1 - \frac{7}{16} = \frac{9}{16}$

$\therefore P(C|A^c) = \frac{3/8}{9/16} = \frac{2}{3}$

Question $\boxed{3}$ X random variable

$$f(x) = \begin{cases} C(1-x^2) & , -1 < x < 1 \\ 0 & , \text{otherwise} \end{cases}$$

X is random continous variable

$\therefore \int_{-1}^{1} f(x)\,dx$ = Area under curve = 1

$\therefore C\int_{-1}^{1} (1-x^2)\,dx = C\left[x - \frac{x^3}{3}\right]_{-1}^{1} = C\left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right]$

$= C \times \frac{4}{3} = 1$

$\therefore C = \frac{3}{4} = 0{,}75$

$f(0) = 0{,}75\,(1-0) = 0{,}75$

$f(-1) = (0{,}75\,(1-1)) = 0$

$f(1) = 0{,}75\,(1-1) = 0$

* value of $P(0 < x < 0{,}75)$

$P(0 < x < 0.75) = \int_{0}^{0{,}75} 0{,}75\,(1-x^2) = 0{,}75\left[x - \frac{x^3}{3}\right]_0^{0{,}75} = 0{,}75\left[{,}75 - \frac{0{,}75^3}{3}\right]$

$= 0{,}45$

(b) X Random variable = minimum of the two numbers

S = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) } , S = 36

X = { 1, 2, 3, 4, 5, 6 }

$F(1) = P(X=1) = P${ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (6,1) (5,1) (4,1) (3,1) (2,1) } $= \frac{11}{36}$

$F(2) = P(X=2) = P${ (2,2) (2,3) (2,4) (2,5) (2,6) (6,2) (5,2) (4,2) (3,2) } $= 9/36$

$F(3) = P(X=3) = P\{(3,3)(3,4)(3,5)(3,6)(4,3)(5,3)(6,3)\} = \frac{7}{36}$

$F(4) = P(X=4) = P\{(4,4),(4,5)(4,6)(5,4)(6,4)\} = \frac{5}{36}$

$F(5) = P(X=5) = P\{(5,5)(5,6)(6,5)\} = \frac{3}{36}$

$F(6) = P(X=6) = P\{(6,6)\} = \frac{1}{36}$

X	1	2	3	4	5	6	Σ
$F(X)$	11/36	9/36	7/36	5/36	3/36	1/36	1

x_i	$F(x_i)$	x_i^2	$x_i F(x_i)$	$x_i^2 F(x_i)$
1	11/36	1	11/36	11/36
2	9/36	4	1/2	1
3	7/36	9	7/12	7/4
4	5/36	16	5/9	20/9
5	3/36	25	5/12	25/12
6	1/36	36	1/6	1
Σ	1		2.5277	8.3611

Expected value : $\mu = \sum_{i=1}^{6} x_i F(x_i) = 2.5277$

variance $= \sum_{i=1}^{6} x_i^2 F(x_i) - \mu^2 = 8.3611 - (2.5277^2)$

$= 1.97184 = \sigma^2$

Standard deviation $= \sigma = +\sqrt{\sigma^2} = 1.4042235$

\#