

نمودج أجابة

ثالثة ل

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ترم اول

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$$(1) (2) \quad P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$\text{Let } D = B \cup C \Rightarrow A \cap D = A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad (7)$$

$$\begin{aligned} \therefore P(A \cap D) &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \end{aligned}$$

$$\begin{aligned} \text{L.H.S.} &= P(A \cup B \cup C) = P(A \cup D) = P(A) + P(D) - P(A \cap D) \\ &= P(A) + P(B \cup C) - P(A \cap D) \end{aligned}$$

$$\begin{aligned} &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \\ &= \text{R.H.S.} (1) \end{aligned}$$

$$(b) (i) \quad A \subset B \Rightarrow A \cap B = A$$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1 \quad (4)$$

$$(ii) \quad A \text{ and } B \text{ are disjoint} \Rightarrow A \cap B = \emptyset$$

(5)

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(\emptyset)}{P(A)} = \frac{0}{P(A)} = 0$$

$$(c) \quad P(A^c \cap B^c) = P(A \cup B)^c$$

$$= 1 - P(A \cup B)$$

(8)

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= [1 - P(B)] - P(B) [1 - P(A)]$$

$$= P(A^c) - P(B) \cdot P(A^c)$$

$$= P(A^c) [1 - P(B)] = P(A^c) \cdot P(B^c)$$

3 τ is a topology of

(a) $X \neq \emptyset \in \tau$,

(ii) τ is closed under arbitrary union,

(iii) τ is closed under finite intersection.

(5)

(b) Fig (1): $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$ is a topology

(5)

Fig (2): $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$ not a topology

(c) $r = \frac{n \sum xy - \sum x \sum y}{\sqrt{(n \sum x^2 - (\sum x)^2)(n \sum y^2 - (\sum y)^2)}}$

(10)

i	X	Y	XY	X ²	Y ²
1	61	165	10065	3721	27225
2	70	170	11900	4900	28900
3	72	170	12240	5184	28900
4	65	165	10985	4225	28900
5	83	170	14110	6889	28900
6	56	154	8624	3136	23716
7	62	164	10168	3844	26895
Σ	469	1158	78892	31899	198762

$= r = \frac{7(78892) - (469)(1158)}{\sqrt{[7(31899) - (469)^2][7(198762) - (1158)^2]}}$

$= \frac{1782}{2191.9726} = 0.81$

و هو الارتباط

2	(a)	x_i	2	3	4	5	6	7	8	9	10	11	12
	(i)	$f(x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) $\mu_x = 7$

$\text{Var}(X) = 5.8$

$\sigma_x = \sqrt{5.8} = 2.4$

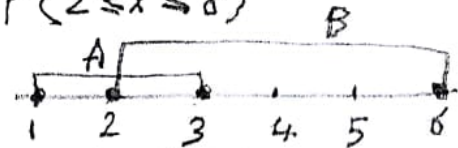
(b) (i) $f(0) = 0, f(5) = 5k$

$\therefore \int_0^5 kx dx = k \left[\frac{x^2}{2} \right]_0^5 = \frac{k}{2} [25 - 0] = \frac{25k}{2} = 1$

$\therefore 25k = 2 \Rightarrow k = \frac{2}{25}$

(ii) $P(1 \leq X \leq 3 | 2 \leq X \leq 6) = \frac{P[(1 \leq X \leq 3) \cap (2 \leq X \leq 6)]}{P(2 \leq X \leq 6)}$

$= \frac{P(2 \leq X \leq 3)}{P(2 \leq X \leq 6)}$



But $P(2 \leq X \leq 3) = \int_2^3 kx dx = \frac{2}{25} \int_2^3 x dx = \frac{1}{25} \left[\frac{x^2}{2} \right]_2^3 = \frac{1}{25} (9 - 4) = \frac{5}{25} = \frac{1}{5}$

$P(2 \leq X \leq 6) = \int_2^5 kx dx + \int_5^6 k dx = k \left[\frac{x^2}{2} \right]_2^5 + k[x]_5^6 = \frac{2}{25} \left[\frac{25}{2} - 2 \right] + \frac{2}{25} [6 - 5]$

$\therefore P(A|B) = \frac{1/5}{21/25} = \frac{1}{5} \times \frac{25}{21} = \frac{5}{21}$

(c) Let A = event that the man is alive in 10 years $\Rightarrow P(A) = \frac{1}{4}$

B = ~ ~ ~ his wife ~ ~ ~ $\Rightarrow P(B) = \frac{1}{3}$

(i) $\because A, B$ are independent $\Rightarrow P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$

(ii) $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) = 1 - \frac{1}{2} = \frac{1}{2}$