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نموذج إجابة

engineering Mathematics (3-B)

رياضيات هندسية (٣-ب)

ترم ثاني

الفرقة الثانية: كهرباء

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Q1 a) Fit the curve $y = \frac{1}{a + b\cos\theta}$ (1)

i	1	2	3
$X_i = \cos\theta$	0.86603	0.7071	0.5
$Y_i = \frac{1}{y_i}$	4.444	3.7037	3.125

$\Longleftarrow$

i	1	2	3
θ_i	30	45	60
y_i	0.225	0.27	0.32

$$y = \frac{1}{a + b\cos\theta} \Rightarrow \frac{1}{y} = a + b\cos\theta \Rightarrow \text{let } \boxed{Y = \frac{1}{y}},\ \boxed{X = \cos\theta}$$

$$\boxed{\therefore Y = a + bX} \Longrightarrow \text{Linear equation}$$

i	X_i	Y_i	X_iY_i	X_i^2
1	0.86603	4.4444	3.8489	0.75
2	0.7071	3.7037	2.61888	0.4999
3	0.5	3.125	1.5625	0.25
$\sum$	2.07313	11.2727	8.0303	1.4999

$$\sum_{i=1}^{n} Y_i = na + b\sum_{i=1}^{n} X_i$$

$$\sum X_i Y_i = a\sum_{i=1}^{n} X_i + b\sum_{i=1}^{n} X_i^2$$

$11.2727 = 3a + 2.7313b \longrightarrow \boxed{1}$

$8.0303 = 2.07313a + 1.4999b \longrightarrow \boxed{2}$

$$\begin{bmatrix} 3 & 2.7313 \\ 2.07313 & 1.4999 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11.2727 \\ 8.0303 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 3 & 2.7313 \\ 2.07313 & 1.4999 \end{bmatrix} = 0.20196$$

$$\Delta a = \begin{vmatrix} 11.2727 & 2.07313 \\ 8.0303 & 1.4999 \end{vmatrix} = 0.2603 \qquad \Delta b = \begin{vmatrix} 3 & 11.272 \\ 2.07313 & 8.0303 \end{vmatrix} = 0.7215$$

$$a = \frac{\Delta a}{\Delta} = 1.28887 \qquad b = \frac{\Delta b}{\Delta} = 3.5725$$

$$\boxed{\therefore y = \frac{1}{1.28887 + 3.5725\cos\theta}}$$

$$RMSE = \sqrt{\frac{\sum (y_i - y(\theta_i))^2}{n \to 3}} = 0.0057427$$

$$\boxed{\therefore y = \frac{1}{1.28887 + 3.5725\cos\theta} \pm 0.0057427}$$

(1)

1b- $F(x) = \frac{\sin x}{1+x^2}$ in interval $0 \le x \le \pi$ ② $\Rightarrow$ Newton's interpola

Solution

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0	0.4373	0.2884	0.1079	0

$h = x_1 - x_0 = \pi/4$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_p \Rightarrow$	x_0 0	0				
			0.4373			
	$\pi/4$	0.4373		-0.5862		
			-0.1489		0.5546	
	$\pi/2$	0.2884		-0.0316		-0.4504
			-0.1805		0.1042	
	$3\pi/4$	0.1079		0.0726		
			-0.1079			
$x_p \Rightarrow$	π x_n	0				

$\Rightarrow$ For $F(0.1)$ $\quad x_p = 0.1 \quad x_p \to x_0$ netwen Forward interpolation

$$P = \frac{x_p - x_0}{h} = \frac{0.1 - 0}{\pi/4} = 0.12732$$

$$F(0.1) = y(x_p) = y_0 + \frac{P}{1!}\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0$$

let $A = 0.12732$

$$F(0.1) = 0 + \frac{A}{1}(0.4373) + \frac{A(A-1)}{2}(-0.5862) + \frac{A(A-1)(A-2)}{3*2}(0.5546) + \frac{A(A-1)(A-2)(A-3)}{4*3*2*1}(-0.4504) = \boxed{0.118693}$$

$\Rightarrow$ For $F(3.1)$ $\quad x_p = 3.1 \quad x_p \to x_n$ netwen Backward

$$P = \frac{x_p - x_n}{h} = \frac{3.1 - \pi}{\pi/4} = -0.05296$$

②

$$F(3.1) = y(x_p) = y_n + \frac{P}{1!}\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + P(P+1)P(+2)(P+3)/4!\ \nabla^4 y_n = \boxed{7.59869 * 10^{-3}}$$

II c) Find the root of equation by secant method $f(x) = x - e^{-x}$ correct to 2 decimal places.

(3.)

Solution

x	0	1
$f(x)$	-1	0.63212

$f(0) = -1 < 0$, $f(1) = 0.63212 > 0$

check $f(0) \cdot f(1) = \square < 0$

$$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

i	x_i	x_{i-1}	$f(x_i)$	$f(x_{i-1})$	x_{i+1}	$f(x_{i+1})$
1	0	1	-1	0.63212	0.6127	0.07081
2	1	0.61270	0.63212	0.07081	0.56384	-0.00518
3	0.6127	0.56384	0.07081	-0.00518	0.56717	0.00004
4	0.56384	0.56717	-0.00518	0.00004	0.56714	✓

$x_{i+1} = 0.56714$

$$|\varepsilon_o| = \left|\frac{\overset{new}{x_{i+1}} - \overset{old}{x_{i+1}}}{\underset{new}{x_{i+1}}}\right| * 100$$

$$= \left|\frac{0.56714 - 0.56717}{0.56714}\right| * 100 = 0.00529\%$$

$\boxed{\therefore \text{The root is } 0.56714 \text{ with error } 0.00529\%}$

Q2 a) Find eigenvalues and egenvectors of matrix A (4)

where $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(-5-\lambda)(4-\lambda)+18\right] + 3\left[12-3\lambda-18\right] + 3\left[-18-30+6\lambda\right] = 0$$

$$\therefore (1-\lambda)\left[-20+\lambda+\lambda^2+18\right] + \left[-9\lambda-18\right] + \left[36+18\lambda\right] = 0$$

$$\left[-20+\lambda+\lambda^2+18+20\lambda-\lambda^2-\lambda^3-18\lambda-9\lambda-18+36+18\lambda\right] = 0$$

$$-\lambda^3+12\lambda+16=0$$

$$\therefore \boxed{\lambda^3-12\lambda-16=0}$$

$\boxed{\lambda_1 = 4}$ $\boxed{\lambda_2 = -2}$ $\boxed{\lambda_3 = -2}$ $\Rightarrow$ check

$\lambda_1+\lambda_2+\lambda_3 =$ مجموع القطر الرئيسي في A

$\lambda_1+\lambda_2+\lambda_3 = 0$

[1] for $\lambda_1 = 4$ not repeated

$$[A-\lambda_1 I]X_1 = \square$$

$$\left[\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

let $X_1 = k$

$-3k - 3X_2 + 3X_3 = 0 \longrightarrow (1)$ (4)

$6k - 6X_2 = 0 \longrightarrow (2)$

From (2) $-6X_2 = -6k \Longrightarrow \boxed{X_2 = k}$

$-3k - 3k + 3X_3 = 0 \qquad 3X_3 = +6k \qquad \boxed{X_3 = 2k}$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} k \\ k \\ 2k \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

[2] for $\lambda_2 = \lambda_3 = -2$ (Repeated eigenvalue)

$$[A - \lambda I] X_{2,3} = 0$$

$$\left[\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \right] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

let $\boxed{X_1 = a}$ & $\boxed{X_2 = b}$

$3a - 3b + 3X_3 = 0 \Longrightarrow 3X_3 = 3b - 3a \quad \therefore \boxed{X_3 = b - a}$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ b-a \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ -a \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

The Eigen values are $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ corresponding

$\lambda_1 = 4$, $\lambda_2 = -2$, $\lambda_3 = -2$ respectively

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2 b- use Runge Kutta to estimate $y(0.4)$, $y' = 2x + y$

$y(0) = 1$

<u>Solution</u>

choosing $h = 0.4$

$y' = f(x,y) = 2x + y \qquad y(x_0) = y_0 \qquad x_0 = 0 \Rightarrow y_0 = 1$

$K_1 = h\, f(x_n, y_n) = h\, f(x_0, y_0) = 0.4[2(0) + 1] = \boxed{0.4}$

$K_2 = h\, f(x_n + h/2,\ y_n + k_1/2) = h\, f(0.2, 1.2)$

$K_2 = 0.4[2(0.2) + 1.2] = \boxed{0.64}$

$K_3 = h\, f(x_n + h/2,\ y_n + k_2/2) = h\, f(0.2, 1.32)$

$K_3 = 0.4[2(0.2) + 1.32] = \boxed{0.688}$

$K_4 = h\, f(x_n + h,\ y_n + k_3) = h\, f(0.4, 1.688)$

$K_4 = 0.4[2(0.4) + 1.688] = \boxed{0.9952}$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y(0.4) = y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\therefore y(0.4) = y_1 = 1 + \frac{1}{6}[0.4 + 2(0.64) + 2(0.688) + 0.995]$

$\boxed{y(0.4) = 1.6752}$

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12 c) by using Trapezoidal and simpson's rules

$$\int_0^{0.6} \frac{1}{\sqrt{4-x^2}}\,dx \quad , n=6 \qquad \text{exact } \int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right)+c$$

solution

x	0 (a)	0.1	0.2	0.3	0.4	0.5	0.6 (b)
y	0.5	0.5006	0.5025	0.5057	0.5103	0.5164	0.5241
	y_0	y_1	y_2	y_3	y_4	y_5	$y_6 = y_n$

$$h = \frac{b-a}{n} = \frac{0.6-0}{6}$$

$$\boxed{h = 0.1}$$

① trapezoidal rule

$$I = \frac{h}{2}\left[y_0 + y_6 + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{0.1}{2}\left[0.5 + 0.5241 + 2(0.5006 + 0.5025 + 0.5057 + 0.5103 + 0.5164)\right] = \boxed{0.304755}$$

exact solution $\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^{0.6} = \sin^{-1}\left(\frac{0.6}{2}\right) - \sin^{-1}\left(\frac{0}{2}\right)$

$$= \boxed{0.304693}$$

$\therefore$ error $= 0.304693 - 0.304755 = \boxed{-6.2 * 10^{-5}}$

② simpson's rule

$$I = \frac{h}{3}\left[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.1}{2}\left[0.5 + 0.5241 + 4(0.5006 + 0.5057 + 0.5164) + 2(0.5025 + 0.5103)\right]$$

$$\boxed{I = 0.3046833}$$

error $= 0.304693 - 0.3046833 = \boxed{9.7 * 10^{-6}}$

3 a- use Gauss elimination method to find solution ⑧

$$5X - 5y + 10Z = -25$$
$$2X + 8Z = 6$$
$$X + y + Z = 9$$

$$\left[\begin{array}{ccc|c} 5 & -5 & 10 & -25 \\ 2 & 0 & 8 & 6 \\ 1 & 1 & 1 & 9 \end{array}\right] \Rightarrow \frac{r_1}{5} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -5 \\ 2 & 0 & 8 & 6 \\ 1 & 1 & 1 & 9 \end{array}\right]$$

$$-2r_1+r_2 \left[\begin{array}{ccc|c} 1 & -1 & 2 & -5 \\ 0 & 2 & 4 & 16 \\ 1 & 1 & 1 & 9 \end{array}\right] \Rightarrow -r_1+r_3 \left[\begin{array}{ccc|c} 1 & -1 & 2 & -5 \\ 0 & 2 & 4 & 16 \\ 0 & 2 & -1 & 14 \end{array}\right]$$

$$-r_2+r_3 \left[\begin{array}{ccc|c} 1 & -1 & 2 & -5 \\ 0 & 2 & 4 & 16 \\ 0 & 0 & -5 & -2 \end{array}\right]$$

$$-5Z = -2 \quad Z = \frac{2}{5} \qquad \boxed{Z = 0.4}$$

$$2y + 4Z = 16 \Rightarrow 2y = 16 - 1.6 = 14.4$$

$$\Rightarrow \boxed{y = 7.2}$$

$$X - y + 2Z = -5$$

$$X = -5 + y - 2Z = -5 + 7.2 - 0.8 = 1.4$$

$$\boxed{X = 1.4}$$

check $1.4 - 7.2 + 0.8 = -5$

⑧

3 b- prove that $\Gamma(n) = (n-1)\,\Gamma(n-1)$

$$\Gamma(n) = \int_0^\infty e^{-t}\, t^{n-1}\, dt \qquad n > 0$$

$$\Gamma(n+1) = \int_0^\infty e^{-t}\, t^{n+1-1}\, dt = \int_0^\infty e^{-t}\, t^{n}\, dt$$

$$= \left[-t^n / e^{t}\right]_0^\infty + n\int_0^\infty e^{-t}\, t^{n-1}\, dt$$

$$\begin{array}{cc} u & dv \\ t^n & e^{-t} \\ n\,t^{n-1} & -e^{-t} \end{array} \qquad -\int$$

$$\Gamma(n+1) = n\,\Gamma(n)$$

$$\Rightarrow \Gamma(n) = (n-1)\,\Gamma(n-1) \qquad \#$$

c- Evaluate $I = \int_0^5 \sqrt{\dfrac{x}{(125 - x^3)}}\, dx = \int_0^5 \dfrac{\sqrt{x}}{\sqrt{125 - x^3}}\, dx$

$$I = \int_0^5 \sqrt{x}\,(125 - x^3)^{-1/2}\, dx$$

$$I = \frac{1}{\sqrt{125}} \int_0^5 \sqrt{x}\,\left(1 - \frac{x^3}{125}\right)^{-1/2} dx$$

let $t = \dfrac{x^3}{125} \Longrightarrow x^3 = 125\,t \Longrightarrow x = 5\sqrt[3]{t} \Rightarrow x = 5\,t^{1/3}$

$$dx = \frac{5}{3}\, t^{-2/3}\, dt \qquad \begin{array}{l} x = 0 \rightarrow t = 0 \\ x = 5 \rightarrow t = 1 \end{array}$$

$$I = \frac{1}{\sqrt{125}} \int_0^1 \sqrt{5}\; t^{1/6}\,(1-t)^{-1/2}\, \frac{5}{3}\, t^{-2/3}\, dt$$

$$= \frac{5\sqrt{5}}{3\sqrt{125}} \int_0^1 t^{-1/2}\,(1-t)^{-1/2}\, dt = \frac{1}{3}\int_0^1 t^{-1/2}\,(1-t)^{-1/2}\, dt$$

$$B(m,n) = \int_0^1 x^{m-1}\,(1-x)^{n-1}\, dx = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

$$m - 1 = -1/2 \Longrightarrow m = 1/2 \qquad n - 1 = -1/2 \Longrightarrow n = 1/2$$

$$I = \frac{1}{3} B(1/2, 1/2) = \frac{1}{3}\,\frac{\Gamma(1/2)\,\Gamma(1/2)}{\Gamma(1)} = \frac{1}{3}\sqrt{\pi}\sqrt{\pi} = \boxed{\frac{\pi}{3}}$$