Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: 1st year Civil

Subject: Engineering Mathematics (2)

Date: 25/5/2016
Time allowed: 3 hours
Full mark: 100
Final Term Exam

All questions are equal marks

Question [1]:(25 marks)

a) Draw the graph of the sequence $\{a_n\} = \{1 + \frac{(-1)^n}{n}\}$

b) Discuss the convergent of the series $\sum_{n=1}^{\infty}(-1)^n.\frac{2n}{4^n - 3}$

c) Evaluate the line integral $\int_{(0,3)}^{(2,4)}[(2y + x^2)dx + (3x - y)dy]$

(i) On the curve x=2t, $y=t^2+3$

(ii) On the straight line from (0,3) to (2,3)

d) Show that the sequence $a_n = \frac{n}{n^2+1}$ is decreasing

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Question [2]: (25 marks)

a) Test the following series by using Cauchy test $\sum_{n=1}^{\infty}\frac{(n)^{n^2}}{(1+n)^{n^2}}$

b) Use Ratio Test to discuss the convergence of the series $\sum_{n=1}^{\infty} n^{\alpha} x^n$

where α is constant.

c) Use Rabbi's Test to discuss the convergence of the series

$$\sum_{n=1}^{\infty}\frac{n!}{(x+1)(x+2).......(x+n)}$$ where $x > 0$

d) Verify Green Theorem in the plane for the line integral:

$\int_C (3x^4 + 2y^2)dx + (4x^2y + y^3)dy$ where C is boundary of the region

between the parabolas: $y = x^2$ and $y^2 = x$

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Question [3]: (25 marks)

a) Sketch the following function, and determine its **Fourier cosine series**.

$$f(x)=\begin{cases} x & , \ 0<x\le 4 \\ 8-x & , \ 4<x<8 \end{cases}$$

b) Find the Fourier series of $f(x)=|x|, \quad -\pi\le x\le\pi$

c) Represent the following as a complex Fourier series:

$$f(x)=x \quad for \quad 0<x<2\pi$$

d) A firm manufactures two types of couplings, **A** and **B**, each of which requires processing time on lathes, grinders and polishers. The machine times needed for each type of coupling are given in the table:

Coupling type	*Time required (hours)*		
	Lathe	*Grinder*	*Polisher*
A	2	8	5
B	5	5	2

The total machine time available is 250 hours on lathes, 310 hours on grinders and 160 hours on polishers. The net profit per coupling of type **A** is **£9** and of type **B** **£10.**

(i) Write down the constraints and the objective function to be *maximized.*

(ii) Use *Autograph* to graph the inequalities, display the feasible region and find the ***number of each type to be produced*** *to maximize* the profit.

Question [4]: (25 marks)

a) Using Method of Separation of Variables (MSV), Solve the following heat equation:

$$u_{xx}=u_t, \quad 0\le x\le 1, \quad t\ge 0,$$
$$where: u(0,t)=u(1,t)=0, \quad \forall t$$
$$u(x,0)=x$$

b) Solve the following P.D.E. $9\frac{\partial^2 u}{\partial x^2}-6\frac{\partial^2 u}{\partial x\,\partial y}+\frac{\partial^2 u}{\partial y^2}=0$, Then find the particular solution which satisfied $u(x,0)=e^x, \quad u(0,y)=9y^3+e^{3y}$

c) Show that the set {1 , {cos(n π x / L) }}, n∈N is orthogonal on the interval [-L,L]

d) By using special functions:

(1) Prove that :

(i) $\Gamma(n+1)=n\Gamma(n)$

(ii) $\beta(m,n)=\beta(m,n+1)+\beta(m+1,n)$

(2) Evaluate $I=\int_a^{\infty} e^{(2ax-x^2)}\,dx$

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