نموذج أجابة

رياضيات هندسية (٢-ب) 2

Engineering Mathematic (2-B)

2015-2016

ترم ثاني

الفرقة : الأولى : كهرباء + مدني + ميكانيكا

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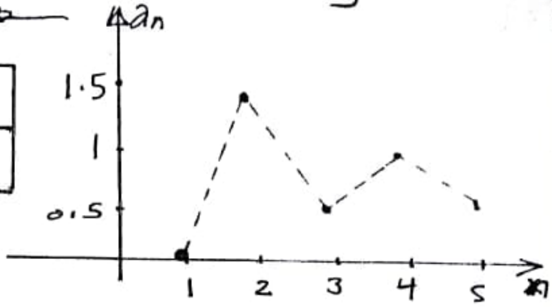
Engineering Mathematics (2)

Question [1]

(a) Draw the graph of sequence $\{a_n\} = \left\{1 + \frac{(-1)^n}{n}\right\}$

— solution —

n	1	2	3	4	5
a_n	0	$\frac{3}{2}$	$2/3$	$5/4$	$4/5$



(b) Discuss the convergent of the series $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n-3}$

— solution —

$$a_n = \frac{2n}{4^n - 3} \quad , \quad a_{n+1} = \frac{2(n+1)}{4(n+1)-3} = \frac{2n+2}{4n+1}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n}{4n-3} = \lim_{n\to\infty} \frac{2}{4 - \frac{3}{n}} = \frac{2}{4} = \frac{1}{2} \neq 0$$

∴ The series is divergence

(c) Evaluate the line integral $\int_{(0,3)}^{(2,4)} \left[(2y+x^2)\,dx + (3x-y)\,dy\right]$

i) on the curve $x = 2t$, $y = t^2 + 3$

ii) on the straight line from $(0,3)$ to $(2,3)$

— solution —

i) $x = 2t$ $\qquad$ $y = t^2 + 3$

$dx = 2\,dt$ $\qquad$ $dy = 2t\,dt$

$$I = \int_0^1 (2t^2 + 6 + 4t^2)(2\,dt) + \left[6t - t^2 - 3\right](2t\,dt)$$

$$= \int_0^1 (12t^2 + 12)\,dt + (12t^2 - 2t^3 - 6t)\,dt$$

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$$I = \int_0^1 (-2t^3 + 24t^2 - 6t + 12)\,dt$$

$$= -\frac{1}{2}t^4 + 8t^3 - 3t^2 + 12t \Big|_0^1 = -\frac{1}{2} + 8 - 3 + 12 = 16.5$$

ii) $y = 3 \rightarrow dy = 0$

$x : 0 \longrightarrow 2$

$$I = \int_0^2 (6 + x^2)\,dx = 6x + \frac{x^3}{3}\Big|_0^2 = 12 + \frac{8}{3} = \frac{44}{3}$$

(d) show that the sequence $a_n = \dfrac{n}{n^2+1}$ is decreasing

$$a_n = \frac{n}{n^2+1} \qquad a_{n+1} = \frac{n+1}{(n+1)^2+1} = \frac{n+1}{n^2+2n+2}$$

$$a_{n+1} - a_n = \frac{n+1}{n^2+2n+2} - \frac{n}{n^2+1}$$

$$= \frac{(n+1)(n^2+1) - n^3 - 2n^2 - 2n}{(n^2+2n+2)(n^2+1)} = \frac{\cancel{n^3} + n + n^2 + 1 - \cancel{n^3} - 2n^2 - 2n}{(n^2+2n+2)(n^2+1)}$$

$$= \frac{-n^2 - n}{(n^2+2n+2)(n^2+1)} = -\left[\frac{n^2+1}{(n^2+2n+2)(n^2+1)}\right] < 0$$

$\therefore$ The sequence is decreasing

Question [2]

a) Test the following series by using caushy test $\sum_{n=1}^{\infty} \dfrac{(n)^{n^2}}{(1+n)^{n^2}}$.

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left[\frac{n}{1+n}\right]^{n^2}} = \left[\frac{n}{1+n}\right]^n$$

$$\lim_{n\to\infty}\left[\frac{n}{1+n}\right]^n = \left(\frac{\infty}{\infty}\right)^\infty$$

$$\lim_{n\to\infty}\left[\frac{1+n-1}{1+n}\right]^n = \lim_{n\to\infty}\left[1 + \frac{-1}{1+n}\right]^n$$

$$\lim_{n\to\infty} \frac{-1}{1+n} = 0 \qquad \lim_{n\to\infty} n = \infty$$

$$\lim_{n\to\infty} \frac{-n}{1+n} = -1 \longrightarrow \therefore \lim_{n\to\infty}\left[\frac{n}{1+n}\right]^n = e^{-1} = \frac{1}{e}$$

$\therefore \lim \sqrt[n]{a_n} < 1$

$\therefore$ The series is convergence

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b) use Ratio Test to discuss the convergence of series $\sum_{n=1}^{\infty} n^{\alpha} x^{n}$ where α is constant

(solution)

$a_n = n^{\alpha} x^{n}$ $\qquad a_{n+1} = (n+1)^{\alpha} x^{n+1}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)^{\alpha} x^{n+1}}{n^{\alpha} x^{n}} = \lim_{n\to\infty} \frac{(n+1)^{\alpha} \cancel{x^{n}} x^{1}}{n^{\alpha} \cancel{x^{n}}}$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{\alpha} x = \lim_{n\to\infty} \left(\frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}}\right)^{\alpha} x = x$$

x:
- $x>1 \rightarrow$ divergence
- $x<1 \rightarrow$ convergence
- $x=1 \rightarrow$ Test is fail

c) use Rabbi's test to discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(x+1)(x+2)\cdots(x+n)}$ where $x>0$

$a_n = \frac{n!}{(x+1)(x+2)\cdots(x+n)}$ $\qquad a_{n+1} = \frac{(n+1)!}{(x+1)(x+2)\cdots(x+n+1)}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)! \leftarrow (n+1)\cancel{n!}}{\cancel{(x+1)(x+2)\cdots}(x+n+1)} * \frac{\cancel{(x+1)(x+2)\cdots(x+n)}}{\cancel{n!}}$$

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{x+n+1}$$

$$\lim_{n\to\infty} n\left(1-\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} n\left[1-\frac{n+1}{x+n+1}\right] = \lim_{n\to\infty} n\left[\frac{x+\cancel{n}+\cancel{1}-\cancel{n}-\cancel{1}}{x+n+1}\right]$$

$$\lim_{n\to\infty} n\left(1-\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} \frac{nx}{x+n+1} = \lim_{n\to\infty} \frac{x}{\frac{x}{n}+1+\frac{1}{n}} = x$$

$\because x>0 \quad \therefore x>1$

$\therefore$ The series is convergence

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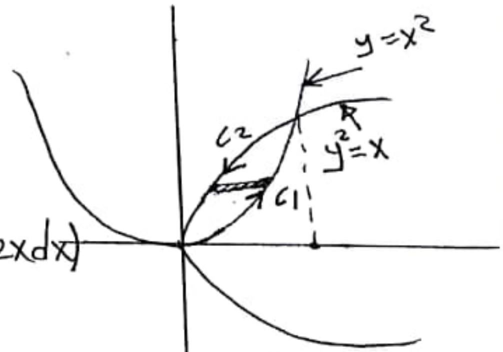
d) Verify Green Theorem in the plane for line integral:

$\int_c (3x^4+2y^2)\,dx + (4x^2y+y^3)\,dy$ where c is boundary of the region between the parabolas: $y=x^2$ and $y^2=x$

solution

$I = I_1 + I_2$



On C1: $y=x^2 \rightarrow dy = 2x\,dx$

$x: 0 \rightarrow 1$

$I_1 = \int_0^1 (3x^4+2x^4)\,dx + (4x^4+x^6)(2x\,dx)$

$= \int_0^1 (5x^2+8x^5+2x^7)\,dx$

$= \frac{5}{3}x^3 + \frac{4}{3}x^6 + \frac{1}{4}x^8 \Big|_0^1$

$= \frac{5}{3}+\frac{4}{3}+\frac{1}{4} = \frac{13}{4}$

$y=x^2$ & $y=x$

$x^2=x$

$x^2-x=0$

$x(x-1)=0$

$x=0$ & $x=1$

On C2: $y^2=x \rightarrow 2y\,dy = dx$

$x: 1 \rightarrow 0$

$I_2 = \int_1^0 (3x^4+2x)\,dx + (4x^{5/2}+x^{3/2})\frac{dx}{2x^{1/2}}$

$= \int_1^0 (3x^4+2x)\,dx + (2x^2+\frac{1}{2}x)\,dx$

$= \int_1^0 3x^4+2x^2+\frac{5}{2}x\,dx = \frac{3}{5}x^5+\frac{2}{3}x^3+\frac{5}{4}x^2 \Big|_1^0$

$= -\frac{3}{5}-\frac{2}{3}-\frac{5}{4} = -\frac{151}{60}$

$I = I_1+I_2 = \frac{11}{15}$

$\oint P\,dx + Q\,dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

$Q = 4x^2y+y^3$ $\qquad$ $P = 3x^4+2y^2$

$\frac{\partial Q}{\partial x} = 8xy$ $\qquad$ $\frac{\partial P}{\partial y} = 4y$

$I = \int_0^1 \int_{y^2}^{\sqrt{y}} (8xy-4y)\,dx\,dy = \int_0^1 4x^2y - 4xy \Big|_{y^2}^{\sqrt{y}} dy$

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$$I = \int_0^1 4y^2 - 4y^{3/2} - 4y^5 + 4y^3 \, dy$$

$$= \frac{4}{3}y^3 - \frac{8}{5}y^{5/2} - \frac{2}{3}y^6 + y^4 \Big|_0^1$$

$$= \frac{4}{3} - \frac{8}{5} - \frac{2}{3} + 1 = \frac{1}{15}$$

Green theorem not verified!

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Q3 a) sketch the Following function and determine its Fourier Cosine series

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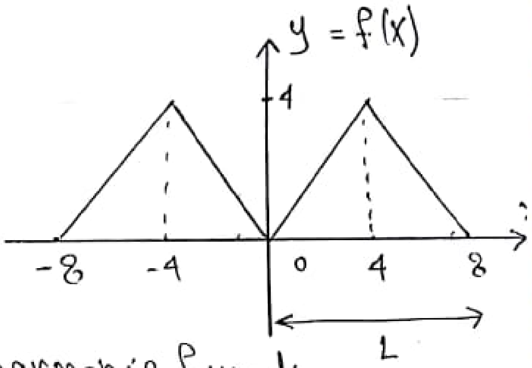
$$f(x) = \begin{cases} x & 0 < x \le 4 \\ 8 - x & 4 < x < 8 \end{cases}$$



solution

① f(x) is periodic function $P = 16,\ L = 8$

② f(x) is piecewise continuous function

③ f(x) even function and even harmonic function

$$b_n = a_{2n+1} = 0$$

$$a_0 = \frac{4}{L}\int_0^{L/2} f(x)\,dx = \frac{4}{8}\int_0^4 x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^4 = \frac{1}{4}[16] = \boxed{4}$$

$$a_{2n} = \frac{4}{L}\int_0^{L/2} f(x)\cos\left(\frac{2n\pi x}{L}\right)dx$$

$$= \frac{4}{8}\int_0^4 x\cos\left(\frac{n\pi x}{4}\right)dx$$

u	dv
x	$\cos\left(\frac{n\pi x}{4}\right)$
(+) 1	$\frac{4}{n\pi}\sin\left(\frac{n\pi x}{4}\right)$
(−) 0	$\frac{-16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)$

$$= \frac{1}{2}\left[\frac{4x}{n\pi}\sin\left(\frac{n\pi x}{4}\right) + \frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)\right]_0^4$$

$$= \frac{1}{2}\left[\frac{16}{n\pi}\underbrace{\sin(n\pi)}_{0} + \frac{16}{n^2\pi^2}\underbrace{\cos(n\pi)}_{(-1)^n} - 0 - \frac{16}{n^2\pi^2}\underbrace{\cos(0)}_{1}\right]$$

$$a_{2n} = \frac{1}{2}\left[\frac{16}{n^2\pi^2}(-1)^n - \frac{16}{n^2\pi^2}\right] = \frac{8}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$f(x) = \frac{4}{2} + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\left[(-1)^n - 1\right]\cos\left(\frac{2n\pi x}{8}\right)$$

OR
$$a_{2n} = \begin{cases} \frac{-16}{n^2\pi^2} & n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}$$

$$\therefore f(x) = 2 - \frac{16}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{4}\right)$$

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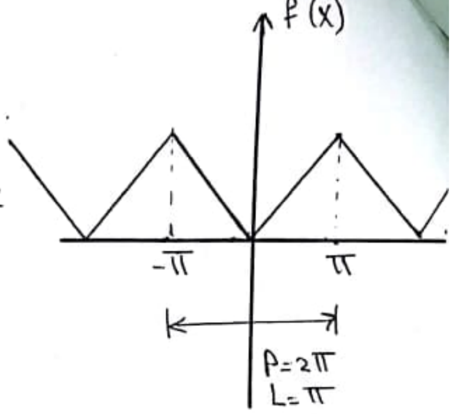
3 b- Find Fourier series of $f(x) = |x| \quad -\pi \leq x \leq \pi$

① $f(x)$ is periodic function $p = 2\pi \longrightarrow L = \pi$

② $f(x)$ is continuous function

③ $f(x)$ is even function and not harmonic



$b_n = 0$

$$a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$a_0 = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left[\pi^2\right] = \boxed{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x \cos\left(\frac{n\pi x}{L}\right)dx$$

u	dv
x	$\cos nx$
1 (+)	$\frac{\sin nx}{n}$
0 (−)	$-\frac{\cos nx}{n^2}$

$$= \frac{2}{\pi}\left[\frac{x}{n}\sin nx + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi}{n}\sin n\pi + \frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \boxed{\frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right]}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$\boxed{f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cdot \cos(nx)}$$

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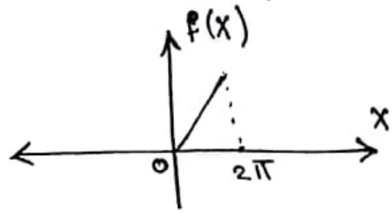
$\boxed{3}$

(c) Represent the following as a complex fourier series;

$f(x) = x$ for $0 < x < 2\pi$

solution

$$C_m = \frac{1}{2L}\int_0^{2L} f(x)\, e^{\frac{-im\pi x}{L}}\, dx = \frac{1}{2\pi}\int_0^{2\pi} x\, e^{-imx}\, dx$$



$$= \frac{1}{2\pi}\left[\frac{-x}{im} e^{-imx} + \frac{1}{m^2} e^{-imx}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi}{im} e^{-i2m\pi} + \cancel{\frac{1}{m^2} e^{-i2m\pi}} - \cancel{\frac{1}{m^2}}\right]$$

$\because e^{i\theta} = \cos\theta + i\sin\theta \equiv e^{-i\theta} = \cos\theta - i\sin\theta$

$e^{-i2m\pi} = \cos(2m\pi) - i\sin(2m\pi) = 1$

$$\begin{array}{cc} u & dv \\ x & e^{-imx} \\ 1 & \frac{e^{-imx}}{-im} \\ 0 & \frac{e^{-imx}}{-m^2} \end{array}$$

$$C_m = \frac{1}{2\pi}\left[\frac{-2\pi}{im}\right] = \frac{-1}{im} \times \frac{i}{i} = \boxed{\frac{i}{m}} \quad m \neq 0$$

at $m = 0$ $\quad C_0 = ???$

$$C_0 = \frac{1}{2\pi}\int_0^{2\pi} x\, dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{4\pi}\left[4\pi^2 - 0\right] = \boxed{\pi}$$

$$f(x) = C_0 + \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} C_m\, e^{\frac{im\pi x}{L}}\, dx$$

$$f(x) = \pi + \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{i}{m}\, e^{imx}$$

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3-
d

coupling type	lathe	Grinder	Polisher
A	2	8	5
B	5	5	2
available	250	310	16

let X = Number of produced of type A
y = " " " " type B

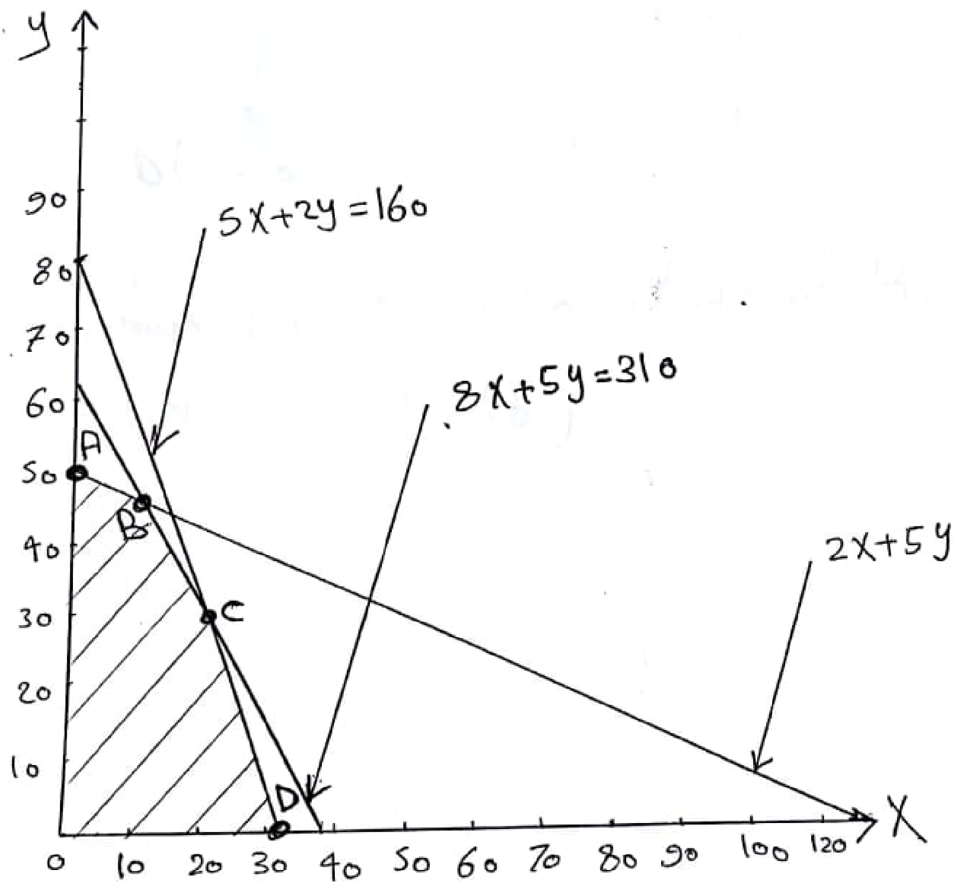
[1] Constraints

$2X + 5y \leqslant 250$
$8X + 5y \leqslant 310$
$5X + 2y \leqslant 160$

[2] objective function

$f = 9X + 6y$

[3] Feasible region



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$A = (0,5)$

$B \begin{cases} 2x+5y=250 \\ 8x+5y=310 \end{cases} \Longrightarrow B=(10,46)$

$C \Rightarrow \begin{cases} 5x+2y=160 \\ 8x+5y=310 \end{cases} \Longrightarrow C=(19.727, 30.682)$

$D = (32,0)$

$f\Big|_{A(0,5)} = 9(0)+10(5) = 50$

$f\Big|_{B(10,46)} = 9(10)+10(46) = 550 \Uparrow$

$f\Big|_{c(19.727,30.682)} = 9(19.727)+10(30.682)$
$= 484.363$

$f\Big|_{D(32,0)} = 9(32)+10(0) = 288$

$\therefore$ max profit $= 550$ at point $B(10,46)$
$(x=10, \ y=46)$

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$\boxed{Q4}$ a) $u_{xx} = u_t$ $\quad 0 \le x \le 1 \quad t \ge 0$

where $u(0,t) = u(1,t) = 0 \quad \forall t$

$u(x,0) = x$

<u>Solution</u>

let $u(x,t) = X(x) \cdot T(t)$

$u_x = X' T \quad | \quad u_t = X \dot{T}$

$u_{xx} = X'' T \quad | \quad u_{tt} = X \ddot{T}$

$X'' T = X \dot{T} \qquad \div XT$

$\frac{X''}{X} = \frac{\dot{T}}{T} = \lambda = \begin{cases} = 0 \\ > 0 \\ < 0 \end{cases}$

Case $\boxed{3}$ $\lambda < 0$ $\quad \boxed{\lambda = -p^2}$

$\frac{X''}{X} = \frac{\dot{T}}{T} = -p^2$

$X'' = -p^2 X$

$X'' + p^2 X = 0$

$(\alpha^2 + p^2) = 0$

$\alpha^2 = -p^2$

$\alpha_{1,2} = \pm i p$ complex not repeated

$\boxed{X(x) = c_1 \cos pX + c_2 \sin pX}$

$\frac{\dot{T}}{T} = -p^2$

$\dot{T} = -p^2 T$

$(\alpha^2 + p^2) = 0$

$\alpha = -p^2$ real notrepeated

$\boxed{T(t) = c_3 e^{-p^2 t}}$

B.C(1) $u(0,t) = 0$

$X(0) T(t) = 0$

$\therefore X(0) = 0$

$\boxed{c_1 = 0}$

B.C2 $u(1,t) = 0$

$X(1) T(t) = 0$

$X(1) = 0$

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$X(1)=0$

$$C_2 \sin p = 0$$

$C_2 \neq 0 \quad \sin p = 0$ at $p = n\pi$

$X(x) = C_2 \sin n\pi x$

$u(x,t) = C_2 C_3 e^{-p^2 t} \sin n\pi x$

$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2 t} \sin n\pi x.$

$\boxed{I.C}$ $u(x,0) = x = f(x)$

$f(x) = x = \sum_{n=1}^{\infty} c_n \sin n\pi x \equiv$ F. sine. series

$c_n = b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L}$

$= 2 \int_0^1 x \sin n\pi x$

u	dv
x	$\sin n\pi$
1 $(+)$	$\frac{-\cos n\pi}{n\pi}$
0 $(-)$	$\frac{-\sin n\pi x}{n^2\pi^2}$

$c_n = 2\left[\frac{-x}{n\pi} \cos n\pi x + \frac{1}{n^2\pi^2} \sin n\pi x\right]_0^1$

$= 2\left[\frac{-(-1)^n}{n\pi}\right]$

$$\boxed{b_n = c_n = \frac{-2(-1)^n}{n\pi}}$$

$u(x,t) = \sum_{n=1}^{\infty} \frac{-2(-1)^n}{n\pi} e^{-n^2\pi^2 t} \sin n\pi x$

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$\boxed{4}$ b- $9\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x\, \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

$u(x,0) = e^x$, $u(0,y) = 9y^3 + e^{3y}$

solution

$\lambda^2 - 6\lambda + 9 = 0$

$(\lambda-3)(\lambda-3) = 0$

$\lambda_1 = \lambda_2 = 3$

$\therefore u(x,y) = F(x+\lambda y) + y\, G(x+\lambda y) \longrightarrow$ general solution

$\therefore u(x,y) = F(x+3y) + y\, G(x+3y)$

since $u(x,0) = e^x$

$\therefore F(x) = e^x \Longrightarrow F(x+3y) = e^{x+3y}$

$u(x,y) = e^{x+3y} + y\, G(x+3y) \longrightarrow \boxed{1}$

Condition $u(0,y) = 9y^3 + e^{3y}$

$\cancel{e^{3y}} + y\, G(3y) = 9y^3 + \cancel{e^{3y}}$

$y\, G(3y) = 9y^3$

$G(3y) = 9y^2 = (3y)^2$

$G(x+3y) = (x+3y)^2$

particular solution

$$\boxed{u(x,y) = e^{x+3y} + y(x+3y)^2}$$

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[4]-c) show that the set $\{1, \{\cos(n\pi x/L)\}\}, n \in N$ is orthogonal on the interval $[-L, L]$

solution

$$\text{set} = \{1, \{\cos(\frac{\pi x}{L}), \cos(\frac{2\pi x}{L}), \ldots\ldots\}\}$$

$$\int_{-L}^{L} \cos(\frac{m\pi x}{L}) \cos(\frac{n\pi x}{L}) dx \Longrightarrow m \neq n$$

$$= \frac{1}{2} \int_{-L}^{L} \left[\cos \frac{(m+n)\pi x}{L} + \cos \frac{(m-n)\pi x}{L} \right] dx$$

$$= \frac{1}{2} \left[\sin \frac{(m+n)\pi x}{L} \cdot \frac{L}{(m+n)\pi} + \sin \frac{(m-n)\pi x}{L} \frac{L}{(m-n)\pi} \right]_{-L}^{L}$$

$= \text{Zero}$

$$\text{at } m = 0 \longrightarrow \int_{-L}^{L} 1 \cos(\frac{n\pi x}{L}) = \left[\sin(\frac{n\pi x}{L}) \frac{L}{n\pi} \right]_{-L}^{L} = \text{Zero}$$

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4 d-(1) prove that

i) $\Gamma(n+1) = n\,\Gamma(n)$

proof: $\Gamma(n+1) = \int_0^\infty e^{-t}\, t^{n+1-1}\, dt = \int_0^\infty e^{-t}\, t^n\, dt$

u	dv
t^n	e^{-t}
$n\,t^{n-1}$	$-e^{-t}$

$+$, $-\int$

$$\Gamma(n+1) = \left[-t^n e^{-t}\right]_0^\infty + n\int_0^\infty e^{-t}\, t^{n-1}\, dt$$

$$\therefore \Gamma(n+1) = n\,\Gamma(n) \quad \#$$

ii) $B(m,n) = B(m,n+1) + B(m+1,n)$

proof: $B(m,n+1) = \dfrac{\Gamma(m)\,\Gamma(n+1)}{\Gamma(m+n+1)}$ $\qquad B(m+1,n) = \dfrac{\Gamma(m+1)\,\Gamma(n)}{\Gamma(m+n+1)}$

$$R.H.S \Rightarrow \frac{\Gamma(m)\,\Gamma(n+1) + \Gamma(m+1)\Gamma(n)}{\Gamma(m+n+1)} = \frac{\Gamma(m)\,n\Gamma(n) + m\Gamma(m)\,\Gamma(}{(m+n)\ \Gamma(m+n)}$$

$$= \frac{\Gamma(m)\,\Gamma(n)\,(m+n)}{(m+n)\ \Gamma(m+n)} = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} = B(m,n) \quad \#$$

2) Evaluate $I = \int_a^\infty e^{2ax - x^2}\, dx$

$$I = \int_a^\infty e^{-(x^2 - 2ax)}\, dx = \int_a^\infty e^{-[(x-a)^2 - a^2]}\, dx$$

$$I = e^{a^2}\int_a^\infty e^{-(x-a)^2}\, dx$$

let $t = (x-a)^2 \Longrightarrow \sqrt{t} = (x-a) \Rightarrow \dfrac{1}{2\sqrt{t}}\,dt = dx$

$\dfrac{1}{2}\, t^{-1/2}\, dt = dx$ $\quad x = a \to t = 0 \qquad x = \infty \longrightarrow t = \infty$

$$I = e^{a^2}\int_0^\infty e^{-t}\, \frac{1}{2}\, t^{-1/2}\, dt = \frac{e^{a^2}}{2}\,\Gamma(1/2)$$

$$I = \frac{e^{a^2}}{2}\sqrt{\pi}$$

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$\boxed{4}$

d) 1) prove that:

i) $F[f(t-a)] = e^{-aiw}\,\hat{f}(w)$

$$F[f(t-a)] = \int_{-\infty}^{\infty} f(t-a)\, e^{-iwt}\, dt$$

let $t-a=u \Rightarrow t=u+a \Rightarrow dt=du$

$$F[f(t-a)] = \int_{-\infty}^{\infty} f(u)\, e^{-iw(u+a)}\, du = \int_{-\infty}^{\infty} f(u)\, e^{-iwu}\, e^{-aiw}\, du$$

$$F[f(t-a)] = e^{-aiw}\,\hat{f}(w) \quad \#$$

ii) $F[f'(t)] = iw\,\hat{f}(w)$

$$F[f'(t)] = \int_{-\infty}^{\infty} f'(t)\, e^{-iwt}\, dt.$$

u	dv
e^{-iwt}	$f'(t)$
$-iw\, e^{-iwt}$	$f(t)$

$(+)$ $(-)\int$

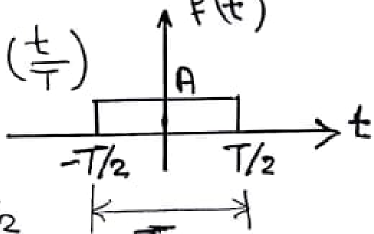
$$= \left[f(t)\, e^{-iwt}\right]_{-\infty}^{\infty} + iw \int_{-\infty}^{\infty} f(t)\, e^{-iwt}\, dt$$

$$= \left[f(\infty)\, e^{-\infty} - f(-\infty)\, e^{\infty}\right] + iw\,\hat{f}(w)$$

at $t \to \pm\infty$ $f(t)=0$

$$\therefore F[f'(t)] = iw\,\hat{f}(w) \quad \#$$

$\boxed{2}$ $f(t) = \begin{cases} A & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \Rightarrow f(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$



solution

$$F[f(t)] = \hat{f}(w) = \int_{-\infty}^{\infty} f(t)\, e^{-iwt}\, dt = \int_{-\infty}^{-T/2} 0 + \int_{-T/2}^{T/2} A\, e^{-iwt}\, dt + \int_{T/2}^{\infty} 0$$

$$= A\left[\frac{e^{-iwt}}{-iw}\right]_{-T/2}^{T/2} = \frac{A}{-iw}\left[e^{-iwT/2} - e^{iwT/2}\right]$$

$$= \frac{2A}{w}\left[\frac{e^{iwT/2} - e^{-iwT/2}}{2i}\right] \quad (10)$$

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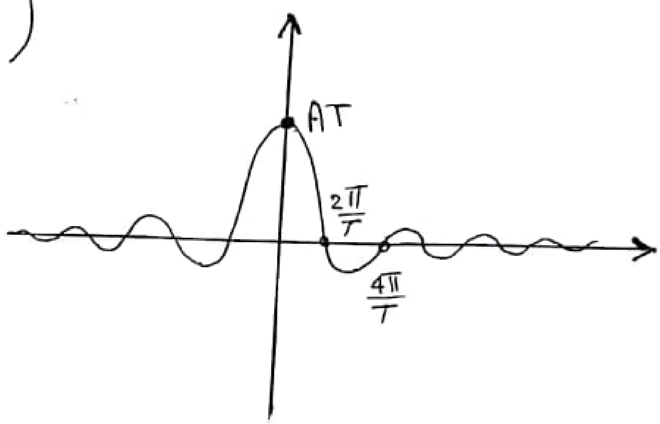
$$= \frac{2A}{w}\left[\sin(wT/2)\right] * T/T$$

$$= \frac{2AT}{w}\left[\frac{\sin(wT/2)}{T}\right] = AT\left[\frac{\sin(wT/2)}{wT/2}\right]$$

$\because \frac{\sin\theta}{\theta} = \text{sinc}\,\theta$

$$\hat{F}(w) = AT\ \text{sinc}\left(\frac{wT}{2}\right)$$



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