نموذج أجابة

رياضيات هندسية (١ - ب)

Engineering Mathematic (1-B)

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(تكامل + تحليلية)

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3 a) 1- Find the following integrals.

(1) $\int \frac{x\cos\sqrt{3x^2-6}}{\sqrt{3x^2-6}}\,dx = \int \frac{2}{6}\frac{6x}{\sqrt{3x^2-6}}\cos\sqrt{3x^2-6}\,dx$

$= \frac{1}{3}\int \frac{6x}{2\sqrt{3x^2-6}}\cos\sqrt{3x^2-6} = \frac{1}{3}\sin\sqrt{3x^2-6} + C$

(2) $\int e^x\left[\frac{1}{x}+\ln x\right]dx = \int\left[\frac{e^x}{x}\cancel{dx} + e^x\ln x\right]dx$

$= \cancel{\int \frac{e^x}{x}dx} + e^x\ln x - \cancel{\int\frac{e^x}{x}dx} = e^x\ln x + C$

u dv

$\ln x$ e^x

$\frac{1}{x}$ e^x

$+$ $-\int$

$\boxed{\text{by using}}$

$f(x)=e^x,\ g(x)=\ln x$

$\int[f(x)g'(x)+g(x)f'(x)]dx = f(x)g(x)+C$

$\int[e^x\frac{1}{x}+e^x\ln x]dx = e^x\ln x + C$

(3) $\int \frac{2}{\sqrt{1-2x-x^2}}dx = \int \frac{2}{\sqrt{-[(x+1)^2-2}}dx$

$= \int \frac{2}{\sqrt{(\sqrt{2})^2-(x+1)^2}}dx = 2\sin^{-1}\frac{(x+1)}{\sqrt{3}} + C$

(4) $\int \sin^3 x\cos^{1/2}x\,dx = \int \sin x\,\sin^2 x\,\cos^{1/2}x\,dx$

$= \int \sin x\,\cos^{1/2}x\,(1-\cos^2 x)\,dx$

$= \int \sin x\,\cos^{1/3}x\,dx - \int \sin x\,\cos^{5/2}x\,dx$

$= \frac{-2}{3}\cos^{3/2}x + \frac{2}{7}\cos^{7/2}x + C$

(5) $\int \frac{dx}{3\sin x - 4\cos x} \Longrightarrow$ let $t = \tan\frac{x}{2}$

$\therefore \frac{x}{2} = \tan^{-1}t \longrightarrow dx = \frac{2\,dt}{1+t^2}$

$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$$\therefore I = \int \frac{2\,dt/1+t^2}{3\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{2}{4t^2+6t-4}\,dt$$

$$= \int \frac{2\,dt}{(4t-2)(t+2)} = \int \left(\frac{A}{4t-2} + \frac{B}{t+2}\right) dt$$

$$= \int \left(\frac{4/5}{4t-2} - \frac{1/5}{t+2}\right) dt = \frac{1}{5}\left[4\ln|4t-2| - \ln|t+2|\right] + c$$

$$= \frac{1}{5}\left[4\ln\left|4\tan\left(\frac{x}{2}\right) - 2\right| - \ln\left|\tan\left(\frac{x}{2}\right) + 2\right|\right] + c$$

$\boxed{6}$ $\displaystyle\int \frac{x^2}{(5-2x)^{5/2}}\,dx$

let $u = 5-2x \qquad dx = \frac{-du}{2}$

$$\therefore \int \frac{\left(\frac{5-2x}{2}\right)^2}{u^{5/2}}(-1/2)\,du = -1/8 \int \frac{25-10u+u^2}{u^{5/2}}\,du$$

$$= \frac{-1}{8}\int \left[25u^{-5/2} - 10u^{-3/2} + u^{-1/2}\right] = \frac{-1}{8}\left[\frac{-50}{3}u^{-3/2} + 20u^{-1/2} + 2u^{1/2}\right] + c$$

(7) $\displaystyle\int \frac{dx}{x^3+x^2} = \int \frac{dx}{x^2(x+1)} = \int \left[\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}\right] dx$

$$= \int \left[\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right] dx = -\ln|x| - \frac{1}{x} + \ln|x+1| + c$$

$\boxed{b}$ Find the reduction formula

$I_n = \int x^{2n+1} e^{x^2}\,dx$ then find $\int x^5 e^{x^2}\,dx$

$$I_n = \int x^{2n}\, x\, e^{x^2}\,dx = \frac{1}{2}x^{2n}e^{x^2} - n\int x^{2n-1} e^{x^2}\,dx$$

u	dv
x^{2n}	xe^{x^2}
$2nx^{2n-1}$	$\frac{1}{2}e^{x^2}$

$+$ $-\int$

$$\boxed{I_n = \frac{1}{2}x^{2n}e^{x^2} - n\,I_{n-2}}$$

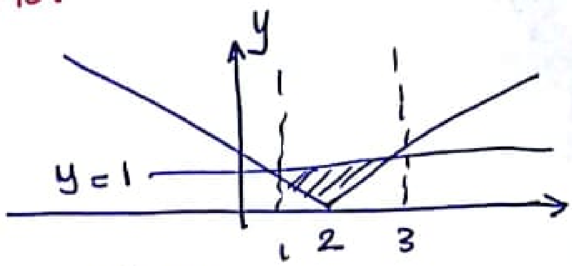
$n=2 \longrightarrow I_2 = \frac{1}{2}x^4 e^{x^2} - 2I_0$

$I_0 = \int x\, e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C$

$I_2 = \frac{1}{2}x^4 e^{x^2} - e^{x^2} + C$

$\boxed{2}$ a) $y = |x-2|$ $\quad y=1$ Find area betwen

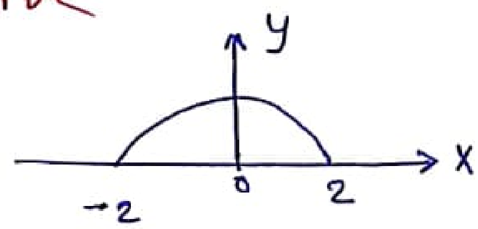
$$y = |x-2| = \begin{cases} x-2 & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$$



$$A = \int_1^2 [1+(x-2)]\,dx + \int_2^3 [1-(x-2)]\,dx = 1$$

b) Find the length of the following curve

$y = \sqrt{4-x^2}$ $\quad x \in [-2,2]$



$$y' = \frac{-2}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$$

$$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx = 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}} = 2\int_0^2 \sqrt{\frac{4}{4-x^2}}\,dx$$

$$= 2\int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx = 4\left[\sin^{-1}\frac{x}{2}\right]_0^2 = 2\pi$$

c) show that volume of sphere is $\frac{4}{3}\pi a^3$

$$V = \pi\int_{-a}^{a} [f(x)]^2\,dx = \pi\int_{-a}^{a} y^2\,dx$$

$$= \pi\int_{-a}^{a} (a^2-x^2)\,dx = 2\pi\int_0^a (a^2-x^2)\,dx$$

$$= 2\pi\left[a^2x - \frac{x^3}{3}\right]_0^a = 2\pi\left[\frac{2a^3}{3}\right] = \frac{4}{3}\pi a^3$$

d) Find $\int_4^{5.2} \ln x\,dx$ by using simpson's method $[4, 5.2]$ divided to 6 subinterval. $\Rightarrow h = 0.2$

x	4	4.2	4.4	4.6	4.8	5	5.2
y	1.38625	1.43509				1.60944	1.64886

$$I = \frac{h}{3}[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4)]$$

$= 1.82785$

21/5/20

Engineering Mathematics

Question [1]

a) change to another form $(x^2+y^2)^2=16(x^2-y^2)$

$\because (x^2+y^2)^2=16(x^2-y^2)$

$(r^2)^2=16\left[(r\cos\theta)^2-(r\sin\theta)^2\right]$

$r^4=16r^2\left[\cos^2\theta-\sin^2\theta\right]$

$r^2=16\cos 2\theta$

b) prove that $r=a\sin\theta+b\cos\theta$, represent a circle equation and find its center and its radius

$\because r=a\sin\theta+b\cos\theta$ $(*r)$

$r^2=ar\sin\theta+br\cos\theta$

$x^2+y^2=ay+bx$

$x^2+y^2-bx-ay=0 \equiv x^2+y^2+2gx+2fy+c=0$

$\therefore$ The eqn. represents a circle equation.

center $=(-g,-f)=\left(\frac{b}{2},\frac{a}{2}\right)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{\frac{b^2}{4}+\frac{a^2}{4}}=\frac{\sqrt{b^2+a^2}}{2}$

c) If the origin is translated to the point $(-2,3)$. Find the new equation $2x^2+4xy+5y^2-4x-22y+7=0$

$x=x'+\alpha$ $\quad y=y'+\beta$

$x=x'-2$ $\quad y=y'+3$

$\because 2x^2+4xy+5y^2-4x-22y+7=0$

$2(x'-2)^2+4(x'-2)(y'+3)+5(y'+3)^2-4(x'-2)-22(y'+3)+7=0$

$\underline{2x'^2}-\cancel{8x'}+8+4x'y'+\cancel{12x'}-\cancel{8y'}-24+\underline{5y'^2}+\cancel{30y'}+45-\cancel{4x'}+8$

(2)

$-\cancel{3x}y' - 66 + 7 = 0$

$\therefore 2x'^2 + 5y'^2 + 4x'y' - 22 = 0$

Question [2]

a) Describe the curve: $2x^2 - y^2 - 2x - 4y = 0$

$$2x^2 - 2x - y^2 - 4y = 0$$

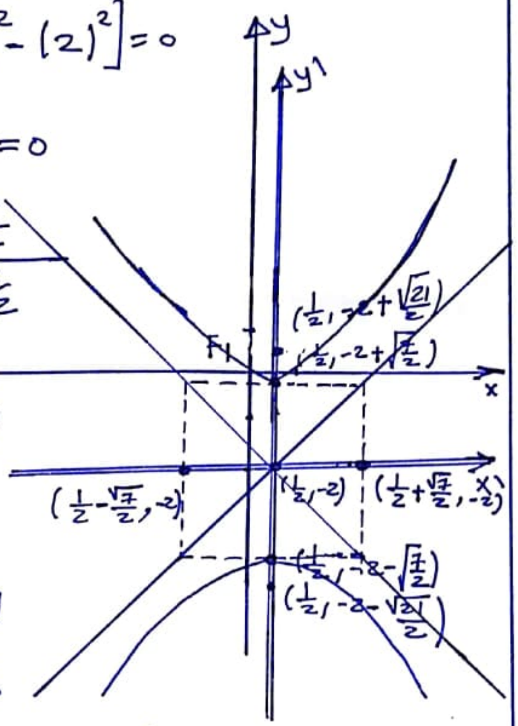
$$2[x^2 - x] - [y^2 + 4y] = 0$$

$$2\left[(x-\tfrac{1}{2})^2 - (\tfrac{1}{2})^2\right] - \left[(y+2)^2 - (2)^2\right] = 0$$

$$2(x-\tfrac{1}{2})^2 - \tfrac{1}{2} - (y+2)^2 + 4 = 0$$

$$\frac{2(x-\frac{1}{2})^2}{-7/2} - \frac{(y+2)^2}{-7/2} = \frac{-\frac{7}{2}}{-\frac{7}{2}}$$

$$\frac{(y+2)^2}{7/2} - \frac{(x-\frac{1}{2})^2}{7/4} = 1$$



$\alpha = \frac{1}{2}$ $\quad \beta = -2$

$a^2 = \frac{7}{4} \longrightarrow a = \frac{\sqrt{7}}{2} = 1.3229$

$b^2 = \frac{7}{2} \longrightarrow b = \sqrt{\frac{7}{2}} = 1.8708$

$c = \sqrt{a^2+b^2} = \sqrt{\frac{7}{4}+\frac{7}{2}} = \frac{\sqrt{21}}{2} = 2.29129$

center $= (\frac{1}{2}, -2)$

Vertices $= (\frac{1}{2}, -2+\sqrt{\frac{7}{2}}), (\frac{1}{2}, -2-\sqrt{\frac{7}{2}})$

Foci $\rightarrow F_1 = (\frac{1}{2}, -2+\frac{\sqrt{21}}{2})$, $F_2 = (\frac{1}{2}, -2-\frac{\sqrt{21}}{2})$

Assympotic lines $\rightarrow (y+2) = \pm\frac{b}{a}(x-\frac{1}{2})$

$$(y+2) = \pm\sqrt{2}\,(x-\tfrac{1}{2})$$

b) Find equation of an ellipse whose foci are $(5,5)$, $(5,-1)$ and Length of major axis is 10 units

$\because 2c = 6 \longrightarrow c = 3$

$2a = 10 \longrightarrow a = 5$

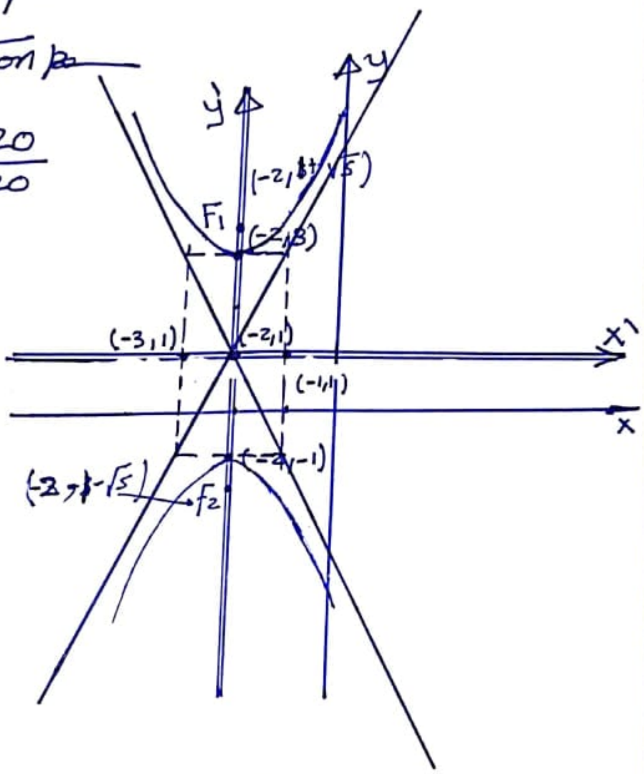
$c^2 = a^2 - b^2 \longrightarrow b^2 = a^2 - c^2$

$b^2 = 25 - 9 = 16$

$\therefore \frac{(x-5)^2}{16} + \frac{(y-2)^2}{25} = 1$

c) put the equation $20(x+2)^2 - 5(y-1)^2 = -20$ in the standard form for a hyperbola and discuss it

solution



$$\frac{20(x+2)^2}{-20} - \frac{5(y-1)^2}{-20} = \frac{-20}{-20}$$

$$\frac{(y-1)^2}{4} - \frac{(x+2)^2}{1} = 1$$

$\alpha = -2 \quad \beta = 1$

$a^2 = 1 \longrightarrow a = 1$

$b^2 = 4 \longrightarrow b = 2$

$c = \sqrt{a^2 + b^2} = \sqrt{5}$

center $= (-2, 1)$

vertices $= (-2, 3), (-2, -1)$

Foci are $F_1 = (-2, 1+\sqrt{5})$, $F_2 = (-2, 1-\sqrt{5})$