نموذج أجابة

رياضيات هندسية (١-P)

Engineering Mathematic (1-A)

اعدادى ترم أول

2016- 2017

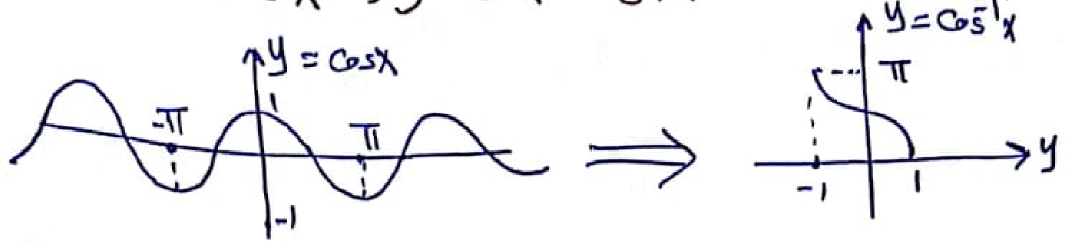
(تفاضل + جبر)

تاريخ الامتحان 15/1/2017

أ.د/ عرفة عبدالظاهر

د/ منال السيد

1] a: $y=\cos^{-1}x$, $y=\cos x$ Draw and Find D_f , R_f



$D_f=[-1,1]$, $R_f=[0,\pi]$

b) Find limits

i) $\lim_{x\to\infty}\left(\frac{x+5}{x+1}\right)^{x+3} \Longrightarrow$ using $\lim_{x\to x_0}[1+f(x)]^{g(x)}=e^m$

$\lim_{x\to\infty}\left(\frac{x+1+4}{x+1}\right)^{x+3}=\lim_{x\to\infty}\left(1+\frac{4}{x+1}\right)^{x+3}$

1) $\lim_{x\to\infty}\frac{4}{x+1}=0$ ✓

2) $\lim_{x\to\infty}(x+3)=\infty$ ✓

3) $\lim_{x\to\infty}\frac{4(x+3)}{x+1}=\lim_{x\to\infty}\frac{4x+12}{x+1}=\frac{\infty}{\infty}$ باستخدام لوبيتال

$\lim_{x\to\infty}\frac{4}{1}=4$ ~~exist~~ $=m$

$\therefore \lim_{x\to\infty}\left(\frac{x+5}{x+1}\right)^{x+3}=e^m=e^4$

ii) $\lim_{x\to 0}(x\cdot\ln x^3)=0\ln 0=0*-\infty$

$\lim_{x\to 0}\frac{\ln x^3}{1/x}=\frac{-\infty}{\infty}$

$\lim_{x\to 0}\frac{3x^2/x^3}{-x^{-2}}=\lim_{x\to 0}-3x^3=0$ exist

iii) $\lim_{x\to 0}\frac{\sin x}{\ln(1+8x)}=\frac{0}{0}$

$\lim_{x\to 0}\frac{\cos x}{\frac{8}{1+8x}}=\frac{1}{8}$ exist

c) Determine point of discontinuity of $f(x)$ then obtain if you can redefine to be continuous at $f(x) = \frac{x - |x|}{x}$

$$f(x) = \begin{cases} \frac{x - x}{x} = 0 & , x \geq 0 \\ \frac{x + x}{x} = 2 & x < 0 \end{cases}$$

$\therefore \lim_{x \to 0} f(x) = ??$ $\quad \lim_{x \to 0^+} 0 = 0$, $\lim_{x \to 0^-} 2 = 2$

$\therefore \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

$\therefore \lim_{x \to 0} f(x)$ is not exist $\therefore$

لا يمكن إعادة تعريف الدالة لتصبح متصلة عند $x=0$ لأن النهاية غير موجودة

d) deduce $\frac{dy}{dx}$ if $y = \sec^{-1} x$

$y = \sec^{-1} x \Longrightarrow \sec y = x$

$y' (\sec y \cdot \tan y) = 1 \qquad y' = \frac{1}{\sec y \ \tan y}$

$\cos^2 y + \sin^2 y = 1 \quad \div \cos^2 y \Longrightarrow 1 + \tan^2 y = \frac{1}{\cos^2 y}$

$\tan y = \pm \sqrt{\sec^2 y - 1} = \sqrt{x^2 - 1}$

$\boxed{2}$ a) Find $y'(x)$

i) $y = 5^{\ln(\operatorname{cosec} x + \tan x)} + e^{\coth^{-1} x} + \sin^{-1}(\tan x^2)$

$$y' = 5^{\ln(\operatorname{cosec} x + \tan x)} (\ln 5) \cdot \frac{-\operatorname{cosec} x \cot x + \sec^2 x}{\operatorname{cosec} x + \tan x} +$$

$$e^{\coth^{-1} x} \frac{1}{1 - x^2} + \frac{2x \cdot \sec^2 x^2}{\sqrt{1 - [\tan x^2]^2}}$$

ii) $y = \cos^2(8^x) + (\tanh x)^{\sin x}$

$y' = 2\cos(8^x)\left(-\sin(8^x)\right)\left(8^x \ln 8\right) +$

$(\tanh x)^{\sin x}\left[\frac{\sin x}{\tan x}\,\text{sech}^2 x + \cos x \cdot \ln(\tanh x)\right]$

iii) $y = \cos^3 t \quad , x = \sqrt{t}$

$$\frac{dy}{dx} = \frac{dy}{dt}\,\frac{1}{dx/dt} = \frac{-3\cos^2 t \,\sin t}{1/2\sqrt{t}} = -6\sqrt{t}\,\sin t\,\cos^2 t$$

b) Deduce the n^{th} derivatives $y = e^{ax}\sin bx$, then find $y = e^{\sqrt{2}x}\sin\sqrt{2}x$

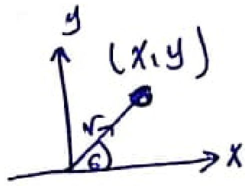
polar form

let $a = r\cos\theta$

$b = r\sin\theta$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\frac{b}{a}$



$y^{(1)} = b e^{ax}\cos bx + a e^{ax}\sin bx$

$y^{(1)} = e^{ax}\left[b\cos bx + a\sin bx\right]$

$y^{(1)} = r e^{ax}\left[\frac{b}{r} \sim + \frac{a}{r} \sim\right]$

$y^{(1)} = r e^{ax}\left[\sin\theta\cos bx + \cos\theta\sin bx\right]$

$y^{(1)} = r e^{ax}\left[\sin(bx+\theta)\right]$

$y^{(2)} = r\left[e^{ax}\, b\cos(bx+\theta) + a e^{ax}\sin(bx+\theta)\right]$

$y^{(2)} = r^2 e^{ax}\left[\frac{b}{r}\cos(bx+\theta) + \frac{a}{r}\sin(bx+\theta)\right]$

$y^{(2)} = r^2 e^{ax}\left[\sin(bx+2\theta)\right]$

$\vdots$

$y^{(n)} = r^n e^{ax}\left[\sin(bx+n\theta)\right]$

$\Rightarrow y = e^{\sqrt{2}x}\sin\sqrt{2}x \Longrightarrow a = \sqrt{2},\ b = \sqrt{2}$

$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2 \quad , \quad \theta = \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}} = 45^\circ = \pi/4$

$y^{(n)} = (2)^n\, e^{\sqrt{2}x}\sin\left(\sqrt{2}x + \frac{n\pi}{4}\right)$

c) Deduce Maclaurin expansion

$f(x) = \sinh x$

$\because f(x) = \frac{e^x - e^{-x}}{2}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} - \cdots$

$\sinh x = \frac{1}{2}\left[2x + \frac{2x^3}{3!} + \frac{2x^3}{5!} + \cdots\right] = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

d) If $y = \ln\sqrt{\sin x}$ show that

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\operatorname{cosec}(2x) = 0$

$\cancel{y = \frac{1}{2}}$ $y = \ln\sqrt{\sin x} = \ln(\sin x)^{1/2} = \frac{1}{2}\ln(\sin x)$

$y' = \frac{1}{2}\frac{\cos x}{\sin x} = \frac{1}{2}\cot x$

$y'' = \frac{-1}{2}\operatorname{cosec}^2 x$

L.H.S =

$= y'' + 2y\operatorname{cosec}(2x) = \frac{-1}{2}\operatorname{cosec}^2 x + 2\,\frac{1}{2}\cot x\operatorname{cosec} 2x$

$= \frac{-1}{2}\frac{1}{\sin^2 x} + \frac{\cos x}{\sin x}\frac{1}{\sin 2x}$

$= \frac{-1}{2}\frac{1}{\sin^2 x} + \frac{\cancel{\cos x}}{\sin x}\frac{1}{2\sin x\cancel{\cos x}}$

$* -\sin^2 x$ بالضرب

$= \frac{-1}{2}\frac{-\sin^2 x}{\sin^2 x} + \frac{-\sin^2 x}{2\sin^2 x} = \frac{1}{2} - \frac{1}{2} = 0 =$ R.H.S

#

$\boxed{Q3}$

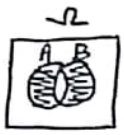
$\boxed{1}$ if $A = \{X : 0 \leq X \leq 2, X \in Z\}$ then $P(A) =$ -----

$A = \{0, 1, 2\}$

$P(A) = \{\phi, A, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}\}$

$\boxed{2}$ $A \Delta B = \{X : X \in (A-B) \vee X \in (B-A)\}$

$= \{X : X \in \{A \cup B\} \wedge X \notin \{A \cap B\}\}$



$\boxed{3}$ $R \subset A \times A$ is an Equivalence relation if R is

$\boxed{1}$ Replexife $\Rightarrow \forall X \in A \Rightarrow (X,X) \in R$

$\boxed{2}$ symmetric $\Rightarrow \forall (X,Y) \in R \Rightarrow (Y,X) \in R$

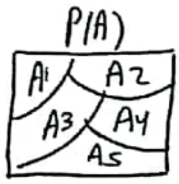
$\boxed{3}$ transtive $\Rightarrow (X,Y) \in R \wedge (Y,Z) \in R \Rightarrow (X,Z) \in R$

$\boxed{4}$ $P = \{A_1, A_2, \ldots A_n\}$ is Partition on A if ---

① $\forall A_i \in P \quad A_i \neq \phi \;, i = 1, 2, \ldots n$

② $A_i \cap A_j = \phi \quad i \neq j$

$\boxed{3}$ $\bigcup_{i=1}^{n} A_i = A$



Scanned by CamScanner

$\boxed{5}$ A is orthogonal Matrix if $AA^T = A^TA = I$

and A is hermitian if $A = (\bar{A})^T$

$\boxed{6}$ if $A = \{ \cdots, -4, -3, 6, 7, 8, \cdots \}$ then $A^c = \cdots$

$A^c = \{-2, -1, 0, 1, 2, 3, 4, 5\}$

$\boxed{7}$ $|e^{i\theta}| = |\cos\theta + i\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta}$

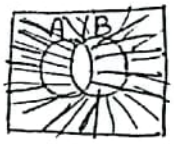
$= \sqrt{1} = 1$

$\boxed{8}$ $x \notin A \cap B \Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A^c \vee x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

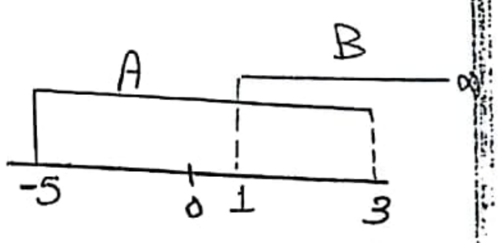
$\Rightarrow x \in (A \cap B)^c$



$\boxed{9}$ if $[-5, 3[$, $B =]1, \infty[$ then:

$A \cap B =]1, 3[$

$A \cup B = [-5, \infty[$



Q4

a) By MI Prove that

$$2^{(n+1)} > n^2 \quad \forall n > 2$$

Solution

1) at $n=3$

$L.H.S = 2^4 = 16$ $\qquad$ $R.H.S = 3^2 = 9$

$\therefore L.H.S > R.H.S$

2) let $n=k$

$$2^{k+1} > k^2 \quad \forall k \in N$$

↳ is true

3) at $n=k+1$

We will prove that $2^{k+2} > (k+1)^2$

من الخطوة رقم (2)

$$2^{k+1} > k^2$$

بالضرب * 2

$$2^{k+2} > 2k^2$$

Scanned by CamScanner

هنقوم بإثبات أن $(K+1)^2 < 2K^2$

$$\Rightarrow 2K^2 - (K+1)^2 = 2K^2 - K^2 - 2K - 1$$

$$= K^2 - 2K - 1$$

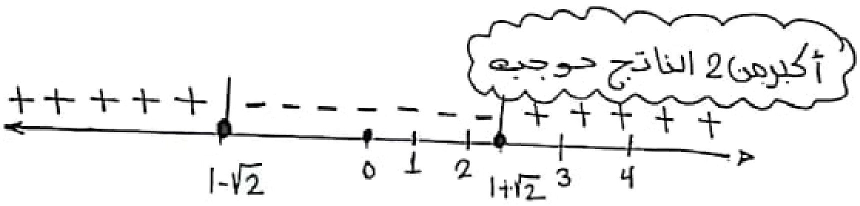
$a = 1$, $b = -2$, $c = -1$

$$K = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$K = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\Rightarrow K = 1 \pm \sqrt{2}$$

$$\therefore 2K^2 - (K+1)^2 = \left(K - (1+\sqrt{2})\right)\left(K - (1-\sqrt{2})\right)$$



$$\therefore 2K^2 - (K+1)^2 > 0 \Rightarrow \forall K > 2$$

$$\therefore 2K^2 > (K+1)^2$$

$$\therefore 2^{K+2} > (K+1)^2$$

#

Scanned by CamScanner

b let $X = \{2, 4, 6, 7\}$ and $R: X \to X$ given by:

$R = \{(x,y) : x, y \in X$ and $x+y$ is an even integer$\}$

Prove that R is Equivalence Relation

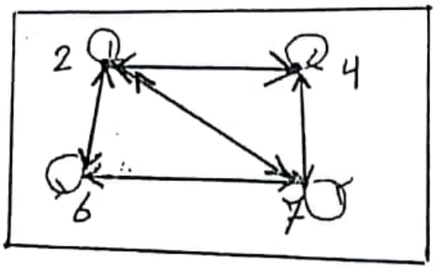
$\underline{S}$

$R = \{(2,2), (2,4), (2,6), (2,7), (4,2), (4,4)$
$, (4,6), (4,7), (6,2), (6,4), (6,6), (6,7), (7,2)$
$, (7,4), (7,6), (7,7)\}$

$R \to$ is Reflexive $\to \forall x \in A \Rightarrow (x,x) \in R$

$R \to$ is symmetric $\to (x,y) \in R \to (y,x) \in R$

$R \to$ is transtive $\to (x,y) \wedge (y,z) \Rightarrow (x,z) \in R$



R is Equivalence

Scanned by CamScanner

C Analysis $I = \dfrac{x^4 + 2x^3 + 6x^2 + 2x - 5}{x^3 - x}$

Solution

* درجه البسط أكبر من درجه المقام
← نقوم بعمل عمل قسمه مطوله

$$
\begin{array}{r|l}
 & x + 2 \\
\hline
x^3 - x & x^4 + 2x^3 + 6x^2 + 2x - 5 \\
 & \ominus x^4 \quad \oplus -x^2 \\
\hline
 & 2x^3 + 7x^2 + 2x - 5 \\
 & \ominus 2x^3 \quad \oplus -2x \\
\hline
 & 7x^2 + 4x - 5
\end{array}
$$

$$I = (x+2) + \frac{7x^2 + 4x - 5}{x(x^2 - 1)}$$

$$PF = \frac{7x^2 + 4x - 5}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$A = \left.\frac{7x^2 + 4x - 5}{(x+1)(x-1)}\right|_{x=0} = \frac{-5}{1 * -1} = 5$$

$$B = \left.\frac{7x^2+4x-5}{x(x-1)}\right|_{x=-1} = \frac{7-4-5}{-1(-2)} = -1$$

$$C = \left.\frac{7x^2+4x-5}{x(x+1)}\right|_{x=1} = \frac{7+4-5}{2} = 3$$

$$I = (x+2) + \frac{5}{x} + \frac{-1}{x+1} + \frac{3}{x-1}$$

$\boxed{d}$ use De Moivre to Find $(\sqrt{3}+i)^5$

Solution

$$Z = \sqrt{3} + i$$

$$r = \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\therefore (\sqrt{3}+i)^5 = \left(2\, e^{i\frac{\pi}{6}}\right)^5 = \left[2\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)\right]^5$$

$$= 32\left[\cos\frac{5\pi}{6} + i \sin\frac{5\pi}{6}\right]$$

$$= 32\left[\frac{-\sqrt{3}}{2} + i\,\frac{1}{2}\right] = -16\sqrt{3} + i16$$

$\#$

Scanned by CamScanner

e if $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 5 & 2 \\ 0 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$

Find the Matrix X which satisfies $AX = B$

Solution

$\Rightarrow X = A^{-1}B$

1 We find A^{-1}:

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 5 & 2 \\ 0 & 3 & 1 \end{vmatrix} = (5-6) + 3 = 2$$

$$\operatorname{Adj}(A) = \begin{bmatrix} +\begin{vmatrix} 5 & 2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 5 \\ 0 & 3 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} \\ +\begin{vmatrix} 0 & 1 \\ 5 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 1 & 5 \end{vmatrix} \end{bmatrix}^T$$

$$\operatorname{Adj}(A) = \begin{bmatrix} -1 & -1 & 3 \\ 3 & 1 & -3 \\ -5 & -1 & 5 \end{bmatrix}^T = \begin{bmatrix} -1 & 3 & -5 \\ -1 & 1 & -1 \\ 3 & -3 & 5 \end{bmatrix}$$

Scanned by CamScanner

$$\therefore A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{3}{2} & \frac{-3}{2} & \frac{5}{2} \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{3}{2} & \frac{-3}{2} & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

Scanned by CamScanner