Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Prep.

Subject: Engineering Mathematics (1)

Date: 9/1/2018
Time allowed: 3 hours
Full Marks: 110
Final-Term Exam: 2 pages

Answer the following questions:

Question 1: [28 marks][ILOs: a1,b1]

a) Use Mathematical Induction to prove that:

$$1+r+r^2+.....+r^n=\frac{1-r^{n+1}}{1-r},\qquad \forall n\in\mathbb{N},\ r\neq 1.$$

b) Resolve $\dfrac{x^2-x}{x^3+x^2-17x+15}$

c) Prove that:

$$(i)\ (A-B)\cup(B-A)=(A\cup B)-(A\cap B)$$

$$(ii)\ A\rightarrow B\equiv\sim(A\wedge\sim B)$$

Question 2: [27 marks] [ILOs: a1,b1,c7]

a) Let $X=\{2,4,6,7\}$ and R be a relation from X to X, defined by:

$$R=\{(x,y):\ x,y\in X \text{ and } x+y \text{ is an even integer}\}.$$

Discuss R?

b) Find the value of $\left(\dfrac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}\right)^{10}$

c) Use the inverse matrix method to calculate the currents I_1, I_2, I_3 of a circuit in the following system:

$$\begin{bmatrix} 4I_1-3I_2+I_3 & 2I_1+I_2-4I_3 \\ I_1+2I_2-2I_3 & 0 \end{bmatrix}=\begin{bmatrix} 11 & -1 \\ 1 & 0 \end{bmatrix}$$

Good luck: Prof. Dr.: Arafa Nasef && Dr.Eng. Manal El-said

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Question 3: [25 marks] [ILOs: a1,c1]

a) Find $\frac{dy}{dx}$ If $i)\ y = \ln\left[\frac{e^{\tan^{-1}x^2}\sqrt{x^2+\text{cosec}\ e^x}}{\ln\sec x}\right]$

$ii)\ y = 3^{\ln\cot x^2} + \cos^2 e^{\sqrt{x^2-1}} + \sinh^{-1}(2x)$

b) If $y = \frac{1}{\sqrt{1-x^2}}\sin^{-1}x$, show that

$i)\ (1-x^2)y^{(1)} = 1 + xy$

$ii)\ (1-x^2)y^{(n+1)} - (2n+1)xy^{(n)} - n^2y^{(n-1)} = 0$

c) Expand the function $f(x) = \sin x$ with the power of $(x - \frac{\pi}{6})$

Question 4: [30 marks] [ILOs: b7,c1]

Choose the correct answer from the following:

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(1) $\lim_{x\to 0}\frac{\sin^2 x}{2x}$ is $(a)\ 1$ $(b)\ 2$ $(c)\ \frac{1}{2}$ $(d)\ 0$

(2) $\lim_{k\to 0}\frac{\ln(e+k)-1}{k}$ is $(a)\ 0$ $(b)\ e$ $(c)\ \frac{1}{e}$ (d) *Non of these*

(3) $\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h}$ is $(a)\ 0$ $(b)\ 1/12$ $(c)\ 1$ $(d)\ \infty$

(4) $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$ is $(a)\ e^{2/\pi}$ $(b)\ 2/\pi$ $(c)\ 0$ (d) Non of these

(5) $\lim_{x\to\infty} x^2\sin\left(\frac{1}{x}\right)$ is $(a)\ 1$ $(b)\ \infty$ (c) Non of these $(d)\ 0$

(6) The points of discontinuity of $f(x) = \frac{\tan x}{x^2-4}$ **are**

$(a)\ \{\pm 4\}$ (b) Non of these $(c)\ \{\pm 2, (n+\frac{1}{2})\pi\},\ n = 0,\pm 1,\pm 2,....$ $(d)\ \{\pm\frac{\pi}{2}\}$

(7) The points of discontinuity of $f(x) = \frac{1}{x} + \sec x$ **are**

(a) Non of these $(b)\ \{\pm 1\}$ $(c)\ \{0\}$ $(d)\ \{0, (2n+1)\frac{\pi}{2}\},\ n = 0,\pm 1,\pm 2,....$

(8) The domain and the range of $f(x) = \cosh x$ **are**

$(a)\begin{bmatrix} D_f = R^+ \\ R_f = [1,\infty[\end{bmatrix}$ (b) Non of these $(c)\begin{bmatrix} D_f = R \\ R_f = R \end{bmatrix}$ $(d)\begin{bmatrix} D_f = R \\ R_f = [1,\infty[\end{bmatrix}$

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(9) The domain and the range of $f(x)=\ln x$ **are**

(a) Non of these $\quad (b)\begin{bmatrix} D_f=R^+ \\ R_f=[0,\infty[\end{bmatrix} \quad (c)\begin{bmatrix} D_f=R \\ R_f=R \end{bmatrix} \quad (d)\begin{bmatrix} D_f=]0,\infty[\\ R_f=]-\infty,\infty[\end{bmatrix}$

(10) The domain and the range of $f(x)=\sqrt{4-x^2}$ **are**

$(a)\begin{bmatrix} D_f=R \\ R_f=R^+ \end{bmatrix} \quad (b)[D_f=R_f=R] \quad (c)\begin{bmatrix} D_f=[-2,2] \\ R_f=[0,2] \end{bmatrix} \quad (d)$Non of these

(11) The domain and the range of $f(x)=\sqrt{x+1}$ **are**

$(a)\begin{bmatrix} D_f=R^+ \\ R_f=[0,\infty[\end{bmatrix} \quad (b)\begin{bmatrix} D_f=[-1,\infty[\\ R_f=[0,\infty[\end{bmatrix} \quad (c)\begin{bmatrix} D_f=R \\ R_f=R \end{bmatrix} \quad (d)$Non of these

(12) If $f(x)=x^3+4$ **and** $g(x)$ **is the inverse function of** $f(x)$ **then** $g'(x)$ **is**

$(a)\ \frac{1}{3(x-4)^{(2/3)}} \quad (b)\ \sqrt[3]{x-4} \quad (c)\ \frac{1}{3(x-2)^{(2/3)}} \quad (d)$ Non of these

(13) If $f(x)=x^2+1$ **and** $g(x)=\sqrt{x}$ **then the derivative of** $g(f(x))$ **is**

$(a)\ \frac{1}{(x+1)^2} \quad (b)\ x(x^2+1)^{\left(\frac{-1}{2}\right)} \quad (c)$ None of these $\quad (d)\ \frac{-2}{\sqrt{(x^2+1)}}$

(14) Find $\frac{dy}{dx}$ **If** $x=t-\sin t$ *and* $y=1-\cos t$

$(a)\ \frac{\sin t}{\cos t-1} \quad (b)\ \frac{1-\cos t}{\sin t} \quad (c)\ \frac{\sin t}{1-\cos t} \quad (d)$ Non of these

(15) Find $\frac{dy}{dx}$ **If** $x=\frac{1}{1-t}$ *and* $y=1-\ln(1-t)$, $t<1$

$(a)\ \frac{1}{1-t} \quad (b)$ Non of these $\quad (c)\ t-1 \quad (d)\ \frac{1}{x}$

(16) Find $\frac{dy}{dx}$ **If** $y=\tan^{-1}\left(\frac{x}{2}\right)$

$(a)\ \frac{2}{x^2+4} \quad (b)\ \frac{4}{4+x^2} \quad (c)\ \frac{2}{\sqrt{4-x^2}} \quad (d)$ Non of these

(17) Find $\frac{dy}{dx}$ **If** $x^3-xy+y^3=1$

$(a)\ \frac{3x^2}{x-3y^2} \quad (b)\ \frac{y-3x^2}{3y^2-x} \quad (c)$ Non of these $\quad (d)\ \frac{3x^2-1}{1-3y^2}$

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(18) If h is the inverse function of f and if $f(x)=\frac{1}{x}$ then $h'(x)$ is

$(a)\ \frac{1}{x}$ (b) Non of these $(c)\ \frac{-1}{x^2}$ $(d)\ x$

(19) If $\sin(xy)=x$ then $\frac{dy}{dx}$ is

$(a)\ \frac{1}{x}\sec(xy)-\frac{y}{x}$ (b) Non of these $(c)\ \sec(xy)$ $(d)\ \frac{\sec(xy)}{x}$

(20) If $y=\sin^{-1}(\tan x)$ then $\frac{dy}{dx}$ is

$(a)\ \frac{1}{1+\tan^2 x}$ $(b)\ \frac{\sec^2 x}{\tan x\sqrt{1-\tan^2 x}}$ $(c)\ \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ (d) Non of these

(21) If $y=(\sin x)^x$ then $\frac{dy}{dx}$ is

$(a)\ x(\sin x)^{x-1}\cos x$ $(b)\ (\sin x)^x[x\cot x+\ln(\sin x)]$ $(c)\ (\sin x)^x$ (d) Non of these

(22) If $y=\sin(ax+b)$ then $y^{(n)}$ is

$(a)\ a^n\sin(ax+b+\frac{n\pi}{2})$ $(b)\ a^n\sin(ax+b-\frac{n\pi}{2})$ $(c)\ a^n\sin(ax+b)$ (d) Non of these

(23) If $y=e^{ax}\sin bx$ then $y^{(n)}$ is

$(a)\ r^n e^{ax}\sin(bx+n\theta)$ $(b)\ e^{ax}\sin(bx+n\theta)$ $(c)\ r^n e^{ax}\sin(bx-n\theta)$ (d) Non of these

(24) If $y=\sin x\cos x$ then $y^{(n)}$ is

$(a)\ \frac{1}{2}[2^n\sin(2x+\frac{n\pi}{2})$ $(b)[2^{n-1}\sin(2x+\frac{n\pi}{2})$ $(c)\ [2^n\cos(2x+\frac{n\pi}{2})$ (d) Non of these

(25) If $y=x\ln^3 x$ then y' is

$(a)\ 3x\ln^2 x+\ln^3 x$ (b) Non of these $(c)3(\ln x+1)$ $(d)3\ln^2 x$

(26) $\lim_{x\to\infty}\frac{3x^2+1}{\sqrt{4x^4+2x^2+1}}$ is $(a)3$ $(b)\infty$ $(c)\ 3/2$ (d) Non of these

(27) If $1+x\le g(x)\le e^x$ then $\lim_{x\to 0}g(x)$ is $(a)\ 2$ $(b)3$ (c)Non of these $(d)1$

(28) If $f(x)=xe^x$ then $f''(x)$ is

$(a)\ xe^x+e^x$ $(b)e^x(x+1)$ $(c)e^x(x+2)$ (d) Non of these

(29) If $f(x)=x^2\ln(2x)$ then $f'(x)$ is

$(a)0$ $(b)2\ln(2x)+3$ $(c)2\ln(2x+\frac{3}{2})$ (d) Non of these

(30) If $f(x)=x^3+2$ *and* $g(x)=\sqrt[3]{x-2}$ then $f(g(x))$ *is*

$(a)\ x$ $(b)\ \sqrt[3]{x}$ $(c)\ 0$ (d) *Non of these*

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