Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Second Year-Electric.

Subject: Engineering Mathematics (3)

Date: 3/1/2019
Time allowed: 3 hours
Full mark: 90
Final Term Exam: 2pages

The following questions measure ILOs a1, b1, b7, c1 and c7.

Answer the following questions:

Question 1: (25 marks)

a) Prove that if $f(z)=u(x,y)+iv(x,y)$ is analytic function, then both $u(x,y)$ and $v(x,y)$ are harmonic functions. **(b1,c1)(5 marks)**

b) Discuss the analyticity of the following functions and which of them is entire:

i. $f(z)=\sin x\cosh y+i\cos x\sinh y$. **(a1,b1)(5 marks)**

ii. $f(z)=\dfrac{x-iy}{x^2+y^2}$. **(a1,b1)(5 marks)**

iii. $f(z)=z\bar{z}$. **(a1,b1)(5 marks)**

c) Is the function $u(x,y)=xe^x\cos y-ye^x\sin y$ harmonic? If so, find its conjugate harmonic function $v(x,y)$ and find the analytic function $f(z)=u(x,y)+iv(x,y)$ and $f'(z)$. **(a1,b1,c1)(5 marks)**

Question 2: : (20 marks)

a) Prove that an analytic function is independent on $\bar{z}$, i.e. for $f(z)=u(x,y)+iv(x,y)$, we have $\dfrac{\partial f}{\partial \bar{z}}=0$. **(b1,c1)(5 marks)**

b) Find the value of $\sin^{-1} i$. **(a1,b1,c1)(5 marks)**

c) Find $\dfrac{d}{dz}f(z)$ when exist and find the points at which $f'(z)$ exist for:

i. $f(z)=\ln z$. **(a1,b1,c1)(5 marks)**

ii. $f(z)=\coth^{-1} z$. **(a1,b1,c1)(5 marks)**

Samah

Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Second Year-Electric.

Subject: Engineering Mathematics (3)

Date: 3/1/2019
Time allowed: 3 hours
Full mark: 90
Final Term Exam: 2pages

The following questions measure ILOs a1, b1, b7, c1 and c7.

Question 3: : (20marks)

a) State Cauchy integral theorem , then evaluate the following integrals around the contour $C:|z|=3$:

i. $\oint \frac{\tan z}{z-1}dz$ **(a1,b1,b7)(5 marks)**

ii. $\oint \frac{e^{-z}}{z^2+4}dz$ **(a1,b1,b7)(5 marks)**

b) Find the series expansion for the functions about the indicated singularity:

i. $f(z)=\ln(\frac{1+z}{1-z})$, about $z=0$. **(b7,c1)(5 marks)**

ii. $f(z)=(z-3)\cos(\frac{1}{z+2})$, about $z=-2$. **(b7,c1) (5 marks)**

Question 4: : (25 marks)

a) Prove that the image of a circle or a line under the mapping $\omega(z)=1/z$, may be a circle or a line. **(c1,c7)(5 marks)**

b) Using the residue theorem evaluate the integrals: **(a1,b7,c1)(15 marks)**

i. $\int_0^{2\pi} \frac{d\theta}{5+\sin\theta}$.

ii. ii. $\int_0^{\infty} \frac{1}{(x^4+4)^2}dx$

c) Find the image of the quarter of the unit circle $|z|=1$ in the first quarter by the mapping :

$\omega=(2+2i)z+(4+i)$. **(c1,c7)((5 marks)**

With our best wishes
Dr Samah El-Kholy
Samah