نظرية الاهتزازات

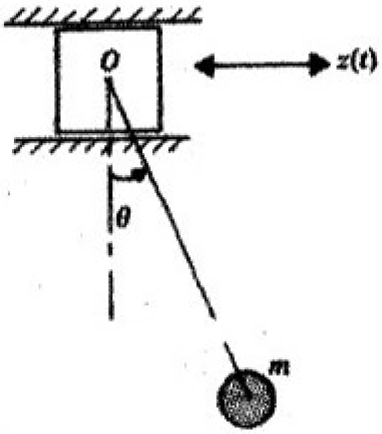
Kafrelsheikh University
Faculty of Engineering
Dept. of Mech. Engineering
Year: Third Year – power
Subject: Theory of vibration

2nd Semester
Final Examination
Date: May 20th, 2018
Time allowed: 3 hours
Full Mark: 100

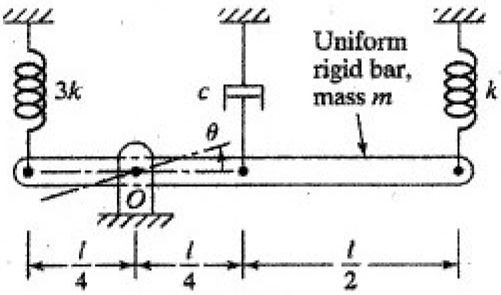
No. of pages: 2 **No. of questions: 7** **Bonus: 20 Marks**

Question #1 (20 Marks):

a) **(ILO b.5.1)** Find the nonlinear and linear differential equations of motion of the system shown in the figure, assuming that $z(t)$ is a specified known displacement.

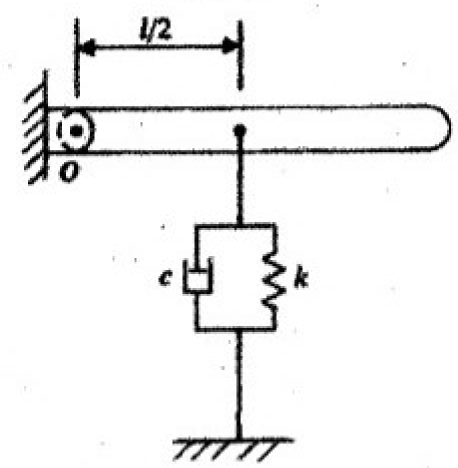


b) **(ILO c.1.1)** Derive the equation of motion and find the natural frequency of vibration of the systems shown in the figure. Assume small oscillation angle θ.



Question #2: (18 Marks) (ILO a.5.3)

A uniform slender rod of mass m and length l is hinged at point O, as shown in the figure. The rod is attached to a spring and damper which have, respectively, a stiffness coefficient k and a damping coefficient c. If $m = 3$ kg, $l = 2$ m, $k = 4000$ N/m, and c = 40 N.s/m, determine the angular displacement and the velocity of the rod after 0.5 s, provided that the initial angular displacement is 5° and the initial angular velocity is zero.



Question #3:(18 Marks) (ILO b.2.2)

It was observed that the damped free oscillations of a single degree of freedom system is such that the amplitude of the twelfth cycle is 48% that of the sixth cycle. Determine the damping factor ξ. If the mass of the system is 5 kg and the spring constant is *1000 N/m*, determine the damping coefficient and the damped natural frequency.

Theory of Vibration – Formulae Sheet

Forced vibration:

$X_o = \frac{F_o}{k}$, $r = \frac{\omega_f}{\omega}$

Forced undamped vibration:

$x_p = X_o\beta \sin \omega_f t$, $\beta = \frac{1}{1-r^2}$

$x(t) = X sin(\omega t + \varphi) + X_o\beta \sin \omega_f t$

At resonance ($r = 1$):

$$x_p = -\frac{\omega X_o t}{2}\cos \omega t$$

$$x(t) = X \sin(\omega t + \varphi) - \frac{\omega X_o t}{2}\cos \omega t$$

Forced vibration of damped system:

$x_p = X_o\beta\sin(\omega_f t - \psi)$, $\psi = tan^{-1}\left(\frac{2r\xi}{1-r^2}\right)$

$\beta = \frac{1}{\sqrt{(1-r^2)^2+(2r\xi)^2}}$, $\beta_{max.} = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Force transmission:

$F_t = F_o\beta_t \sin(\omega_f t - \bar{\psi})$

Transmissibility: $\beta_t = \beta\sqrt{1 + (2r\xi)^2}$

$\bar{\psi} = \psi - \psi_t$, $\psi_t = \tan^{-1}(2r\xi)$

Work:

$W_e = \pi F_o X_o \beta \sin \psi$, $W_d = \pi c X_o^2 \beta^2 \omega_f$

$W_s = 0$

Rotating unbalance:

$$x_p(t) = \left(\frac{me}{M}\right)\beta_r \sin(\omega_f t - \psi)$$

$$\beta_r = \frac{r^2}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

$F_t = (me\omega^2)\beta_t \sin(\omega_f t - \bar{\psi})$

$\beta_t = \beta_r\sqrt{1 + (2r\xi)^2}$

Base motion:

$x_p(t) = Y_o\beta_b \sin(\omega_f t - \psi + \psi_b)$

$\beta_b = \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2+(2r\xi)^2}}$, $\psi_b = \tan^{-1}(2r\xi)$

$F_t = Y_o k\beta_b r^2 \sin(\omega_f t - \psi + \psi_b)$

Relative motion:

$z = Y_o\beta_r \sin(\omega_f t - \psi)$

$$dB = 20Log(\frac{x_1}{x_2})$$

$$t_{p_1} = \frac{\pi / 2 - \phi}{\omega_n}$$

$$E = \frac{1}{2}kX^2$$

$$K_t = \frac{GJ}{L}$$

$\xi = \frac{c}{C_c}$, $C_C = 2m\omega$

$X_i = \sqrt{1-\xi^2}Xe^{-\xi\omega t_i}$

logarithmic Decrement:

$$\ln\frac{X_i}{X_{i+n}} = n\xi\omega\tau_d = n\delta$$

$$\xi = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

Energy Loss:

$$\frac{\Delta U}{U_i} = 1 - e^{-2\delta}$$

IMPACT DYNAMICS:

$m_1v_1' + m_2v_2' = m_1v_1 + m_2v_2$

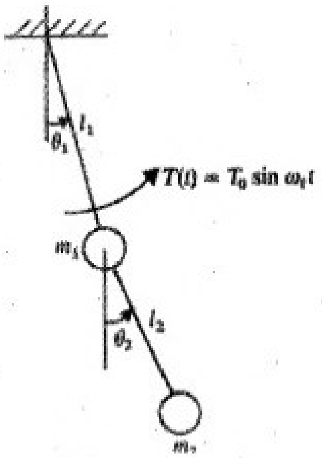
$v_2' - v_1' = e(v_1 - v_2)$

Question #4: (18 Marks) (ILO c.1.2)

A vibrometer is used to measure the amplitude of a vibrating machine. It was observed that the machine is vibrating at a frequency of 12 Hz. The natural frequency and damping ratio of the vibrometer are, respectively, 8 Hz and 0.7. The reading of the vibrometer indicates that the amplitude of vibration is 1.5 cm. Determine the correct value for the amplitude of vibration of the machine. Explain how to reduce the error between the measured value by the vibrometer and the correct value?

Question #5 (18 Marks) (ILO b.2.1)

Use Lagrange equation to derive the differential equations of motion of the two degree of freedom system shown in the figure. Then, assuming small oscillations, determine the response of the system, as a function of time, to the harmonic forcing function $T(t)$.
What condition will make the amplitude of the forced response of $\theta_1(t)$ equal zero.



Question #6 (18 Marks) (ILO b.5.2)

Using modal analysis, find the steady-state response of a two-degree-of-freedom system with equations of motion:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} sin\, \omega_f t$$

$m_1 = 10$ kg, $m_2 = 1$ kg, $k_1 = 30$ N/m, $k_2 = 5$ N/m, $F_1 = 10$ N, $\omega_f = 5$ rad/s.

Question #7(10 Marks):

Give brief answers to the following:

a) **(ILO a.19.1)** Mention two different methods that can be used to derive the equations of motion of vibrating systems.
b) What will be the frequency of the applied force with respect to the natural frequency of the system if the magnification factor is less than unity?
c) Will the force transmitted to the base of a spring-mounted machine decrease with the addition of damping?
d) How can we make a system vibrate in one of its natural modes?
e) What is meant by static and dynamic coupling? How can you eliminate coupling of the equations of motion?
f) How many distinct natural frequencies and natural modes of vibrations can exist for an n-degree-of-freedom system?