مادة نظرية الآلات د/ رشدي

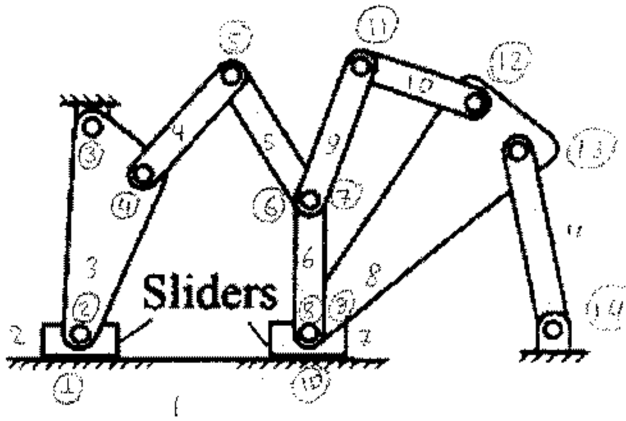
Kafrelsheikh University
Faculty of Engineering
Dept. of Mech. Engineering
Year: second Year
Subject: Theory of Machines

1st Semester
Final Examination
Date: Jan. 17th, 2017
Time allowed: 3 hours
Full Mark: 100

1. No. of pages: 8
2. No. of questions: 8

نموذج اجابة

1. **(10 Marks)** For the linkage shown in the figures, determine the number of degrees of freedom of the mechanisms.



Sol.:-

$n = 11$

$j = 14$, $\sum_{i=1}^{j} f_i = 14$

$DoF = 3(n-j-1) + \sum_{i=1}^{j} f_i$

$DoF = 3(11-14-1) + 14$

$= -12 + 14 = 2 \Rightarrow \boxed{DoF = 2}$

2. **(10 Marks)** Assume that you have a set of the following lengths: 20 mm, 30 mm, 45 mm, 56 mm, and 73 mm. Design a four-bar linkage that can be driven with a continuous-rotation electric motor. Justify your answer with appropriate equations, and make a freehand sketch of the resulting linkage. Label the crank, frame, coupler, and rocker (follower).

Sol.:-

Choose the links to be: 20 mm, 45 mm, 56 mm, and 73 mm

$S = 20$ mm , $L = 73$ mm

$P = 45$ mm , $Q = 56$ mm

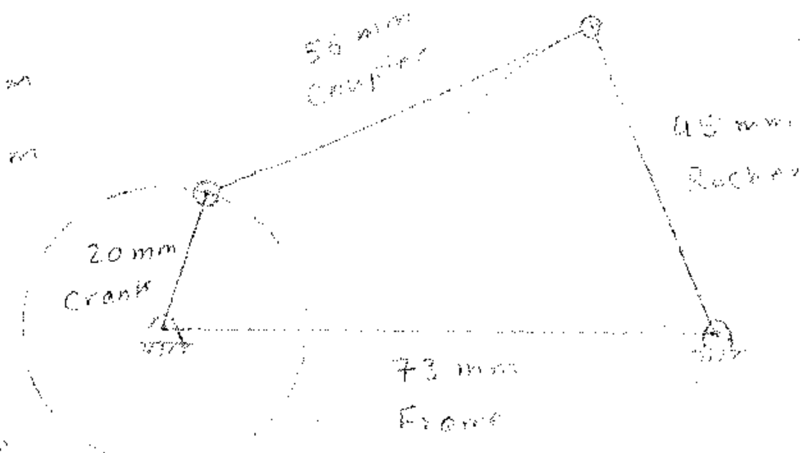
$S + L = 20 + 73 = 93$ mm

$P + Q = 45 + 56 = 101$ mm

$S + L < P + Q$

Grashof type 1

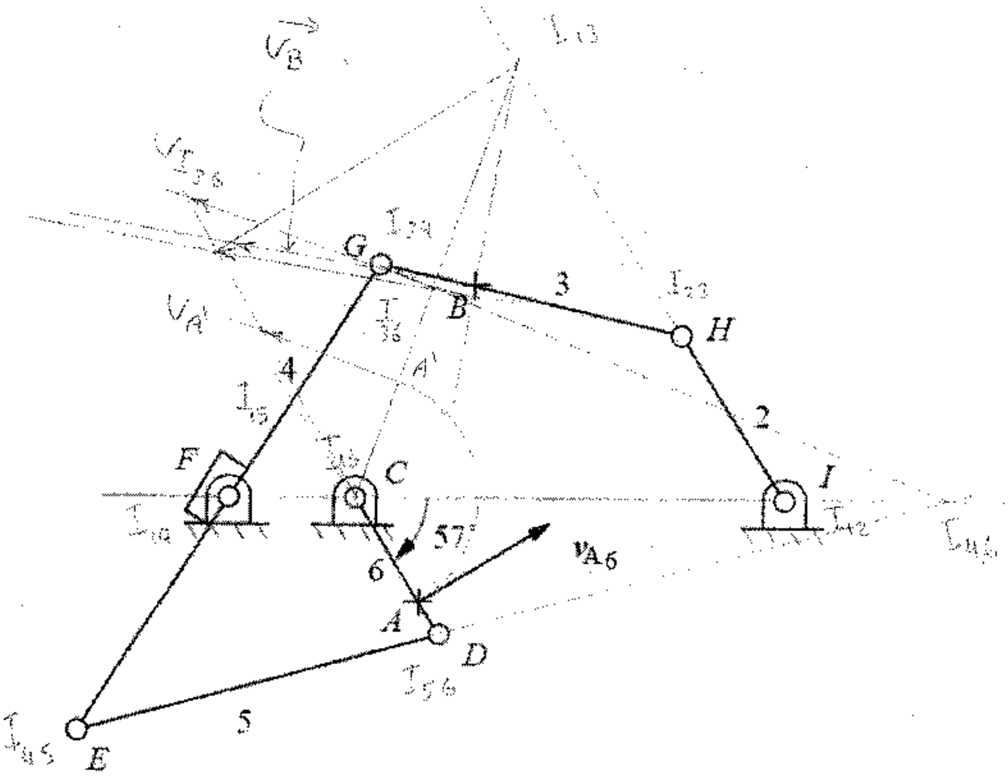
∴ The mechanism can be driven with a continuous rotation electric motor.



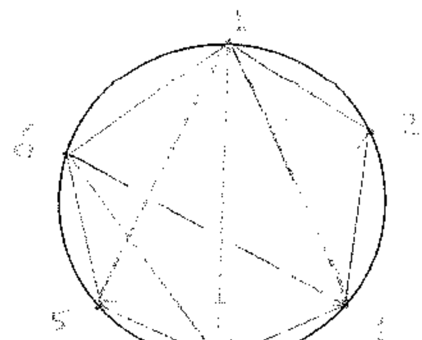
Crank - Rocker mechanism

1 (**16 Marks**) If $v_A = 10$ in/s, determine the velocity vector (magnitude and direction) for point B on link 3 using an instant center method.

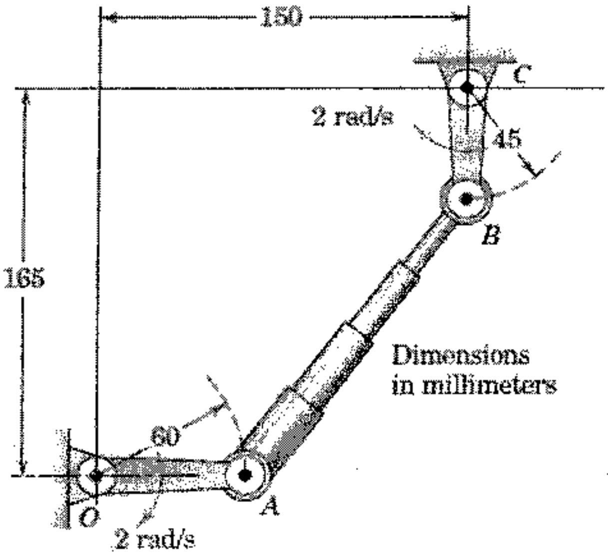
$CD = 0.8"$
$CA = 0.6"$
$ED = 1.85"$
$EF = FG = 1.35"$
$GH = 1.5"$
$HI = 0.95"$
$CI = 2.1"$
$CF = 0.65"$



$V_B = 3.3 * 5 = 16.5$ in/s



4. **(16 Marks)** for the mechanism shown, write the appropriate vector equations, solve them using vector polygon, and determine the angular velocity of the telescoping link AB for the position shown where the driving links have the angular velocities indicated. Use velocity scale 1 cm:20 mm/s.



Sol. :-

$V_A = 2 * 60 = 120$ mm/s ↓

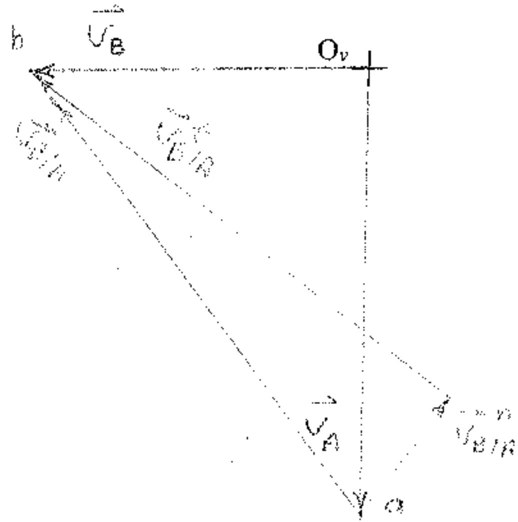
$V_B = 2 * 45 = 90$ mm/s ←

Link AB is telescopic, therefore, the velocity of point B relative to A has two components, one in the direction of the link and one perpendicular to it

$\vec{V}_{B/A} = \vec{V}^n_{B/A} + \vec{V}^t_{B/A}$

$\vec{V}_B = \vec{V}_A + \vec{V}^n_{B/A} + \vec{V}^t_{B/A}$

$\vec{V}_B - \vec{V}^t_{B/A} = \vec{V}_A + \vec{V}^n_{B/A}$



$V^n_{B/A} = 2 * 20 = 40$ mm/s

$V^t_{B/A} = 7.1 * 20 = 142$ mm/s

$\therefore \omega_{AB} = \dfrac{V^t_{B/A}}{AB}$, $AB = \sqrt{(150-60)^2 + (165-45)^2}$

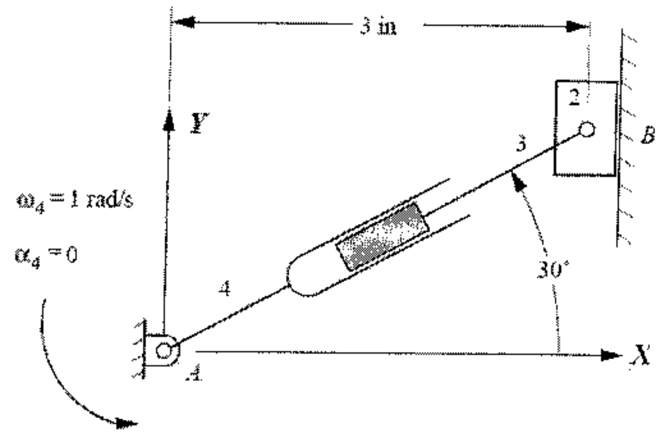
$= \sqrt{(90)^2 + (120)^2} = 150$ mm

$= \dfrac{142}{150}$

$= 0.947$ rad/s ccw

5. **(20 Marks)** In the mechanism shown, $\omega_4 = 1$ rad/s CW (constant). Using the analytical method, determine the velocity and acceleration of point B.

CCW



Sol. :-

Loop closure equation

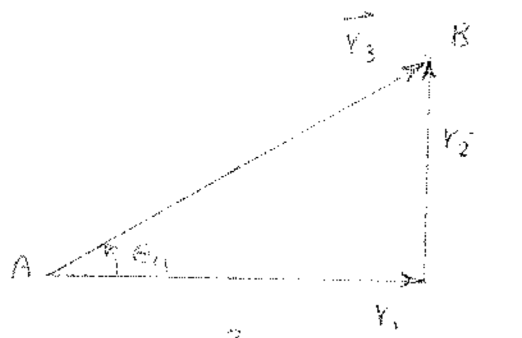
$\vec{r_3} = \vec{r_1} + \vec{r_2}$, $\omega_4 = 1$ rad/s CCW

$\vec{r_1}$ is constant

$\vec{r_2} = r_2 e^{j\theta_2}$, r_2 variable, $\theta_2 = 90°$ constant

$\vec{r_3} = r_3 e^{j\theta_4}$, r_3 and θ_4 are variables

$r_3 e^{j\theta_4} = r_1 e^{j0} + r_2 e^{j90°}$



(1) ← $\dot r_3 e^{j\theta_4} + r_3 \omega_4 e^{j(\theta_4+90°)} = \dot r_2 e^{j90°}$, $\dot r_2 = V_B$

$\dot r_3 \cos\theta_4 + r_3\omega_4 \cos(\theta_4+90°) = 0$ → (2)

$\dot r_3 \sin\theta_4 + r_3\omega_4 \sin(\theta_4+90°) = \dot r_2$ → (3)

$r_3 = \frac{3}{\cos\theta_4} = \frac{3}{\cos 30°} = 3.464$

From (2) $\dot r_3 = -r_3\omega_4 \frac{\cos(\theta_4+90°)}{\cos\theta_4} = -3.464 * 1 * \frac{\cos 120}{\cos 30} = 2$ in/s

From (3) $\dot r_2 = V_B = \dot r_3 \sin\theta_4 + r_3\omega_4 \sin(\theta_4+90°)$

$V_B = 2\sin 30° + 3.464 * 1 * \sin 120° \Longrightarrow V_B = 4$ in/s ↑

Differentiate Equation (1) w.r.t. time , $\alpha_4 = 0$

$\ddot r_3 e^{j\theta_4} + 2\dot r_3 \omega_4 e^{j(\theta_4+90°)} + r_3\alpha_4 e^{j(\theta_4+90°)} + r_3\omega_4^2 e^{j(\theta_4+180°)} = \ddot r_2 e^{j90°}$

$\ddot r_3 \cos\theta_4 + 2\dot r_3\omega_4 \cos(\theta_4+90°) + r_3\omega_4^2 \cos(\theta_4+180°) = 0$ → (4)

$\ddot r_3 \sin\theta_4 + 2\dot r_3\omega_4 \sin(\theta_4+90°) + r_3\omega_4^2 \sin(\theta_4+180°) = \ddot r_2 = a_B$ → (5)

From (4) $\ddot r_3 = \frac{1}{\cos 30°}\left[-2*2*1*\cos 120° - 3.464*(1)^2\cos 210°\right] \Rightarrow \ddot r_3 = 5.7734$ in/s²

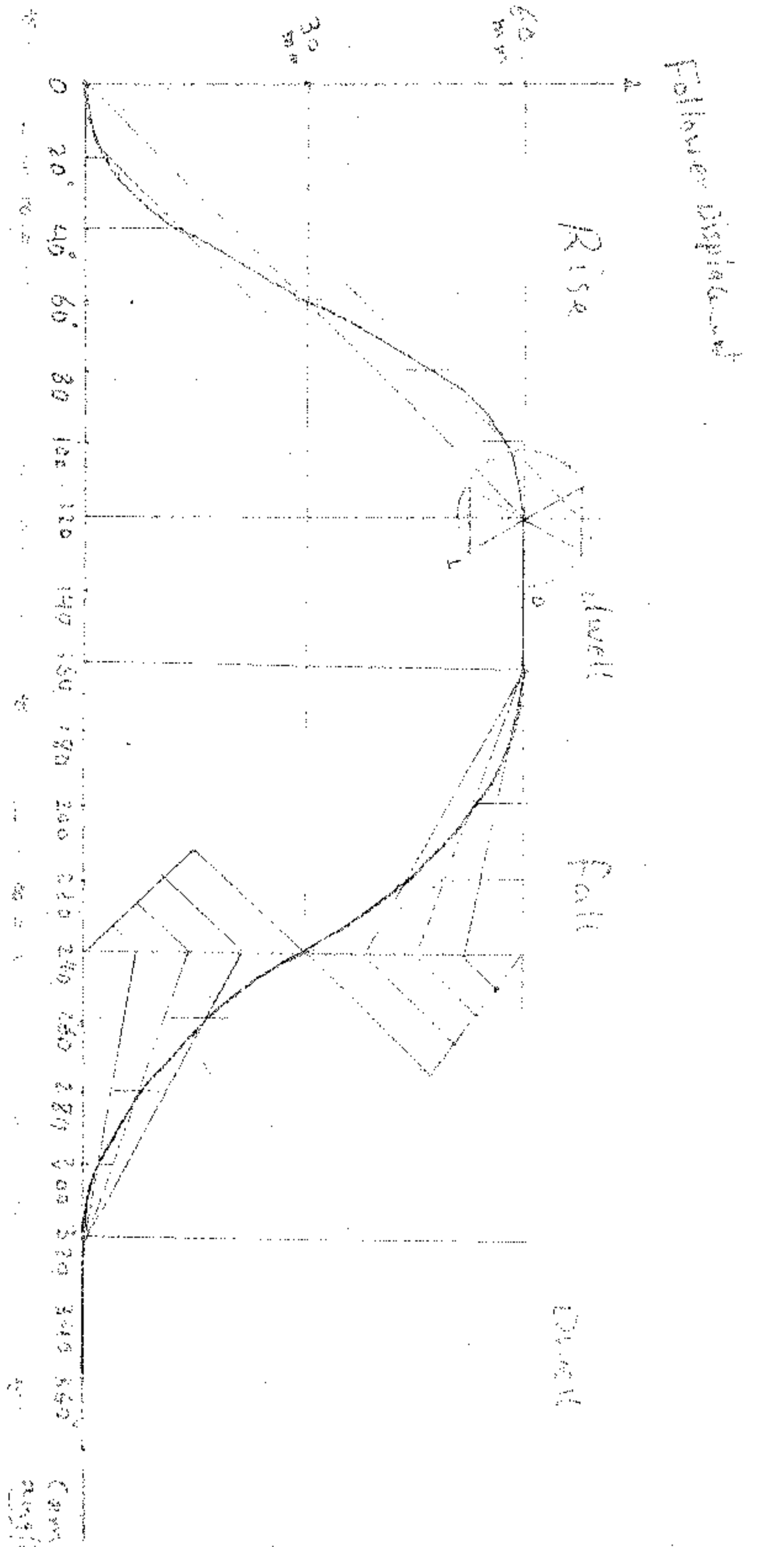
From (5)

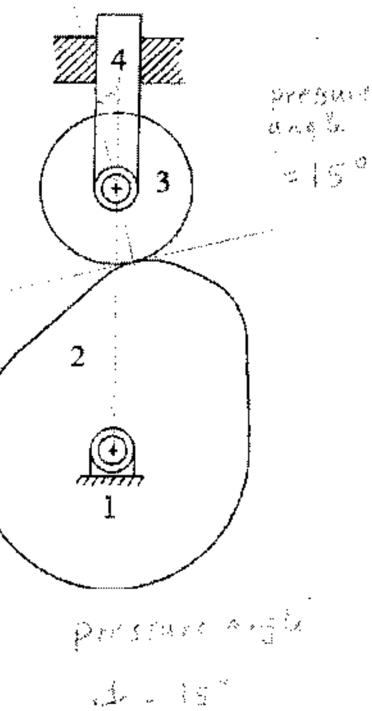
$a_B = 5.7734\sin 30° + 2*2*1*\sin(120°) + 3.464*(1)^2\sin 210°$

$a_B = 4.62$ in/s²

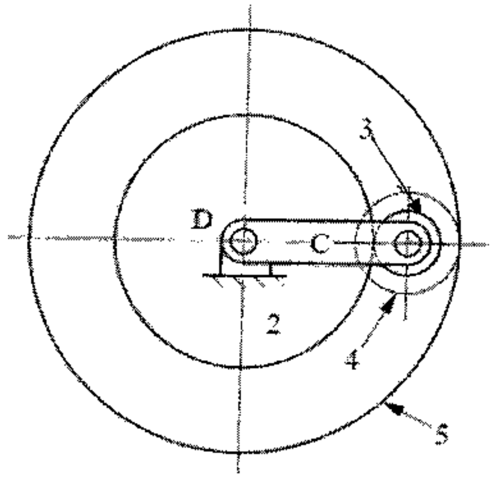
a. **(6 Marks)** For the cam mechanism shown, find the pressure angle at the position shown.

b. **(12 Marks)** Draw the displacement schedule for a follower that rises through a total displacement of 60 mm with cycloidal motion in 120° of cam rotation. The follower then dwells for 40° and falls 30 mm with constant acceleration in 80° followed by a fall of 30 mm with constant deceleration in 80°. The follower then dwells for 40° before repeating the cycle.





1. **(16 Marks)** In the gear train shown, gears 3 and 4 are integral (compound gears). Gear 3 meshes with gear 2, and gear 4 meshes with gear 5. If gear 2 is fixed and $\omega_5 = 100$ rpm CCW, determine the angular velocity of the carrier.



$N_2 = 60T$
$N_3 = 16T$
$N_4 = 24T$
$N_5 = 100T$

Sol.:-

$^{c}\omega_2 = \omega_2 - \omega_c$

$^{c}\omega_5 = \omega_5 - \omega_c$

$\therefore \frac{^{c}\omega_2}{^{c}\omega_5} = \frac{\omega_2 - \omega_c}{\omega_5 - \omega_c}$

and $\frac{^{c}\omega_2}{^{c}\omega_5} = \frac{^{c}\omega_2}{^{c}\omega_3} * \frac{^{c}\omega_4}{^{c}\omega_5}$

$= -\frac{N_3}{N_2} * \frac{N_5}{N_4} = -\frac{16*100}{60*24} =$

$= -\frac{16*100}{60*24} = -\frac{10}{9}$

$\therefore \frac{^{c}\omega_2}{^{c}\omega_5} = \frac{\omega_2 - \omega_c}{\omega_5 - \omega_c} = -\frac{10}{9}$

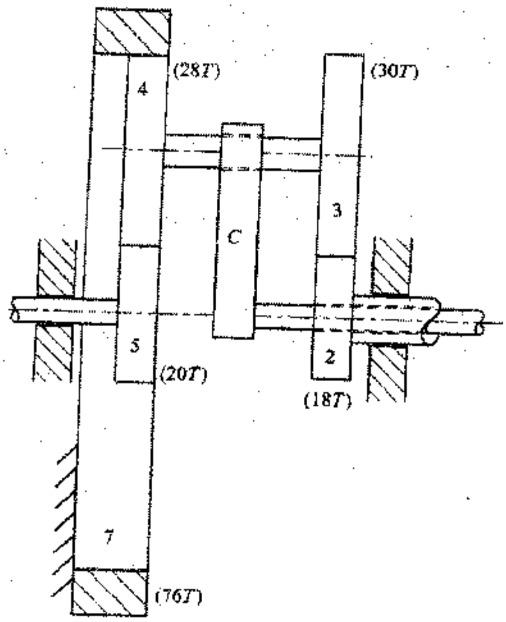
$\frac{0 - \omega_c}{100 - \omega_c} = -\frac{10}{9}$

$9\omega_c = 1000 - 10\omega_c$

$\therefore 19\omega_c = 1000$

$\therefore \omega_c = 52.63$ rpm CCW

5. **(12 Marks)** Assume that gear 5 in the figure is driven at a speed of 200 rpm in the CCW direction viewed from the left end. Gear 4 meshes with a fixed ring gear and with gear 5 as shown. Find the magnitude and direction of the angular velocity of gear 2.



Sol.:

${}^{c}\omega_5 = \omega_5 - \omega_c$

${}^{c}\omega_7 = \omega_7 - \omega_c$

$\therefore \dfrac{{}^{c}\omega_5}{{}^{c}\omega_7} = \dfrac{\omega_5 - \omega_c}{\omega_7 - \omega_c}$

but $\dfrac{{}^{c}\omega_5}{{}^{c}\omega_7} = -\dfrac{N_7}{N_5} = -\dfrac{76}{20}$

$\therefore \dfrac{{}^{c}\omega_5}{{}^{c}\omega_7} = \dfrac{\omega_5 - \omega_c}{\omega_7 - \omega_c} = -\dfrac{76}{20}$

$\therefore \dfrac{200 - \omega_c}{0 - \omega_c} = -\dfrac{76}{20} \Rightarrow 76\,\omega_c = 200 - 20\,\omega_c$

$\therefore \omega_c = \dfrac{4000}{96} = 41.67$ rpm

${}^{c}\omega_2 = \omega_2 - \omega_c$

$\dfrac{{}^{c}\omega_2}{{}^{c}\omega_5} = \dfrac{\omega_2 - \omega_c}{\omega_5 - \omega_c}$

but $\dfrac{{}^{c}\omega_2}{{}^{c}\omega_5} = \dfrac{{}^{c}\omega_2}{{}^{c}\omega_3} \times \dfrac{{}^{c}\omega_4}{{}^{c}\omega_5} = -\dfrac{N_3}{N_2} \times -\dfrac{N_5}{N_4}$

$= \dfrac{30 \times 20}{18 \times 28} = \dfrac{25}{21}$

$\therefore \dfrac{\omega_2 - \omega_c}{\omega_5 - \omega_c} = \dfrac{25}{21} \Rightarrow \dfrac{\omega_2 - 41.67}{200 - 41.67} = \dfrac{25}{21}$

$\therefore \omega_2 = 230.76$ rpm CCW direction viewed from the left end.

End of Questions.
Best wishes.