عزيز إطاعة

د. سمير سرحان

حارة اصطبل [illegible]

كوم الشقا [illegible]

شارع الكهرباء

1- We find the three spherical components by dotting **G** with the appropriate unit vectors, and we change variables during the procedure:

$$G_r = G.a_r = \frac{xz}{y} a_x.a_r = \frac{xz}{y} sin\theta cos\emptyset$$
$$= rsin\theta cos\theta \frac{cos^2\emptyset}{sin\emptyset}$$
$$G_\theta = G.a_\theta = \frac{xz}{y} a_x.a_\theta = \frac{xz}{y} cos\theta cos\emptyset$$
$$= rcos^2\theta \frac{cos^2\emptyset}{sin\emptyset}$$
$$G_\emptyset = G.a_\emptyset = \frac{xz}{y} a_x.a_\emptyset = \frac{xz}{y}(-sin\emptyset)$$
$$= -rcos\theta\ cos\emptyset$$

$$\mathbf{G} = rcos\theta cos\emptyset(sin\theta cot\emptyset\ \mathbf{a}_r + cos\theta cot\emptyset\ \mathbf{a}_\theta - \mathbf{a}_\emptyset)$$

2-

$$dQ = \rho_L dl = \rho_L dZ$$

and hence the total charge Q is

$$Q = \int_{Z_A}^{Z_B} \rho_L dZ \qquad \text{(line charge)} \qquad 2.17$$

The electric field intensity E at an arbitrary point P(x, y, z) $dl = dZ'$

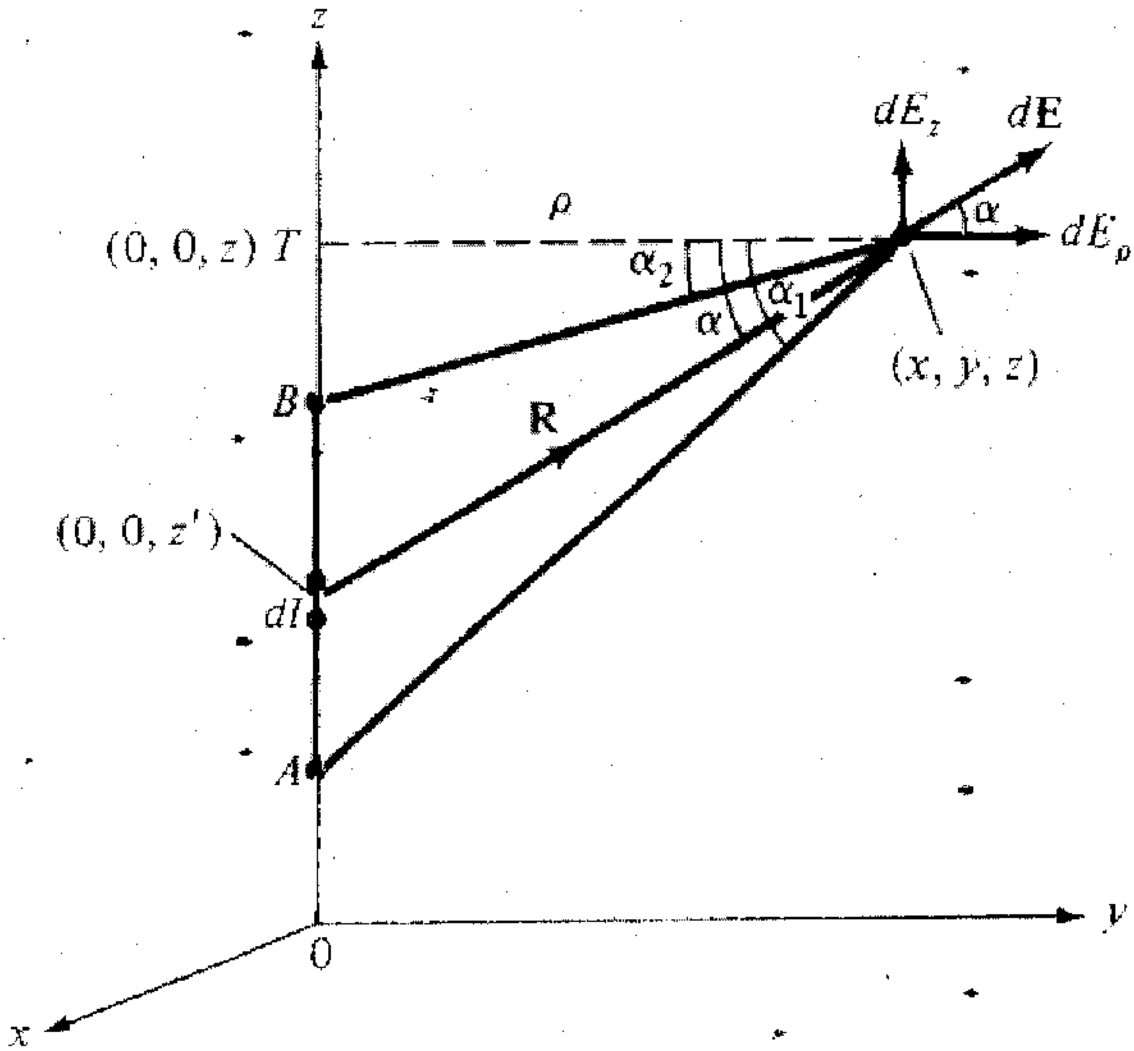
$$\mathbf{R} = (x, y, z) - (0,0,z') = x\mathbf{a}_x + y\mathbf{a}_y + (z - z')\mathbf{a}_z$$

Or

$$\mathbf{R} = \rho\mathbf{a}_\rho + (z - z')\mathbf{a}_z$$
$$R^2 = |\mathbf{R}|^2 = x^2 + y^2 + (z - z')^2 = \rho^2 + (z - z')^2$$

$$\frac{\mathbf{a}_R}{R^2} = \frac{\mathbf{R}}{|\mathbf{R}|^3} = \frac{\rho\mathbf{a}_\rho + (z - z')\mathbf{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}$$

1



Substituting all this into eq. (2.14), we get

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{\rho\mathbf{a}_\rho + (z - z')\mathbf{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}\,dz' \qquad 2.18$$

$$R = [\rho^2 + (z - z')^2]^{\frac{1}{2}} = \rho\sec\alpha$$

$$z' = OT - \rho\tan\alpha, \qquad dz' = -\rho\sec^2\alpha\, d\alpha$$

$$\mathbf{E} = \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\rho\sec^2\alpha\,[cos\alpha\;\mathbf{a}_\rho + sin\alpha\;\mathbf{a}_z]d\alpha}{\rho^2\sec^2\alpha} \qquad 2.19$$

2

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}\int_{\alpha_1}^{\alpha_2}[\cos\alpha\,\mathbf{a}_\rho + \sin\alpha\,\mathbf{a}_z]d\alpha$$

Thus for a finite line charge,

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_0\rho}[-(\sin\alpha_2 - \sin\alpha_1)\,\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\,\mathbf{a}_z] \quad 2.20$$

As a special case, for an infinite line charge, point B is at (0, 0,∞) and A at (0, 0, -∞) so that $\alpha_1 = \pi/2$, $\alpha_2 = -\pi/2$; the z-component vanishes and eq. (2.20) becomes

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho \qquad 2.21$$

3-

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3}$$

where $\mathbf{E_1}$, $\mathbf{E_2}$, and $\mathbf{E_3}$ are, respectively, the contributions to **E** at point (1, 1, - 1) due to the infinite sheet 1, infinite sheet 2, and infinite line 3 as shown in Figure 2.9(a). Applying eqs. (4.26) and (4.21) gives

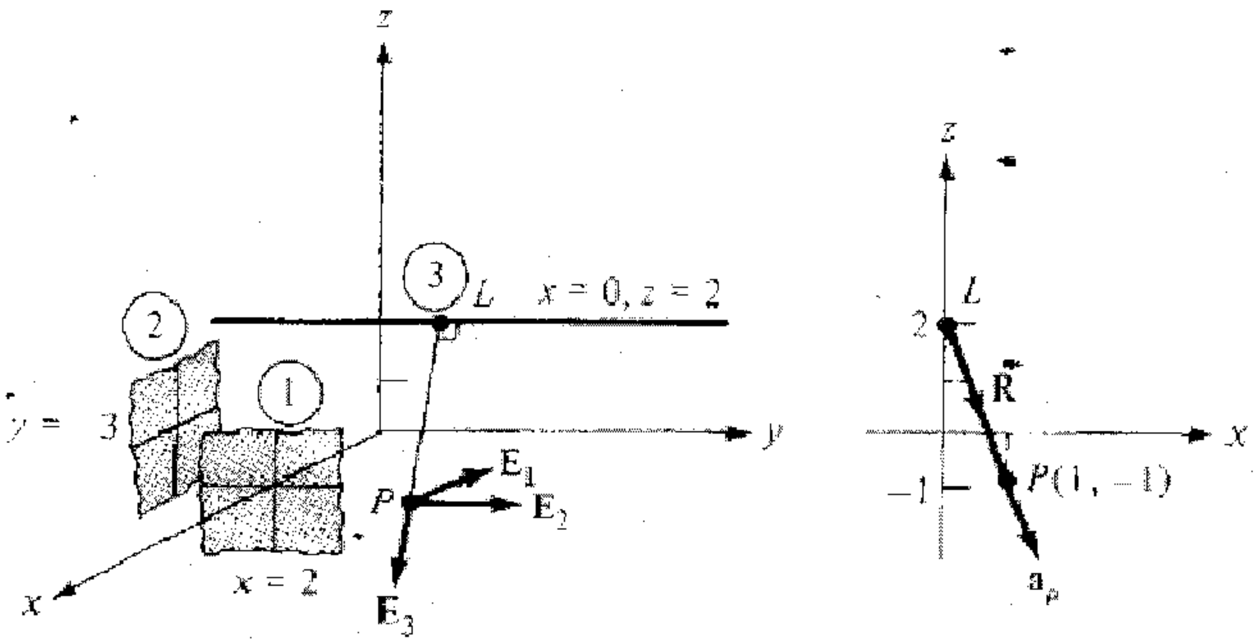
$$\mathbf{E}_1 = \frac{\rho_{S_1}}{2\varepsilon_o}(-\mathbf{a}_x) = -\frac{10\cdot 10^{-9}}{2\cdot\frac{10^{-9}}{36\pi}}\mathbf{a}_x = -180\pi\mathbf{a}_x$$

$$\mathbf{E}_2 = \frac{\rho_{S_2}}{2\varepsilon_o}\mathbf{a}_y = \frac{15\cdot 10^{-9}}{2\cdot\frac{10^{-9}}{36\pi}}\mathbf{a}_y = 270\pi\,\mathbf{a}_y$$

and

$$\mathbf{E_3} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho$$

3



where $\mathbf{a}_\rho$ (not regular $\mathbf{a}_\rho$ but with a similar meaning) is a unit vector along LP perpendicular to the line charge and ρ is the length LP to be determined from Figure 2.9(b). Figure 2.9(b) results from Figure 2.9(a) if we consider plane y = 1 on which $\mathbf{E}_3$ lies. From Figure 2.9(b), the distance vector from L to P is

$$\mathbf{R} = -3\mathbf{a}_z + \mathbf{a}_x$$

$$\rho = |\mathbf{R}| = \sqrt{10}, \qquad \mathbf{a}_\rho = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{1}{\sqrt{10}}\mathbf{a}_x - \frac{3}{\sqrt{10}}\mathbf{a}_z$$

$$\mathbf{E}_3 = \frac{10\pi.10^{-9}}{2\pi.\frac{10^{-9}}{36\pi}}.\frac{\mathbf{1}}{\mathbf{10}}(\mathbf{a}_x - 3\mathbf{a}_z) = 10\pi(\mathbf{a}_x - 3\mathbf{a}_z)$$

Thus by adding $\mathbf{E_1}$, $\mathbf{E_2}$, and $\mathbf{E_3}$, we obtain the total field as

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3} = -\mathbf{162}\pi\,\mathbf{a}_x + 270\pi\,\mathbf{a}_y - 54\pi\,\mathbf{a}_z\ V/m$$

4-

$$\rho_v = \nabla.\mathbf{D} = \frac{\partial D_z}{\partial z} = \boldsymbol{\rho}\,\mathbf{cos^2\varphi}$$

At $(1, \frac{\pi}{4}, 3)$, $\rho_v = \mathbf{1}.\mathbf{cos}^2\left(\frac{\pi}{4}\right) = 0.5\ C/m^3$. The total charge enclosed by the cylinder can be found in two different ways.

Method 1: This method is based directly on the definition of the total volume charge.

$$Q = \int \rho_v \, dv = \int \rho cos^2\varphi \; \rho d\varphi d\rho dz$$

$$= \int_{z=-2}^{2} dz \int_{\varphi=0}^{2\pi} cos^2\varphi \, d\varphi \int_{\rho=0}^{1} \rho^2 d\rho = 4(\pi)\left(\frac{1}{3}\right) = \frac{4\pi}{3} C$$

we can use Gauss's law.

$$Q = \Psi = \oint \mathbf{D}.d\mathbf{s} = [\int_s + \int_t + \int_b]\mathbf{D}.d\mathbf{s}$$

$$= \Psi_s + \Psi_t + \Psi_b$$

where Ψ_s, Ψ_t and Ψ_b are the flux through the sides, the top surface, and the bottom surface of the cylinder, respectively. Since **D** does not have component along $\mathbf{a}_\rho$, $\Psi_s = 0$, for Ψ_t, $d\mathbf{s} = \rho d\varphi d\rho \, \mathbf{a}_z$ so

$$\Psi_t = \int_{\rho=0}^{1} \int_{\varphi=0}^{2\pi} z\rho cos^2\varphi \; \rho d\varphi d\rho|_{z=2} = 2\int_0^1 \rho^2 \, d\rho \int_0^{2\pi} cos^2\varphi d\varphi$$

$$\Psi_t = 2\left(\frac{1}{3}\right)\pi = \frac{2\pi}{3}$$

and for Ψ_b, $d\mathbf{s} = -\rho d\varphi d\rho \, \mathbf{a}_z$, so

$$\Psi_b = -\int_{\rho=0}^{1} \int_{\varphi=0}^{2\pi} z\rho cos^2\varphi \; \rho d\varphi d\rho|_{z=-2} = 2\int_0^1 \rho^2 \, d\rho \int_0^{2\pi} cos^2\varphi d\varphi$$

$$= \frac{2\pi}{3}$$

Thus

$$Q = \Psi = 0 + \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3} \; C$$

5- the field of the uniform line charge with $\rho_L = 2\pi\varepsilon_0$

$$\mathbf{E} = \frac{1}{\rho}\mathbf{a}_\rho$$

In rectangular coordinates,

$$\mathbf{E} = \frac{x}{x^2+y^2}\mathbf{a}_x + \frac{y}{x^2+y^2}\mathbf{a}_y$$

Thus we form the differential equation

$$\frac{E_y}{E_x} = \frac{dy}{dx} = \frac{y}{x} \qquad or \quad \frac{dy}{y} = \frac{dx}{x}$$

Therefore

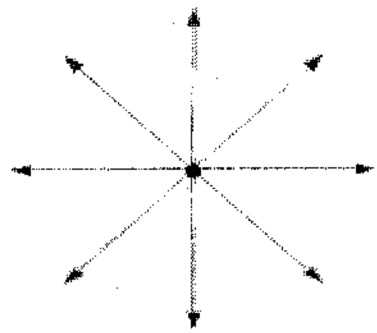
$$\ln y = \ln x + C_1 \qquad or \quad \ln y = \ln x + \ln C$$

from which the equations of the streamlines are obtained,

$$y = C\,x$$

If we want to find the equation of one particular streamline, say one passing through P(-2, 7, 10), we merely substitute the coordinates of that point into our equation and evaluate C. Here, 7 = C(-2), and C =-3.5, so y =-3.5x. Each streamline is associated with a specific value of C, and the radial lines shown in Figure 2.9d are obtained when C = 0, 1, -1, and 1/C = 0.

The equations of streamlines may also be obtained directly in cylindrical or spherical coordinates.



6- The potential at P(-4, 5, 6) is

$$V_P = 2(-4)^2(3) - 5(6) = 66 \text{ V}$$

Next, we may use the gradient operation to obtain the electric field intensity.

$$\mathbf{E} = -\nabla V = -4xy\,\mathbf{a}_x - 2x^2\,\mathbf{a}_y + 5\mathbf{a}_z \quad V/m$$

The value of E at point P is

$$\mathbf{E}_p = 48\,\mathbf{a}_x - 32\,\mathbf{a}_y + 5\mathbf{a}_z \quad V/m$$

and

$$\mathbf{E}_p = \sqrt{48^2 + (-32)^2 + 5^2} = 57.9\,V/m$$

The direction of E at P is given by the unit vector

$$\mathbf{a}_{\mathbf{E},\mathbf{p}} = (48\,\mathbf{a}_x - 32\,\mathbf{a}_y + 5\mathbf{a}_z)/57.9$$
$$= 0.829\,\mathbf{a}_x - 0.553\,\mathbf{a}_y + 0.086\mathbf{a}_z$$

If we assume these fields exist in free space, then

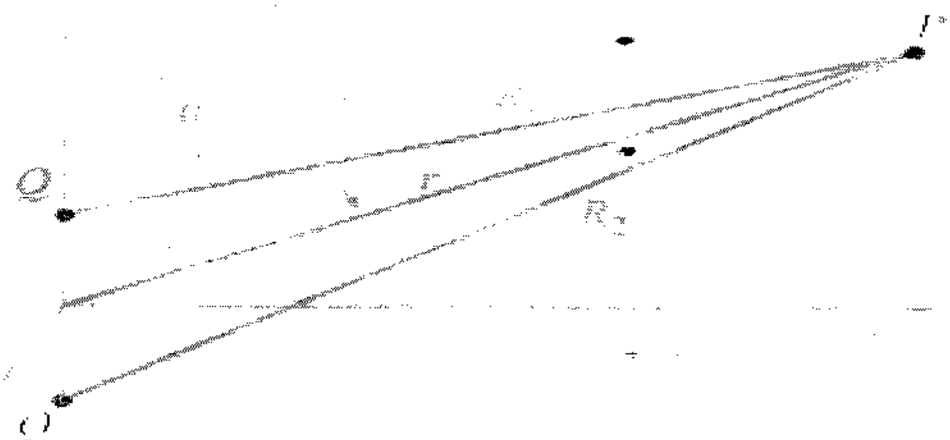
$$\mathbf{D} = \epsilon_0 \mathbf{E} = -35.4xy\,\mathbf{a}_x - 17.71x^2\,\mathbf{a}_y + 44.3\,\mathbf{a}_z \ \mathrm{pC/m^2}$$

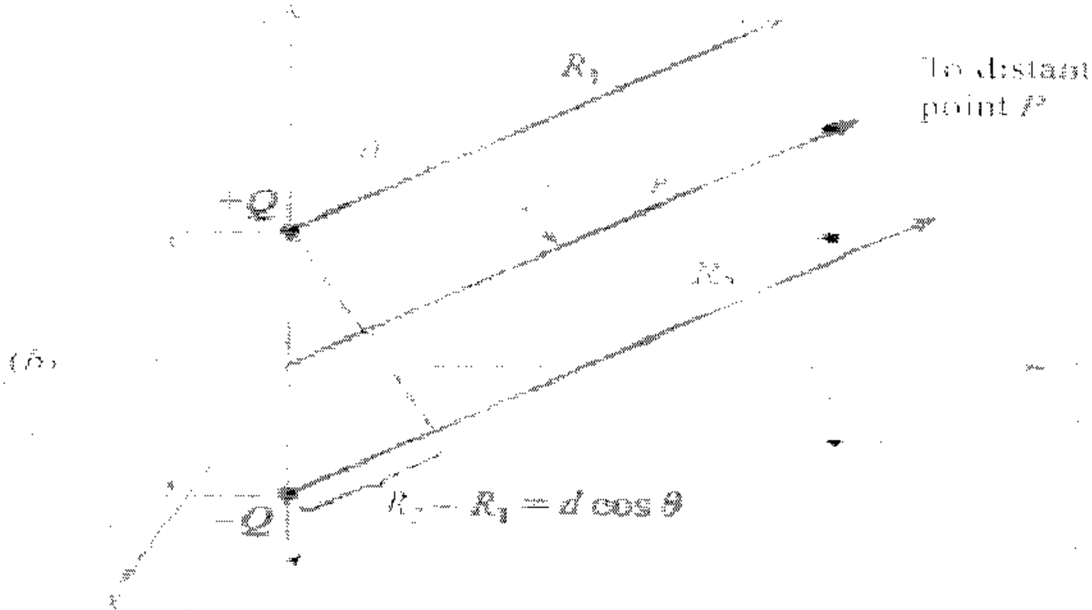
Finally, we may use the divergence relationship to find the volume charge density that is the source of the given potential field,

$$\rho_v = \nabla . \mathbf{D} = -35.4y \frac{PC}{m^3}$$

At P, $\rho_v = -106.2 \frac{PC}{m^3}$

7-





$$R_2 - R_1 = d\ cos\theta$$

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

Again, we note that the plane z = 0(θ =90°) is at zero potential.
Using the gradient relationship in spherical coordinates,

$$\mathbf{E} = -\nabla V = -(\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\mathbf{a}_\varphi)$$

we obtain

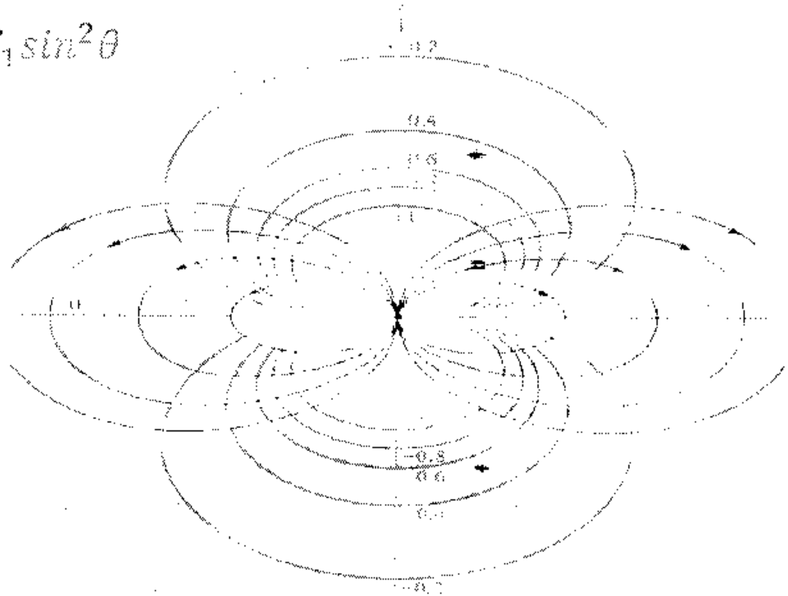
$$\mathbb{E} = -\left(-\frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\mathbf{a}_r - \frac{Qd\sin\theta}{2\pi\varepsilon_0 r^3}\mathbf{a}_\theta\right)$$

Or

$\mathbb{E} = \frac{Qd}{2\pi\varepsilon_0 r^3}(2\cos\theta\ \mathbf{a}_r - \sin\theta\ \mathbf{a}_\theta)$; $\quad \frac{E_\theta}{E_r} = \frac{r\ d\theta}{dr} = \frac{\sin\theta}{2\cos\theta}$

Or

$\frac{dr}{r} = 2\ cot\theta\ d\theta$; $\quad r = C_1 sin^2\theta$



8-

a) Find the total current crossing the plane $y = 1$ in the $\mathbf{a}_y$ direction in the region $0 < x < 1$, $0 < z < 2$. This is found through

$$I = \int\int_S \mathbf{J}\cdot\mathbf{n}\Big|_S da = \int_0^2\int_0^1 \mathbf{J}\cdot\mathbf{a}_y\Big|_{y=1} dx\,dz = \int_0^2\int_0^1 -10^4\cos(2x)e^{-2}\,dx\,dz$$

$$= -10^4(2)\frac{1}{2}\sin(2x)\Big|_0^1 e^{-2} = \underline{-1.23\,\text{MA}}$$

b) Find the total current leaving the region $0 < x, y < 1$, $2 < z < 3$ by integrating $\mathbf{J} \cdot d\mathbf{S}$ over the surface of the cube. Note first that current through the top and bottom surfaces will not exist, since $\mathbf{J}$ has no z component. Also note that there will be no current through the $x = 0$ plane, since $J_x = 0$ there. Current will pass through the three remaining surfaces, and will be found through

$$I = \int_2^3 \int_0^1 \mathbf{J} \cdot (-\mathbf{a}_y)\Big|_{y=0} \, dx\, dz + \int_2^3 \int_0^1 \mathbf{J} \cdot (\mathbf{a}_y)\Big|_{y=1} \, dx\, dz + \int_2^3 \int_0^1 \mathbf{J} \cdot (\mathbf{a}_x)\Big|_{x=1} \, dy\, dz$$

$$= 10^4 \int_2^3 \int_0^1 \left[\cos(2x)e^{-0} - \cos(2x)e^{-2}\right] dx\, dz - 10^4 \int_2^3 \int_0^1 \sin(2)e^{-2y} \, dy\, dz$$

$$= 10^4 \left(\frac{1}{2}\right) \sin(2x)\Big|_0^1 (3-2)\left[1 - e^{-2}\right] + 10^4 \left(\frac{1}{2}\right) \sin(2)e^{-2y}\Big|_0^1 (3-2) = \underline{0}$$