Kafrelsheikh University
Faculty of Engineering
Dept. Mech. Engineering
Year: Preparatory
Subject: Mechanics (I)_Dynamics

Semester: 2nd Semester
Final Examination
Date: May 28th, 2017
Time allowed: 3 hours
Full Mark: 70

Model Answer (نموذج إجابـــــة)

(a) No. of questions: 8-No. of pages: 9. Only page no. [9/9] is blank.
(b) This is a close book exam. Only calculator is permitted
(c) Clear, systematic answers and solutions are required. In general, marks will not be assigned for answers and solutions that require unreasonable (in the opinion of the instructor) effort to decipher.
(d) Solve all questions.
(e) The exam will be marked out of 70. There are 20 marks bonus.

1. During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. **Determine** the maximum height Y reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s² due to gravity. Neglect the effect of air resistance. **(10 Marks)**

Solution

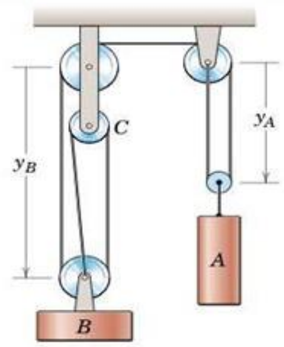
- For upward motion

$$v = 0$$
$$v_0 = 75 \text{ m/s}$$
$$v^2 = v_0^2 - 2gS$$
$$S = \frac{v_0^2}{2g} = \frac{(75)^2}{2\times 9.81} = 286.7 \text{ m}$$
$$S_{max} = 40 + 286.7 = 326.7 \text{ m from ground}$$

- For downward motion

$$v_0 = 0 \text{ m/s}$$
$$S = 326.7 \text{ m}$$
$$v^2 = v_0^2 + 2gS$$
$$v^2 = 0 + 2\times 9.81\times 326.7 = 6409.9 \text{ m}^2/\text{s}^2$$
$$v = \sqrt{6409.9} = 80.1 \text{ m/s} \qquad \text{(velocity of rocket just before hitting the ground)}$$

2. In the pulley configuration shown, cylinder A has a downward velocity of o.3 m/s and an upward acceleration of 1 m/s². **Determine** the velocity and acceleration of cylinder B. **(10 Marks)**

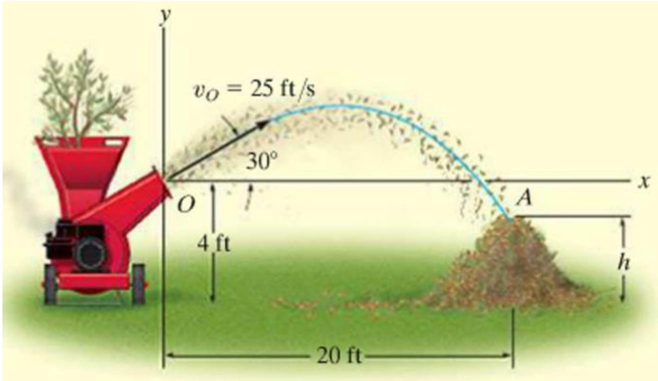


Solution

$$3y_B + 2y_A = C$$
$$3v_B + 2v_A = 0$$
$$3a_B + 2a_A = 0$$

$$3y_B + 2y_A = C$$
$$v_B = -\frac{2}{3}v_A = -\frac{2}{3}\times 0.3 = -0.2 \text{ m/s} = 0.2 \text{ m/s}\uparrow$$
$$a_B = -\frac{2}{3}a_A = -\frac{2}{3}\times 1 = -\frac{2}{3} \text{ m/s}^2 = 0.667 \text{ m/s}^2\uparrow$$

3. The chipping machine is designed to eject wood chips at v_0 = 25 ft/s. If the tube is oriented at 30° from the horizontal, **determine** how high *h*, the chips strike the pile if at this instant they land on the pile 20 ft from the tube **(10 Marks)**



Solution

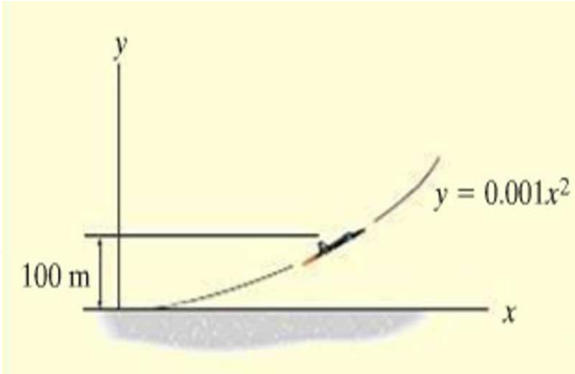
- For horizontal motion

$$x_{AB} = v_0 \cos(30)t$$
$$t = \frac{x_{AB}}{v_0 \cos(30)} = \frac{20}{25\cos(30)} = 0.923 \text{ sec}$$

- For vertical motion

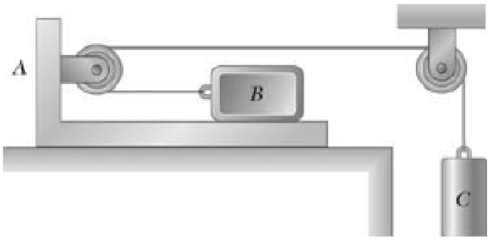
$$y_{OA} = v_0 \sin(30)t - \frac{1}{2}gt^2 = 25\sin(30)\times(0.923) - \frac{1}{2}\times 32.2\times(0.923)^2 = -2.18\,\text{ft}$$
$$h = 4 + y_{OA} = 4 - 2.18 = 1.82\,\text{ft}$$

4. The path of the plane is described by **y=(.001x²)**m If the plane is rising with a constant upward Velocity of 10 m/s, **Determine** the magnitudes of the velocity and acceleration of the plane When it reaches an altitude of y =100m. **(10 Marks)**



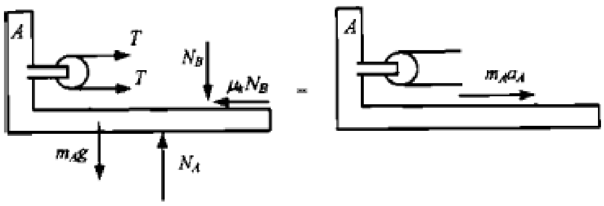
Solution

5. Objects A, B, and C have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between A and B is μ_k, and the friction between A and the ground is negligible and the pulleys are massless and frictionless. Assuming B slides on A draw the FBD (Free Body Diagram) and equation of motion for each of the three masses A, B and C. **(10 Marks)**

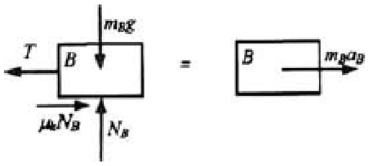


Solution

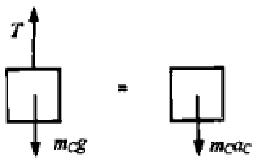
Block A



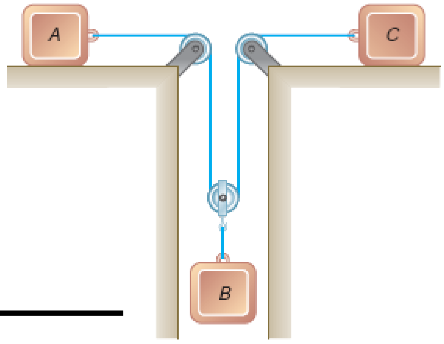
Block B



Block C



6. The coefficients of friction between blocks A and C and the horizontal surfaces are $\mu_s = 0.25$ and $\mu_k = 0.2$. Knowing that m_A = 4 kg, m_B = 8 kg, and m_C = 8 kg. Using Newton's second law to determine: **(10 Marks)**
   a. the tension in the cord
   b. the acceleration of blocks B and C.

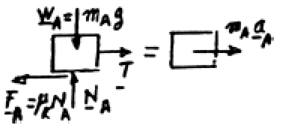


Solution

We first check that static equilibrium is not maintained:

$$(F_A)_m + (F_C)_m = \mu_s (m_A + m_C) g$$
$$= 0.24(5+10)g$$
$$= 3.6g$$

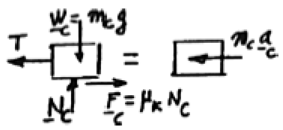
Since $W_B = m_B g = 10\text{g} > 3.6\text{g}$, equilibrium is *not* maintained.



Block A: $\quad \Sigma F_y: \quad N_A = m_A g$

$$F_A = \mu_k N_A = 0.2 m_A g$$

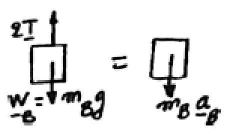
$$\xrightarrow{+} \Sigma F_x = m_A a_A: \quad T - 0.2 m_A g = m_A a_A \qquad (1)$$



Block C: $\quad \Sigma F_y: \quad N_C = m_C g$

$$F_C = \mu_k N_C = 0.2 m_C g$$

$$\xleftarrow{+} \Sigma F_x = m_C a_C: \quad T - 0.2 m_C g = m_C a_C \qquad (2)$$



Block B: $\quad +\downarrow \Sigma F_y = m_B a_B$

$$m_B g - 2T = m_B a_B \qquad (3)$$

From kinematics: $\quad a_B = \dfrac{1}{2}(a_A + a_C) \qquad (4)$

(*a*) Tension in cord. Given data: $\quad m_A = 5$ kg
$\qquad m_B = m_C = 10$ kg

Eq. (1): $\quad T - 0.2(5)g = 5a_A \qquad a_A = 0.2T - 0.2g \qquad (5)$

Eq. (2): $\quad T - 0.2(10)g = 10a_C \qquad a_C = 0.1T - 0.2g \qquad (6)$

Eq. (3): $\quad 10g - 2T = 10a_B \qquad a_B = g - 0.2T \qquad (7)$

Substitute into (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g + 0.1T - 0.2g)$$

$$1.2g = 0.35T \qquad T = \frac{24}{7}g = \frac{24}{7}(9.81 \text{ m/s}^2) \qquad T = 33.6 \text{ N} \blacktriangleleft$$

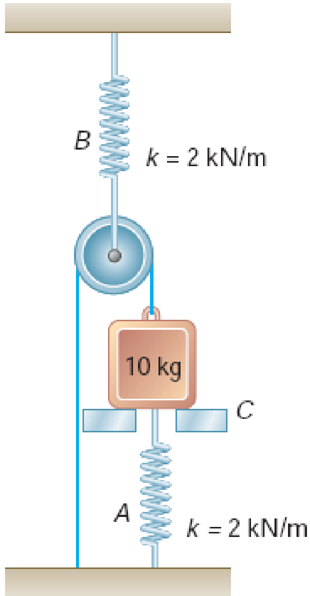
(*b*) Substitute for T into (5), (7), and (6):

$$a_A = 0.2\left(\frac{24}{7}g\right) - 0.2g = 0.4857(9.81 \text{ m/s}^2) \qquad \mathbf{a}_A = 4.76 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

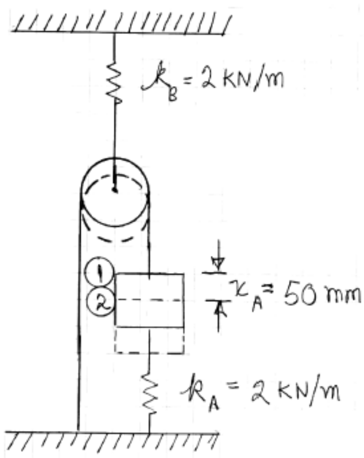
$$a_B = g - 0.2\left(\frac{24}{7}g\right) = 0.3143(9.81 \text{ m/s}^2) \qquad \mathbf{a}_B = 3.08 \text{ m/s}^2 \downarrow \blacktriangleleft$$

$$a_C = 0.1\left(\frac{24}{7}g\right) - 0.2g = 0.14286(9.81 \text{ m/s}^2) \qquad \mathbf{a}_C = 1.401 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

7. A 10-kg block is attached to spring A and connected to spring B by a cord and pulley. The block is held in the position shown with both springs unstretched when the support is removed and the block is released with no initial velocity. Knowing that the constant of each spring is 2 kN/m, and using **work and energy principle**, determine: **(15 Marks)**
   (a) velocity of the block after it has moved down 50 mm.
   (b) maximum velocity achieved by the block.
   (c) sketch on v-y diagram shown below the velocity of block C in the range $0 \le y \le 87.2$ mm.



Solution



(a) W = weight of the block = 10 (9.81) = 98.1 N

$$x_B = \frac{1}{2}x_A$$

$$U_{1-2} = W(x_A) - \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

(Gravity) (Spring A) (Spring B)

$$U_{1-2} = (98.1\text{ N})(0.05\text{ m}) - \frac{1}{2}(2000\text{ N/m})(0.05\text{ m})^2$$

$$-\frac{1}{2}(2000\text{ N/m})(0.025\text{ m})^2$$

$$U_{1-2} = \frac{1}{2}(m)v^2 = \frac{1}{2}(10\text{ kg})v^2$$

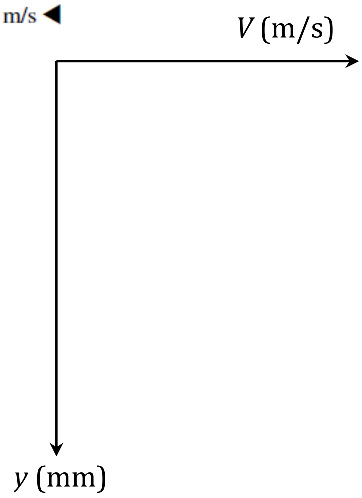
$$4.905 - 2.5 - 0.625 = \frac{1}{2}(10)v^2$$

v = 0.597 m/s ◀

(b) Let x = distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2}k_A(x)^2 - \frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(m)v^2$$

$$\frac{d}{dx}\left[\frac{1}{2}(m)v^2\right] = 0 = W - k_A(x) - \frac{k_B}{8}(2x)$$



$$0 = 98.1 - 2000\,(x) - \frac{2000}{8}(2x) = 98.1 - (2000 + 250)x$$

x = 0.0436 m (43.6 mm)

For $x = 0.0436$, $U = 4.2772 - 1.9010 - 0.4752 = \frac{1}{2}(10)v^2$

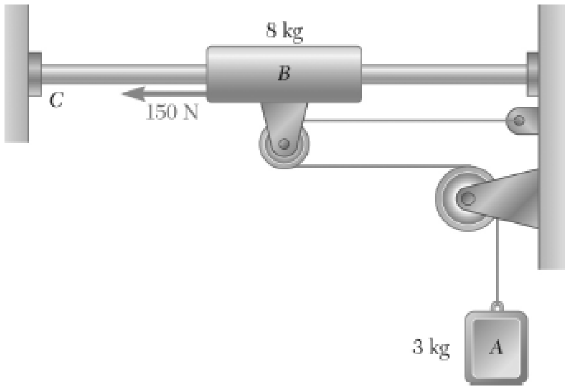
v_{max} = 0.6166 m/s

v_{max} = 0.617 m/s ◀

8. The system shown is at rest when a constant 150-N force is applied to collar B. By neglecting the effect of friction and using **impulse and momentum principle**, determine: **(15 Marks)**

  (a) the time at which the velocity of collar B will be 2.5 m/s to the left,

  (b) the corresponding tension in the cable.



Solution

Let F be the cable tension and v_B be the velocity of collar B

Its movement is horizontal so only horizontal forces acting on B do work. Let d be the distance through which the 30 lb applied force moves.

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + 30d - (2F)(2) = \frac{1}{2}\frac{18}{32.2}v_B^2$$

$$30d - 4F = 0.27950 v_B^2 \qquad (1)$$

Now consider the weight A. When the collar moves 2 ft to the left, the weight moves 4 ft up, since the cable length is constant. Also, $v_A = 2v_B$.

$$(T_1)_A + (U_{1-2})_A = (T_2)_B$$

$$0 + (F - W_A)(4) = \frac{1}{2}\frac{W_A}{g}v_A^2$$

$$4F - (6)(4) = \frac{1}{2}\frac{6}{32.2}(2v_B)^2$$

$$4F - 24 = 0.37267 v_B^2 \qquad (2)$$

Add Eqs. (1) and (2) to eliminate F.

$$30d - 24 = 0.65217 v_B^2 \qquad (3)$$

(a) Case a: $d = 2$ ft, $v_B = ?$

$$(30)(2) - (24) = 0.65217 v_B^2$$

$$v_B^2 = 55.2 \text{ ft}^2/\text{s}^2 \qquad v_B = 7.43 \text{ ft/s} \blacktriangleleft$$

(b) Case b: $d = ?$, $v_B = 0$.

$$30d - 24 = 0 \qquad d = 0.800 \text{ ft} \blacktriangleleft$$