Kafrelsheikh University
Faculty of Engineering
Electrical Engineering Department
Final Exam, 2015-2016

2nd Year (Electrical Engineering)
Automatic Control (1)
Time: 180 minutes
Mark: 90
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Answer all the following questions:

Problem 1: (12 points)

a) Use block diagram reduction to simplify the block diagram below into a single block relating Y(s) to R(s), [7 Marks].

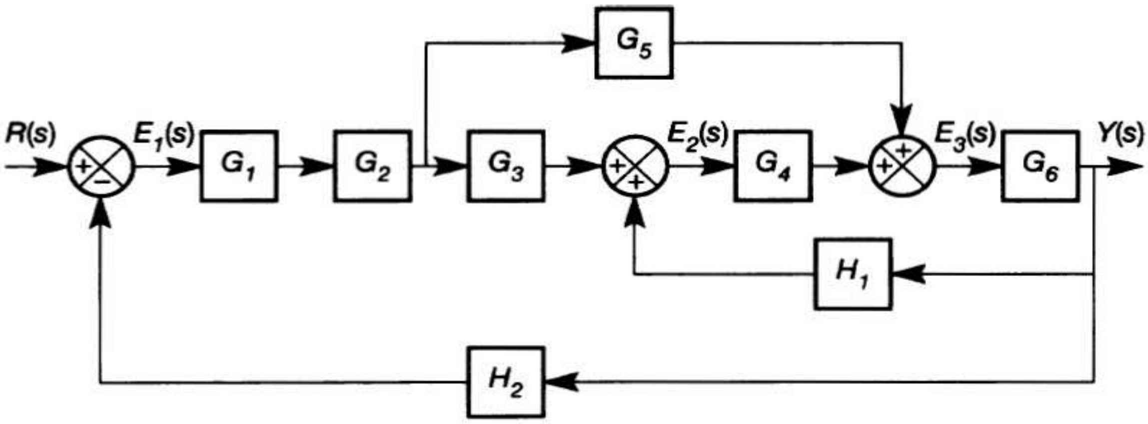

Fig. 1

b) Check whether the following system given by its characteristic equation is stable or not and shows the location of roots on the s-plane. **[7 Marks]**

$$q(s) = S^5 + 10S^4 + 45S^3 + 90S^2 + 164S + 200 = 0$$

Problem 2: (13 points)

a) A closed loop negative feeback system has an open loop transfer function:

$$G(s)H(s) = \frac{K(s+10)}{s(s^2+6s+13)}$$

Sketch the Root Locus for K > 0.

$a(s) = s^5 + 10s^4 + 45s^3 + 90s^2 + 164s + 200$

All coefficients are positive ($a_i > 0$)

→ necessary condition passed. ✓

Row	5	s^5	1	45	164	0
	4	s^4	10	90	200	0
	3	s^3	36	144	0	
	2	s^2	50	200	0	
	1	s^1	0	0	← Special case #2	
New row (dp/ds)	1	s^1	100	0		
	0	s^0	200			

Aux. poly:

$p(s): 50s^2 + 200 = 0$

$s^2 = -4$

$\boxed{s = \pm j2}$

No sign changes

→ no RHP poles.

However, there is a pair of non-repeated imaginary poles at $\pm j2$.

⇒ System is neutrally stable.

Or, with Special Case #1

Row 2	s^2	(+)	50	200	0
Row 1	s^1		$0 \to \varepsilon > 0$	0	
Row 0	s^0	(+)	200		

$$(+) \Rightarrow (\varepsilon) \Rightarrow (+)$$

Not a sign change, pair of imaginary roots.

No sign change in 1st column of Routh array.

$\to$ No RHP poles.

$\Rightarrow$ Neutrally stable system.

(due due roots on Imaginary axis)

Example 6-10: A closed-loop negative feedback system has an open-loop transfer function of

$$G(s)H(s) = \frac{K(s+10)}{s(s^2+6s+13)}$$

Sketch the Root Locus for K > 0.
Solution:
There are three poles and one zero, i.e., n=3 and m=1. Therefore, the Root Locus has three branches (n=3), one branch starting from one of the poles and approaching to the zero while the other two branches starting from the two $(n-m=3-1=2)$ remaining poles and approaching to their Asymptotes $(n-m=2)$ going to infinite when K increases.

The poles are located at 0, $-3+j2$ and $-3-j2$ while the zero is located at -10. The Root Locus is on the Real Axis between 0 and -10 since the number of pole(s) is an odd number on the right hand side of the zero at -10.

The angle of Asymptotes can be calculated as follows

$$\phi_A = \frac{\pi + 2k\pi}{n-m} = \begin{cases} \dfrac{\pi}{3-1} = \dfrac{\pi}{2}, & \text{for } k=0 \\ \dfrac{\pi+2\pi}{3-1} = \dfrac{3\pi}{2} = -\dfrac{\pi}{2}, & \text{for } k=1 \end{cases}$$

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The Centroid can be obtained by

$$\alpha = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \quad -\frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} = -\frac{(0+(3-2j)+(3+2j))-(10)}{3-1} = 2$$

A Centroid with a positive value implies two branches of the Root Locus would travel from the left-half-plane to the right-half-plane of the s-plane. Therefore, those branches would cross the Imaginary Axis and the locations can be obtained as follows:

The characteristic equation: $s(s^2+6s+13)+K(s+10)=s^3+6s^2+(13+K)s+10K=0$

Routh Table

s^3	1	$13+K$
s^2	6	$10K$
s^1	b_1	
s^0	c_1	

$$b_1 = \frac{6\times(13+K)-1\times 10K}{6} = \frac{78-4K}{6}$$

$c_1 = 10K$

To have a stable system, $K>0$ and $78-4K>0$. Therefore, $0<K<19.5$.

When K=19.5, the Root Locus intersects the Imaginary Axis which can be obtained from the equation starting with s^2 term in the Routh Table, namely, $6s^2+10K=0$ with K=19.5.

$$6s^2+195=0$$

$$(s-j5.7009)(s+j5.7009)=0$$

Therefore, the two branches of the Root Locus intersect the following two conjugate points at $\pm 5.7009\ \text{rad/sec}$ with K=19.5.

The angle of departure from the pole at 0 (i.e., $p_1=0$):

$$\angle(0+10)-(\theta_{dp_1}+\angle(0+3-2j)+\angle(0+3+2j))=180^\circ$$

$$\angle(10)-(\theta_{dp_1}+\angle(3-2j)+\angle(3+2j))=180^\circ$$

$$0^\circ-(\theta_{dp_1}+146.3^\circ+(-146.3^\circ))=180^\circ$$

$$\theta_{dp_1}=0^\circ-146.3^\circ-(-146.3^\circ)-180^\circ=-180^\circ=180^\circ$$

The angle of departure from the pole at $-3+2j$ $(\text{i.e., } p_2 = 3-2j)$:

$$\angle(-3+2j+10)-\left(\angle(-3+2j+0)+\theta_{dp_2}+\angle(-3+2j+3+2j)\right)=180^\circ$$

$$\angle(7+2j)-\left(\angle(-3+2j)+\theta_{dp_2}+\angle(4j)\right)=180^\circ$$

$$\theta_{dp_2}=-180^\circ+15.9^\circ-146.3^\circ-90^\circ=-400.4^\circ=-40.4^\circ$$

The angle of departure from the pole at $-3-2j$ $(\text{i.e., } p_3 = 3+2j)$:

$$\angle(-3-2j+10)-\left(\angle(-3-2j+0)+\angle(-3-2j+3-2j)+\theta_{dp_3}\right)=180^\circ$$

$$\angle(7-2j)-\left(\angle(-3-2j)+\angle(-4j)+\theta_{dp_3}\right)=180^\circ$$

$$\theta_{dp_2}=-180^\circ-15.9^\circ-(-146.3^\circ)-(-90^\circ)=400.4^\circ=40.4^\circ$$

The angle of arrival at the zero -10 $(\text{i.e., } z_1 = 10)$

$$\theta_{az_1}-\left(\angle(-10+0)+\angle(-10+3-2j)+\angle(-10+3+2j)\right)=180^\circ$$

$$\theta_{az_1}-\left(\angle(-10)+\angle(-7-2j)+\angle(-7+2j)\right)=180^\circ$$

$$\theta_{az_1}=180^\circ+(180^\circ)+(-164.1^\circ)+164.1^\circ=360^\circ=0^\circ$$

With all the calculations, one might be able to conclude that there is no Breakaway Point for the given system. One might pursuit evaluating the Breakaway Point anyway and should conclude the same thing.

$$K = -\frac{1}{(s+10) \Big/ s(s^2+6s+13)} = -\frac{s(s^2+6s+13)}{(s+10)} = -(s^3+6s^2+13s)(s+10)^{-1}$$

$$\frac{dK}{ds} = \frac{d\left((s^3+6s^2+13s)(s+10)^{-1}\right)}{ds} = (3s^2+12s+13)(s+10)^{-1} + (-1)(s+10)^{-2}(s^3+6s^2+13s) = 0$$

$$\frac{(3s^2+12s+13)}{(s+10)} - \frac{s^3+6s^2+13s}{(s+10)^2} = \frac{(3s^2+12s+13)(s+10) - s^3 - 6s^2 - 13s}{(s+10)^2} = \frac{2s^3+36s^2+120s+130}{(s+10)^2} = 0$$

$$2s^3+36s^2+120s+130 = s^3+18s^2+60s+65 = 0$$

Therefore, $s = -1.9691 \pm 0.8632j$ and $s = -14.0619$. However, there is no Root Locus on the Real Axis at -14.0619. So, no Breakaway Point exists as expected.

The Root Locus for the given system is sketched below using the information obtained from following the calculations provided in the Guidelines.