Kafrelsheikh University
Faculty of Engineering
Electrical Engineering Department
Final Exam, 2016 -2017

2nd Year (Electrical Engineering)
Automatic Control
Time: 180 minutes
Mark: 90
Dr. Abdel-Fattah Heliel

Problem 1: (15 points)

a) What are the advantages and disadvantages of open-loop and closed-loop control systems? **[5 points]**

b) Determine the transfer function C/R for the block diagram shown in Fig. 1 by **signal flow graph (SFG) techniques.** **[10 points]**

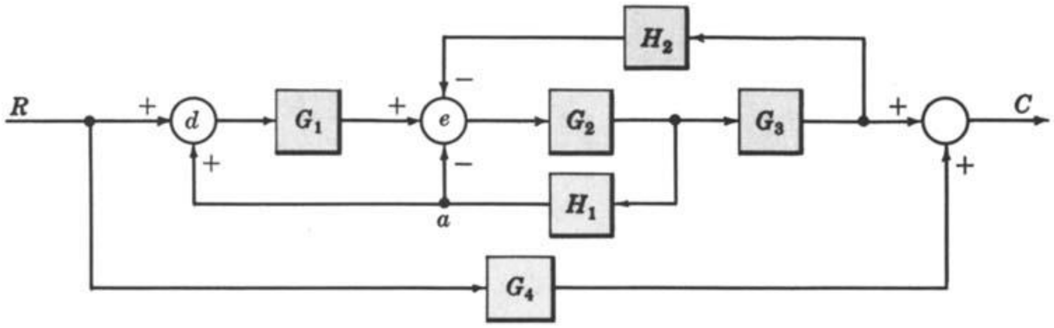
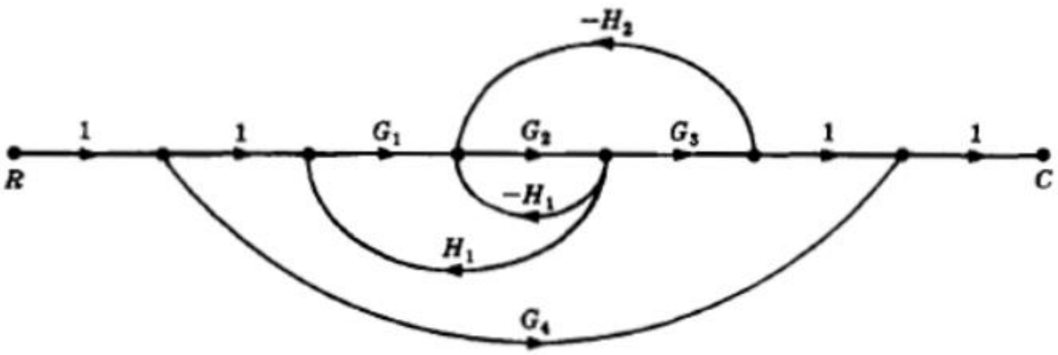


Fig. 1

The signal flow graph, Fig. 8-46, is drawn directly from Fig. 7-44. There are two forward paths. The path gains are $P_1 = G_1G_2G_3$ and $P_2 = G_4$. The three feedback loop gains are $P_{11} = -G_2H_1$, $P_{21} = G_1G_2H_1$, and $P_{31} = -G_2G_3H_2$. No loops are nontouching. Hence $\Delta = 1 - (P_{11} + P_{21} + P_{31})$. Also, $\Delta_1 = 1$; and since no loops touch the nodes of P_2, $\Delta_2 = \Delta$. Thus

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_1 - G_1G_2G_4H_1 + G_2G_3G_4H_2}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$



Problem 2: (30 points)

a) Obtain the transfer functions X1(s) / U (s) and X2(s)/U(s) of the mechanical system shown in Fig. 2. [10 points]

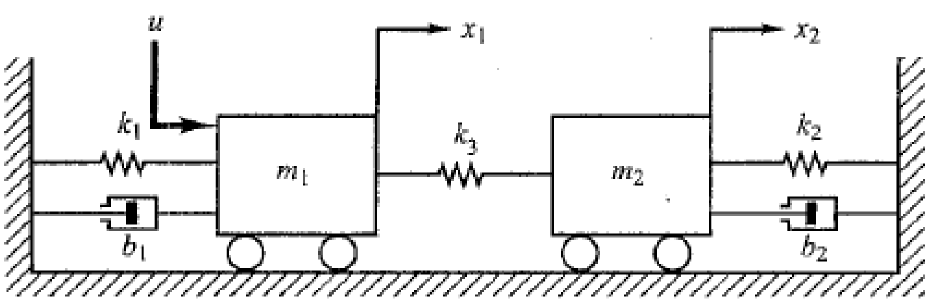


Fig. 2

$$m_1\ddot{x}_1 = -k_1 x_1 - b_1\dot{x}_1 - k_3(x_1 - x_2) + u$$
$$m_2\ddot{x}_2 = -k_2 x_2 - b_2\dot{x}_2 - k_3(x_2 - x_1)$$

Rewriting, we have

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1 x_1 + k_3 x_1 = k_3 x_2 + u$$
$$m_2\ddot{x}_2 + b_2\dot{x}_2 + k_2 x_2 + k_3 x_2 = k_3 x_1$$

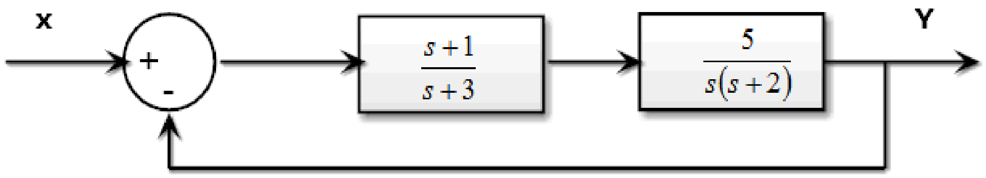
Assuming the zero initial condition and taking the Laplace transforms of these two equations, we obtain

$$(m_1 s^2 + b_1 s + k_1 + k_3)X_1(s) = k_3 X_2(s) + U(s) \qquad (1)$$
$$(m_2 s^2 + b_2 s + k_2 + k_3)X_2(s) = k_3 X_1(s) \qquad (2)$$

By eliminating $X_2(s)$ from Equations (1) and (2), we get

$$(m_1 s^2 + b_1 s + k_1 + k_3)X_1(s) = \frac{k_3^2}{m_2 s^2 + b_2 s + k_2 + k_3}X_1(s) + U(s)$$

b) Find the steady-state error for the system given below for (a) a unit-step input, (b) a unit-ramp input, and (c) a unit-parabolic input.. **(10 points)**



$$\frac{E(s)}{X(s)} = \frac{1}{1+GH} = \frac{1}{1+\frac{5(s+1)}{s(s+2)(s+3)}}$$

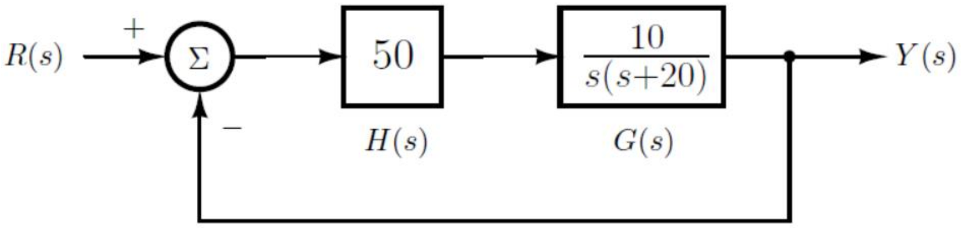
$$E(s) = \frac{s(s+2)(s+3)}{s(s+2)(s+3)+5(s+1)}X(s) \qquad e_{ss} = \lim_{s\to 0} s\,E(s)$$

(a) for $x(t) = u(t)$, $X(s) = \frac{1}{s}$ $\quad \therefore\ e_{ss} = \lim_{s\to 0} s\left[\frac{s(s+2)(s+3)}{s(s+2)(s+3)+5(s+1)}\cdot\frac{1}{s}\right] = 0$

(b) for $x(t) = t\,u(t)$, $X(s) = \frac{1}{s^2}$ $\quad \therefore\ e_{ss} = \lim_{s\to 0} s\left[\frac{s(s+2)(s+3)}{s(s+2)(s+3)+5(s+1)}\cdot\frac{1}{s^2}\right] = \frac{6}{5}$

(c) for $x(t) = \frac{1}{2}t^2 u(t)$, $X(s) = \frac{1}{s^3}$ $\quad \therefore\ e_{ss} = \lim_{s\to 0} s\left[\frac{s(s+2)(s+3)}{s(s+2)(s+3)+5(s+1)}\cdot\frac{1}{s^3}\right] = \infty$

c) You are given the block diagram of a control system shown below. Find the unit-impulse response of this system. In other words, find $y(t)$ when $r(t) = \delta(t)$. **(10 points)**



$$\frac{Y(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{50\,\frac{10}{s(s+20)}}{1+50\,\frac{10}{s(s+20)}} = \frac{500}{s^2+20s+500}$$

if $r(t)=\delta(t)$, $R(s)=1$; therefore,

$$Y(s) = \frac{500}{s^2+20s+500}$$

$$= \frac{500}{(s+10)^2+400}$$

$$= \frac{500}{(s+10)^2+(20)^2}$$

Recall:

$$\left\{\begin{array}{l} \Rightarrow s^2+as+b = \left(s+\frac{a}{2}\right)^2 + b-\left(\frac{a}{2}\right)^2 \\ \frac{\omega}{(s+\alpha)^2+\omega^2} \xleftrightarrow{\mathcal{L}} e^{-\alpha t}\sin(\omega t)\,u(t) \\ \frac{s+\alpha}{(s+\alpha)^2+\omega^2} \xleftrightarrow{\mathcal{L}} e^{-\alpha t}\cos(\omega t)\,u(t) \end{array}\right.$$

$$Y(s) = 25\,\frac{20}{(s+10)^2+(20)^2} = A\,\frac{\omega}{(s+\alpha)^2+\omega^2}$$

where
$A=25$
$\omega=20$
$\alpha=10$

$$y(t) = A\,e^{-\alpha t}\sin(\omega t)\,u(t)$$

$$y(t) = 25\,e^{-10t}\sin(20t)\,u(t)$$

Problem 3: (20 points)

a) What is PID control? Explain the control effects by P, I, and D, respectively. **[5 points]**

b) Determine the range of values of K such that the following closed-loop system is stable. **[15 points]**

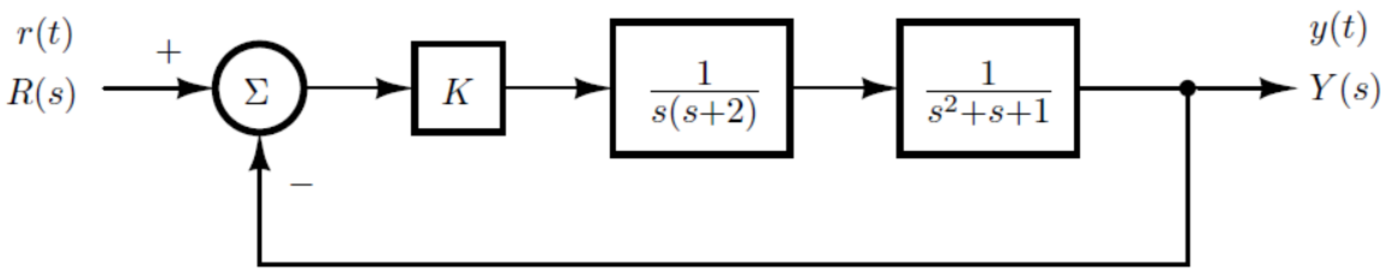


Figure 3

$$\frac{Y}{R} = \frac{K\,\frac{1}{s(s+2)(s^2+s+1)}}{1+K\,\frac{1}{s(s+2)(s^2+s+1)}} = \frac{K}{\underbrace{s(s+2)}_{(s^2+2s)}(s^2+s+1)+K}$$

characteristic equation $\underbrace{(s^2+2s)(s^2+s+1)}_{s^4+s^3+s^2+2s^3+2s^2+2s}+K = 0$

$$s^4+3s^3+3s^2+2s+K=0$$

Routh Table

each mistake -5

s^4	1	3	K
s^3	3	2	
s^2	$\frac{(3)(3)-(2)(1)}{3} \to \frac{7}{3}$	$\frac{(3)(K)-1(0)}{3} \to K$	
s^1	$\frac{\frac{7}{3}(2)-3K}{7/3}$		
s^0	K		

s^4	1	3	K
s^3	3	2	
s^2	$\frac{7}{3}$	K	
s^1	$\frac{14/3-3K}{7/3}$		
s^0	K		

For the closed-loop system to be stable, there should not be any poles on the RHS of s-plane.

$\Rightarrow$ no sign changes in the first column

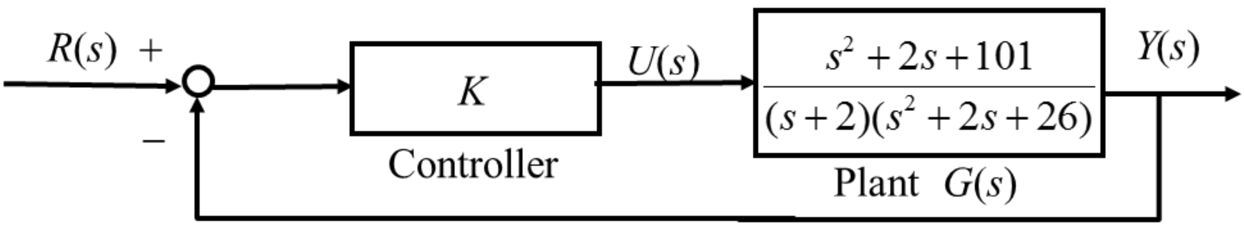
both need to be satisfied
$$\begin{cases} ① \ \frac{14}{3}-3K>0 \Rightarrow 3K<\frac{14}{3} \Rightarrow K<\frac{14}{9} \\ ② \ K>0 \end{cases}$$

$$\therefore \boxed{0<K<14/9}$$

Problem 4: (20 points)

a) What is root locus? Why it is used? And summarize the steps for constructing the root loci? **(5 points)**

b) A feedback control system is proposed. The corresponding block diagram is: **(15 points)**



Sketch the root locus of the closed-loop poles as the controller gain K varies from 0 to ∞.

Step 1: Formulate the (closed-loop) characteristic equation into the standard form for sketching root locus:

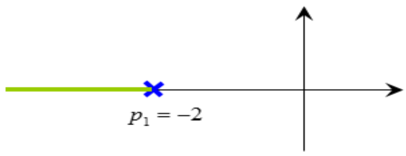
$$1+K\frac{\overbrace{s^2+2s+101}^{N(s)}}{\underbrace{(s+2)(s^2+2s+26)}_{D(s)}}=0$$

Step 2: Find the open-loop zeros, z_i , and the open-loop poles, p_i :

$\boxed{\text{open-loop zeros}}$ $\quad s^2+2s+101=(s+1)^2+100=0,\ z_{1,2}=-1\pm 10j$

$\boxed{\text{open-loop poles}}$ $\quad (s+2)((s+1)^2+25)=0,\ p_1=-2,\ p_{2,3}=-1\pm 5j$

Step 3: Determine the real axis segments that are to be included in the root locus by applying Rule 4.



Step 4: Determine the number of asymptotes and the corresponding intersection σ_0 and angles θ_k by applying Rules 2 and 5.

$N_p - N_z = 1$ $\quad$ One asymptote $\quad$ $\theta_k = (2k+1)\times 180^\circ = 180^\circ$

Step 5: (If necessary) Determine the break-away and break-in points using Rule 6.

Step 6: (If necessary) Determine the departure and arrival angles using Rule 7.

$z_1=-1+10j$ $\quad \theta_{z_1}+90^o-\tan^{-1}(10)-90^o-90^o=180^o$

$\theta_{z_1}=354^o=-6^o$

$z_2=-1-10j$ $\quad \theta_{z_2}=6^o$

$p_1=-2$ $\quad \theta_{p_1}=180^o$

$p_2=-1+5j$ $\quad \theta_{p_2}=11^o$

$p_3=-1-5j$ $\quad \theta_{p_2}=-11^o$

Step 7: (If necessary) Determine the imaginary axis crossings using Rule 8.

$$(s+2)(s^2+2s+26)+K(s^2+2s+101)=0$$

$$\Leftrightarrow s^3+(4+K)s^2+(30+2K)s+(52+101K)=0$$

$$\stackrel{s=j\omega}{\Rightarrow}\left[(52+101K)-(4+K)\omega^2\right]+\left[(30+2K)-\omega^2\right]\omega j=0$$

$$\begin{cases}(52+101K)-(4+K)\omega^2=0\\ \left[(30+2K)-\omega^2\right]\omega=0\end{cases}\Rightarrow\begin{cases}\omega_1=0\\ K_1=-\dfrac{52}{101}\end{cases},\begin{cases}\omega_2=9.5\\ K_2=30.4\end{cases},\begin{cases}\omega_3=5.7\\ K_3=1.1\end{cases}$$

Step 8: Use the information from Steps 1-7 and Rules 1-3 to sketch the

Stability condition

$0 < K < 1.1$

or

$K > 30.4$

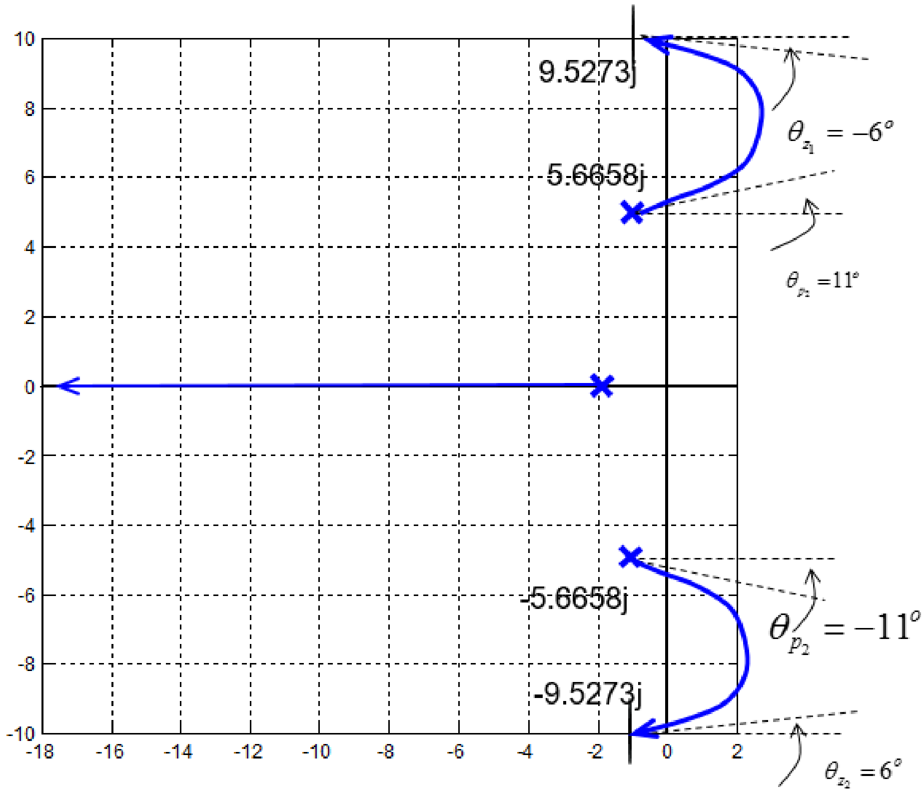
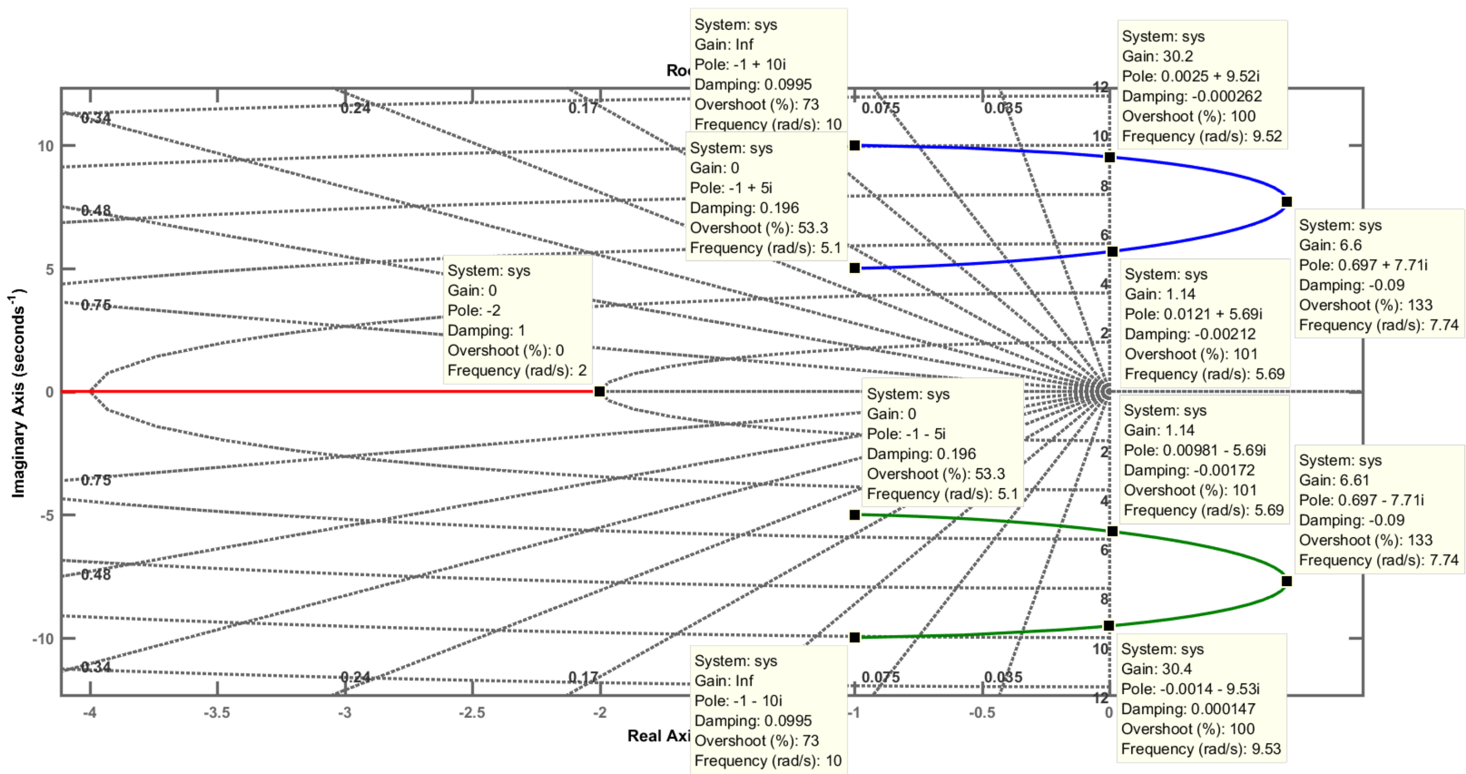


Fig. 3



مع تمنياتي لكم بالتوفيق والنجاح،،،