Kafrelsheikh University
Faculty of Engineering
Electrical Engineering Department
Answer Final Exam, 2017-2018
The exam covered course ILOS specifications

2nd Year (Electrical Engineering)
Automatic Control (1)
Time: 180 minutes
Mark: 90
Dr. Abdel-Fattah Heliel

Answer all the following questions:

Problem 1: (25 Marks)

a) **What are the advantages and disadvantages of open-loop and closed-loop control systems? [7 Marks]** [ILOS: a1]

Advantages open-loop:

- Simple construction and ease of maintenance.
- There is no stability concern.
- Convenient when output is hard to measure or measuring the output precisely is economically not feasible. (For example, in the washer system, it would be quite expensive to provide a device to measure the quality of the washer's output, cleanliness of the clothes).

Disadvantages open-loop::

- Disturbances and changes in calibration cause errors, and the output may be different from what is desired.
- Recalibration is necessary from time to time.

The effect of Feedback (closed-loop control systems)

- Reduce the error between the actual and desired value.
- Change the stability of the system.
- Change the overall the system gain.
- Change the sensitivity of the system gain.
- Change the bandwidth of the system.
- Reduce the effect of external disturbances and noise.
- Reduce the effect of variations of system parameters.

b) **Obtain the transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the mechanical system shown in Fig. 1. [18 Marks]** [ILOS: a13]

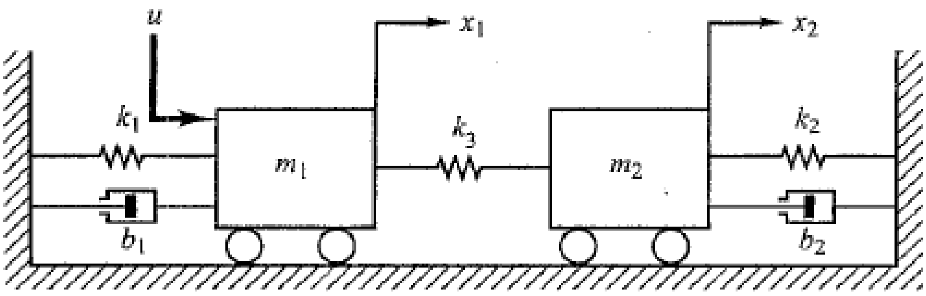


Fig. 1

$$m_1 \ddot{x}_1 = -k_1 x_1 - b_1 \dot{x}_1 - k_3 (x_1 - x_2) + u$$
$$m_2 \ddot{x}_2 = -k_2 x_2 - b_2 \dot{x}_2 - k_3 (x_2 - x_1)$$

Rewriting, we have

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_3 x_1 = k_3 x_2 + u$$
$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_3 x_2 = k_3 x_1$$

Assuming the zero initial condition and taking the Laplace transforms of these two equations, we obtsin

$$(m_1 s^2 + b_1 s + k_1 + k_3) X_1(s) = k_3 X_2(s) + U(s) \quad (1)$$

$$(m_2 s^2 + b_2 s + k_2 + k_3) X_2(s) = k_3 X_1(s) \quad (2)$$

By eliminating $X_2(s)$ from Equations (1) and (2), we get

$$(m_1 s^2 + b_1 s + k_1 + k_3) X_1(s) = \frac{k_3^2}{m_2 s^2 + b_2 s + k_2 + k_3} X_1(s) + U(s)$$

Hence

$$\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + b_2 s + k_2 + k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}$$

From Equation (2), we obtain

$$\frac{X_2(s)}{X_1(s)} = \frac{k_3}{m_2 s^2 + b_2 s + k_2 + k_3}$$

Hence

$$\frac{X_2(s)}{U(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = \frac{k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}$$

Problem 2: (20 Marks)

a) For the control system shown below: [ILOS: b7]

i. Draw the corresponding Signal Flow Graph. (3 Marks)

ii. Determine the transfer function C(s)/R(s) using Signal Flow Graph. (3 Marks)

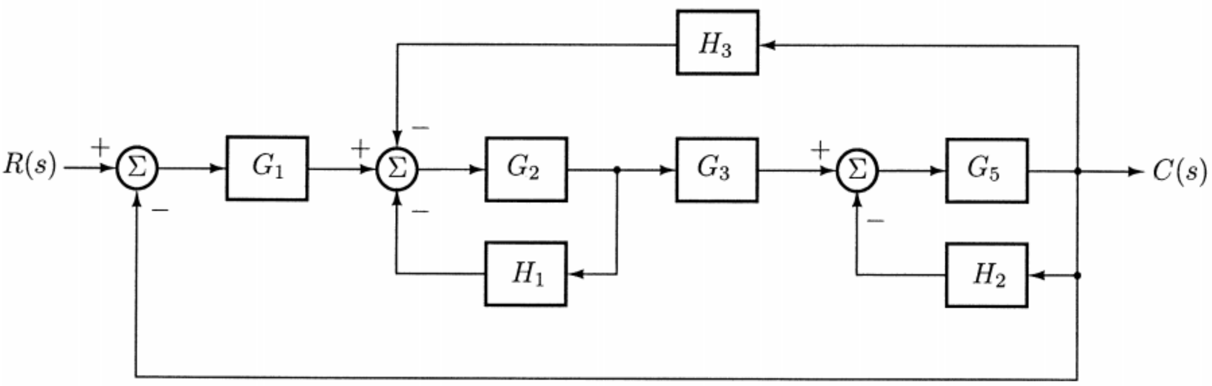
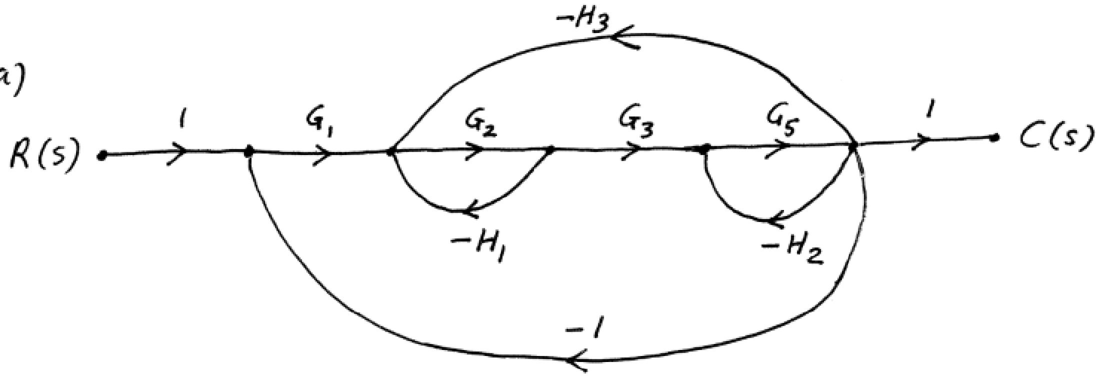


Fig. 2

Solution:

(a)



(b) <u>Loop gains</u>

$LG_1 = -G_2H_1$

$LG_2 = -G_5H_2$

$LG_3 = -G_1G_2G_3G_5$

$LG_4 = -G_2G_3G_5H_3$

<u>Forward gains</u>

$M_1 = G_1G_2G_3G_5\ ,\quad \Delta_1 = 1$

$$\frac{C(s)}{R(s)} = \frac{M_1\Delta_1}{1 - \sum_{i=1}^{4} LG_i + \underbrace{(LG_1)(LG_2)}_{\text{non touching loops}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5}{1 + G_2H_1 + G_5H_2 + G_1G_2G_3G_5 + G_2G_3G_5H_3 + G_2G_5H_1H_2}}$$

b) **Check whether the following system given by its characteristic equation is stable or not and shows the location of roots on the s-plane. [14 Marks] [ILOS: a1.2, b5.1,]**

$$q(s) = S^5 + 10S^4 + 45S^3 + 90S^2 + 164S + 200 = 0$$

Solution

$a(s) = s^5 + 10s^4 + 45s^3 + 90s^2 + 164s + 200$

All coefficients are positive $(a_i > 0)$

$\rightarrow$ necessary condition passed. ✓

Row	5	s^5	1	45	164	0
	4	s^4	10	90	200	0
	3	s^3	36	144	0	
	2	s^2	50	200	0	
	1	s^1	0	0	$\leftarrow$ Special case #2	
New row (dp/ds)	1	s^1	100	0		
	0	s^0	200			

Aux. poly:

$p(s): 50s^2 + 200 = 0$

$s^2 = -4$

$\boxed{s = \pm j2}$

No sign changes

$\rightarrow$ no RHP poles.

However, there is a pair of non-repeated imaginary poles at $\pm j2$.

$\Rightarrow$ System is neutrally stable.

Or, with Special Case #1

			(+)		
Row	2	s^2	50	200	0
Row	1	s^1	$0 \to \varepsilon > 0$	0	
Row	0	s^0	(+) 200		

$(+) \Rightarrow (\varepsilon) \Rightarrow (+)$

Not a sign change, pair of imaginary roots.

No sign change in 1^{st} Column of Routh array.

$\to$ No RHP poles.

$\Rightarrow$ Neutrally stable system.

(due due roots on Imaginary axis)

Problem 3: [25 Marks]

a) **Obtain analytically the rise time, peak time, maximum overshoot, and settling time in the unit-step response of a closed-loop system given by: $\frac{C(s)}{R(s)} = \frac{36}{s^2+2s+36}$, and show locations of poles and zeros on the pole-zero plot. [10 Marks]. [ILOS: b2.1]**

Solution:

$$\frac{C(s)}{R(s)} = \frac{36}{s^2+2s+36} = \frac{36}{(s+1)^2+(\sqrt{35})^2}$$

we find that $\omega_n = 6$, $\zeta = \frac{1}{6}$, and $\omega_d = \sqrt{35}$.

Rese time:

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where

$$\beta = \tan^{-1}\frac{\omega_d}{\zeta\omega_n} = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{\frac{35}{36}}}{\frac{1}{6}}$$

$$= \tan^{-1} 5.9161 = 1.4034 \text{ rad}$$

Hence

$$t_r = \frac{3.1416 - 1.4034}{\sqrt{35}} = 0.2938 \text{ sec}$$

Peak time:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.1416}{\sqrt{35}} = 0.5310 \text{ sec}$$

Maximum overshoot:

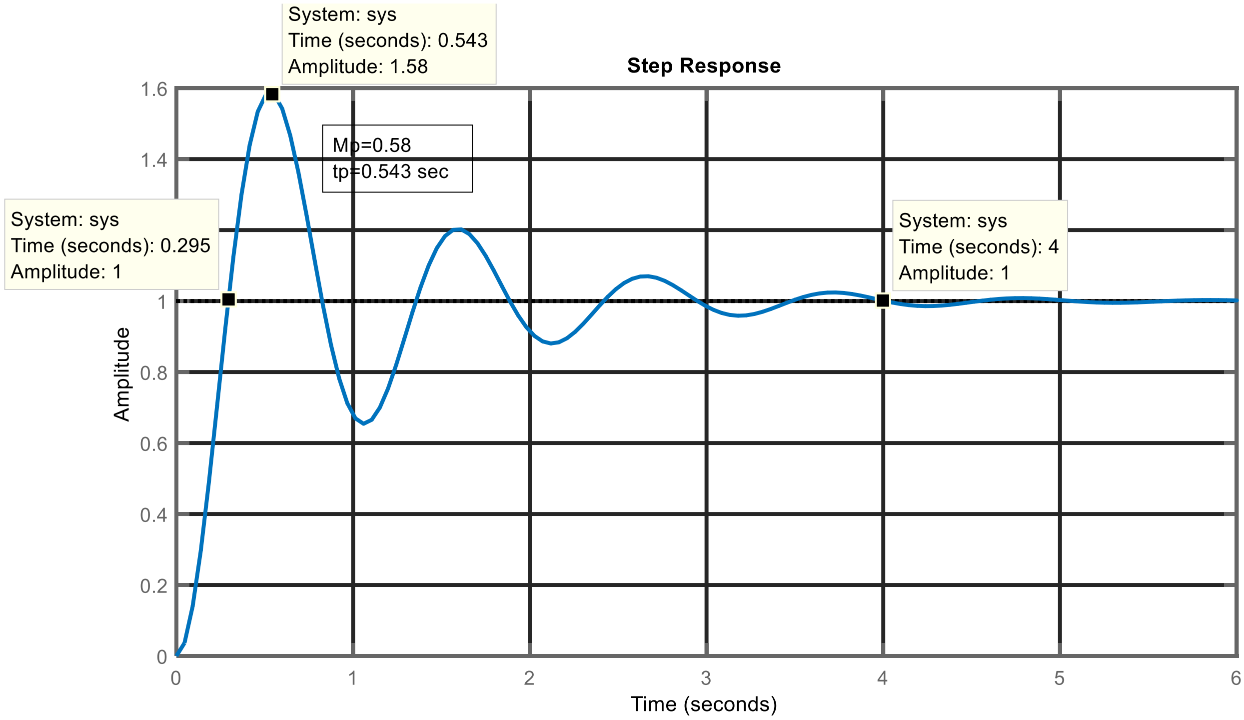
$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = e^{-\frac{\pi}{\sqrt{35}}} = e^{-0.5310}$$

$$= 0.5880$$

Settling time (2% criterion):

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{1}{6}\times 6} = 4 \text{ sec}$$

b) **Draw output response $C(t)$ for the unit-step response. [7 Marks].** **[ILOS: b1.1, c7.1]**



c) **Consider the following electrical system with the applied voltage V_i as the input and V_o as the output. [ILOS: a5.1]**

1. **Write the loop equations. (2 Marks)**
2. **Write the node equations. (2 Marks)**
3. **Find the transfer function of the system. (2 Marks)**
4. **What is the order of this system? Notice that $V_1 = V_o$ (2 Marks)**

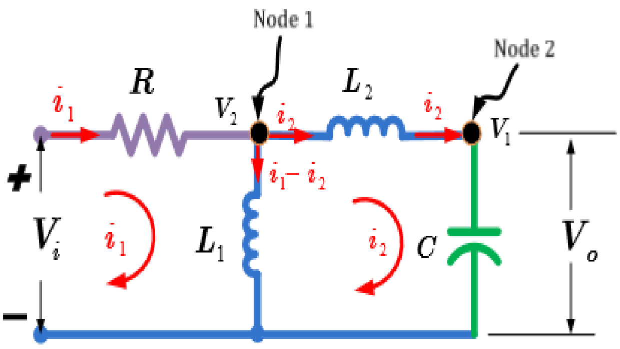


Fig. 3

Solution

1. **Loop Equations**:

Loop 1 (LHS)

$$R\,i_1 + L_1\frac{d}{dt}\left(i_1 - i_2\right) = v_i\,(t)$$

or

$$R\,i_1 + L_1\left(i_1 - i_2\right)s = V_i\,(s) \tag{1}$$

Loop 2 (RHS)

$$L_1\frac{d}{dt}\left(i_1 - i_2\right) + L_2\frac{d}{dt}\left(i_2 - i_1\right) + \frac{1}{C}\int i_2 dt = 0$$

or

$$L_1\left(i_1 - i_2\right)s + L_2 i_2 s + \frac{1}{Cs}i_2 = 0 \tag{2}$$

Outer loop

$$\frac{1}{C}\int i_2 dt = V_o$$

2. **Node Equations**:

Node 1

$$i_1 + i_2 + \left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad \frac{V_2 - V_i}{R} + \frac{V_2 - V_1}{L_2 s} + \frac{V_2}{L_1 s} = 0 \tag{3}$$

Node 2

$$\frac{V_1 - V_2}{L_2 s} + \frac{V_1}{\left(1/Cs\right)} = 0 \tag{4}$$

3. **Transfer Function**:

Remember we are trying to find $\frac{V_0\,(s)}{V_i\,(s)} = \frac{V_1\,(s)}{V_i\,(s)}$? Because $V_1(s)= V_o(s)$

From Eq. (4), $\Rightarrow \frac{V_1}{L_2 s} + \frac{V_1}{\left(1/Cs\right)} = \frac{V_2}{L_2 s} \qquad \Rightarrow V_1\left(\frac{1}{L_2 s} + Cs\right) = \frac{V_2}{L_2 s}$

$$\Rightarrow V_2 = V_1\left(\frac{1}{L_2 s} + Cs\right)L_2 s = V_1\left(1 + L_2 Cs^2\right) \tag{5}$$

Substitution of Eq. (5) into Eq. (3) gives

$$\frac{\left(1 + L_2Cs^2\right)V_1}{R} - \frac{V_i}{R} + \frac{\left(1 + L_2Cs^2\right)V_1}{L_2 s} - \frac{V_1}{L_2 s} + \frac{\left(1 + L_2Cs^2\right)V_1}{L_1 s} = 0$$

$$\left(\frac{\left(1 + L_2Cs^2\right)_1}{R} + \frac{\left(1 + L_2Cs^2\right)}{L_2 s} - \frac{1}{L_2 s} + \frac{\left(1 + L_2Cs^2\right)}{L_1 s}\right)V_1 = \frac{V_i}{R}$$

$$\left(\frac{L_1L_2 s\left(1 + L_2Cs^2\right) + RL_1\left(1 + L_2Cs^2\right) - RL_1 + RL_2\left(1 + L_2Cs^2\right)}{RL_1L_2 s}\right)V_1 = \frac{V_i}{R}$$

Therefore,

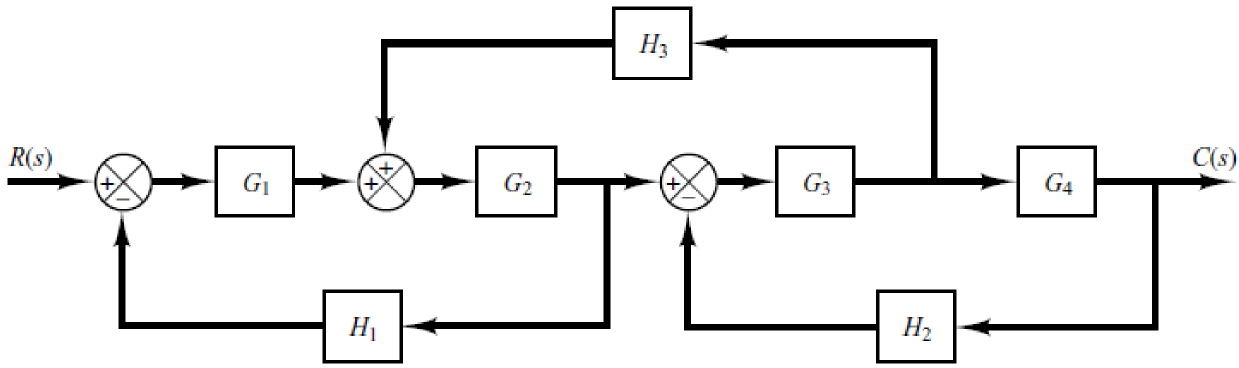
$$\frac{V_1\,(s)}{V_i\,(s)} = \frac{L_1L_2 s}{L_1L_2 s\left(1 + L_2Cs^2\right) + RL_1\left(1 + L_2Cs^2\right) - RL_1 + RL_2\left(1 + L_2Cs^2\right)}$$

$$\frac{V_1\,(s)}{V_i\,(s)} = \frac{L_1 s}{L_1L_2Cs^3 + RC\left(L_1 + L_2\right)s^2 + L_1 s + R}$$

4. The above TF represents **a third order system**.

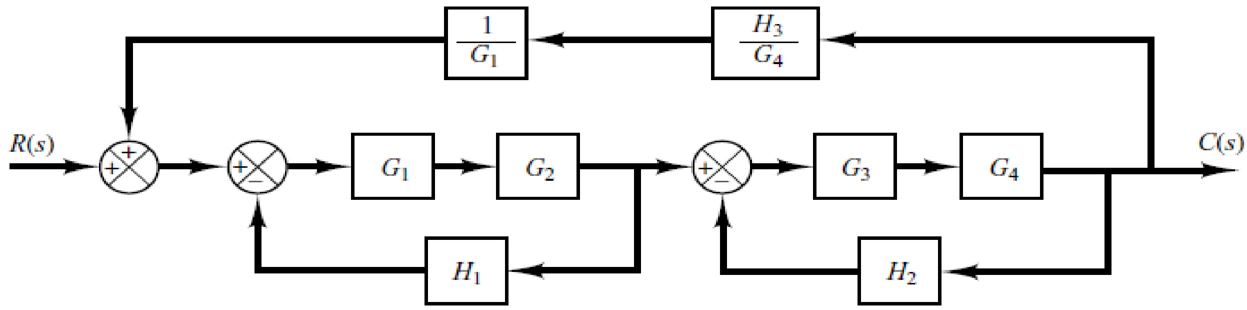
Problem 4: (20 Marks)

a) **Simplify the block diagram then obtain the close-loop transfer function $C(S)/R(S)$. [5 Marks]** **[ILOS: b7.1]**

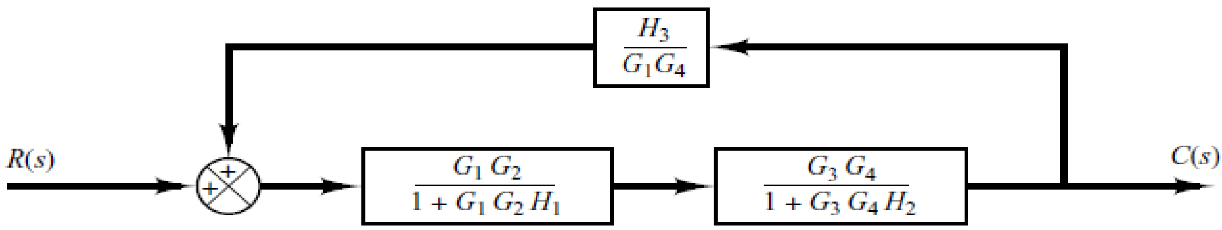


Solution:

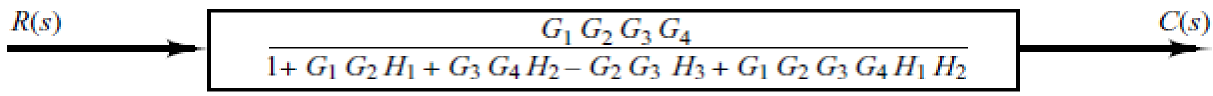
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



By simplifying each loop, the block diagram can be modified as



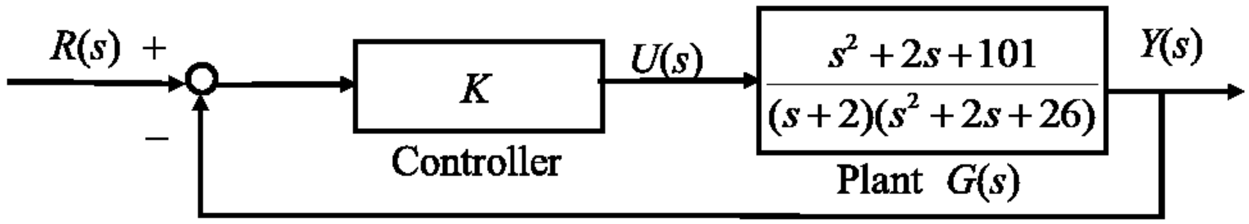
Further simplification results in



the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

b) A feedback control system is proposed. The corresponding block diagram is: (15 Marks) **[ILOS: a4.1, b5.1]**



Sketch the root locus of the closed-loop poles as the controller gain *K* varies from 0 to ∞.

Solution :

Find closed-loop characteristic equation:

$$1+K\frac{s^2+2s+101}{(s+2)\left(s^2+2s+26\right)}=0$$

Step 1: Formulate the (closed-loop) characteristic equation into the standard form for sketching root locus:

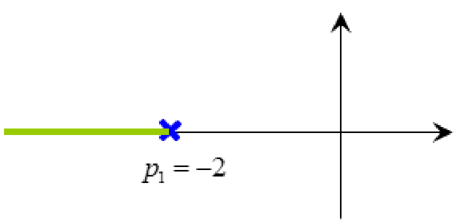
$$1+K\frac{\underbrace{s^2+2s+101}_{N(s)}}{\underbrace{(s+2)\left(s^2+2s+26\right)}_{D(s)}}=0$$

Step 2: Find the open-loop zeros, z_i, and the open-loop poles, p_i :

open-loop zeros $\quad s^2+2s+101=(s+1)^2+100=0,\ z_{1,2}=-1\pm 10j$

open-loop poles $\quad (s+2)\left((s+1)^2+25\right)=0,\ p_1=-2,\ p_{2,3}=-1\pm 5j$

Step 3: Determine the real axis segments that are to be included in the root locus by applying Rule 4.



Step 4: Determine the number of asymptotes and the corresponding intersection σ_0 and angles θ_k by applying Rules 2 and 5.

$N_p - N_z = 1$ One asymptote $\theta_k = (2k+1)\times 180^\circ = 180^\circ$

Step 5: (If necessary) Determine the break-away and break-in points using Rule 6.

Step 6: (If necessary) Determine the departure and arrival angles using Rule 7.

$z_1 = -1+10j$ $\quad \theta_{z_1} + 90^o - \tan^{-1}(10) - 90^o - 90^o = 180^o$

$\theta_{z_1} = 354^o = -6^o$

$z_2 = -1-10j$ $\quad \theta_{z_2} = 6^o$

$p_1 = -2$ $\quad \theta_{p_1} = 180^o$

$p_2 = -1+5j$ $\quad \theta_{p_2} = 11^o$

$p_3 = -1-5j$ $\quad \theta_{p_2} = -11^o$

Step 7: (If necessary) Determine the imaginary axis crossings using Rule 8.

$$(s+2)(s^2+2s+26)+K(s^2+2s+101)=0$$

$$\Leftrightarrow s^3+(4+K)s^2+(30+2K)s+(52+101K)=0$$

$$\stackrel{s=j\omega}{\Rightarrow} [(52+101K)-(4+K)\omega^2]+[(30+2K)-\omega^2]\omega j=0$$

$$\begin{cases}(52+101K)-(4+K)\omega^2=0\\ [(30+2K)-\omega^2]\omega=0\end{cases} \Rightarrow \begin{cases}\omega_1=0\\ K_1=-\dfrac{52}{101}\end{cases}, \begin{cases}\omega_2=9.5\\ K_2=30.4\end{cases}, \begin{cases}\omega_3=5.7\\ K_3=1.1\end{cases}$$

Step 8: Use the information from Steps 1-7 and Rules 1-3 to sketch the root locus.

Stability condition

$0 < K < 1.1$

or

$K > 30.4$