

**Answer as much as you can:****Question 3: (25 Marks)**

- (a) Fit the curve $y = \frac{1}{a+b \cos \theta}$ to the following data, and find the root mean square error.

i	1	2	3
θ_i	30	45	60
y_i	0.225	0.27	0.32

- (b) Calculate $\sqrt{120}$ by using Lagrange interpolation.

- (c) By using Bisection method, find the root of $3x - e^{-x} = 0$ in the interval $[0.25, 0.27]$, (correct to 3 decimal places)

Question 4: (25 Marks)

- (a) Use Euler's method to approximate the solution for the initial value problem $y' = x^2 + y$, $0 \leq x \leq 0.1$, $y(0) = 1$ with $h = 0.02$.

- (b) Find Eigen value and Eigen vectors, then solve the given initial value problem

$$X' = \begin{bmatrix} 1 & 9 \\ 1 & 1 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

- (c) Evaluate:

$$(i) \int_{-\infty}^{\infty} (e^{5x} - e^x) dx$$

$$(ii) \int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$$

..... Good luck: Dr. Eng./ Manal El-Sayed

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Answer as much as you can:

Question 1: (25 Marks)

- a) Deduce Parseval's identity for Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi x}{T} + b_n \sin \frac{2n\pi x}{T} \right)$$

of the periodic function $f(x)$ with period T .

- b) Prove that Fourier series of x^2 on the interval $[-\pi, \pi]$ is given by:

$$x^2 = \frac{\pi^2}{3} - 4 \left(\cos x - \frac{1}{2^2} \cos 2x + \frac{1}{3^2} \cos 3x - \frac{1}{4^2} \cos 4x + \dots \right)$$

and then find the sum of the following series :

i) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

ii) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

iii) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

Question 2: (25 Marks)

- a) Reduce the PDE $u_{tt} = c^2 u_{xx}$ to its canonical form and find its general solution using the canonical transformation $\eta = x + ct$, $\theta = x - ct$.

- b) Using the result obtained in part (a), deduce D'Alembert solution of the following wave equation:

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty$$

with initial deflection $f(x)$ and initial velocity $g(x)$ and then solve the PDE:

$$4u_{tt} = u_{xx}, \quad -\infty < x < \infty$$

with initial deflection $f(x) = \cos x$ and initial velocity $g(x) = \cos 3x$.

- c) Solve the one dimensional heat equation:

$$u_t = u_{xx}$$

in a rod of length $\ell = \pi$ when the ends are at zero temperature ($u(0, t) = u(\pi, t) = 0$)

and the initial temperature is given by:

$$f(x) = \begin{cases} x & 0 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$$

..... Good luck: Dr. Eng./ Samah El-Kholy

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