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2016-2017

Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: 1st year Civl. Eng.

Subject: Engineering Mathematics (2)

Date: /1/2017
Time allowed: 3 hours
Full mark: 100
Final Term Exam: 1page

Question 1: (20 Marks)

1. Discuss the continuity of the function: $f(x,y)=\begin{cases} \dfrac{x^2+y^2}{xy}, & (x,y)\neq(0,0) \\ 0, & (x,y)\neq(0,0) \end{cases}$.

2. Let $w=f(x,y)$, $u^2-v=3x+y$ and $u-2v^2=x-2y$ then find $\dfrac{\partial w}{\partial u}$.

3. Expand in Maclurin series the function $f(x,y)=e^{2x+3y}$.

4. Discuss the maximum and the minimum of $f(x,y)=x^2+2y^2+6x+1$.

Question 2: (20 Marks)

1. Prove that if $z=f(x,y)$ is homogeneous of degree k, then $xz_x+yz_y=kz$.

2. If $z=\sec^{-1}\dfrac{x^3+y^3+xy^2}{x+y}$ then find the value of xz_x+yz_y.

3. Verify Green's theorem for the integral $I=\oint_C (x+2y)dx+(y+3x)dy$

when C is the ellipse $4x^2+9y^2=36$.

Question 3: (20 Marks)

1. Prove that the integrating factor for the differential equation $y'+p(x)y=q(x)$ is $\mu=\exp(\int p(x)dx)$ and then deduce that the general solution is $y\mu=\int \mu q(x)dx+c$.

2. Solve the D. Eqs. :

A) $y'=\sqrt{1-(\frac{y}{x})^2}+(\frac{y}{x})$ B) $y\sin(xy)dx+(1+x\sin(xy))dy=0$

3. Prove that $\mu(x,y)=x+y$ is an integrating factor for $(3xy+y^2)dx+(3xy+x^2)dy=0$ and solve this equation.

4. Find the orthogonal trajectories of $r=c\sin\theta$.

Question 4: (20 Marks)

Solve the D. Eqs. : 1) $D^2(D^4-16)y=e^x+\sin x$. 2) $(D^3+3D^2-D-3)y=2+\cos x$.

3) $y''+9y=\operatorname{cosec}3x$. 4) $(x^2D^2-9xD+25)y=\ln x$.

Question 5: (20 Marks)

1. Evaluate A) $L(\int_0^t e^{2u}\sin 5u\,\sin u\,du)$. B) $L(t^2e^{2t}u(t-1))$.

C) $L^{-1}(\ln\dfrac{s+3}{s-2})$. D) $L^{-1}(e^{-2s}(\dfrac{6-s}{s^2-4s+3}))$.

2. Solve the following D.E. using Laplace transform $y''+y'-4y=2e^t$, $y(0)=y'(0)=0$.

With my best wishes
Dr. Samah El-kholy

أوى ترم أول

كورباد

رياضيات هندسية (٢ - م)

11 / 2017

د. سماح

Q.1 (1) $f(x,y) = \begin{cases} \dfrac{x^2+y^2}{xy} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

(1) $f(0,0) = 0$

(2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy} = \dfrac{0}{0}$

let $y = mx$

$$\lim_{x\to 0} \frac{x^2+m^2x^2}{mx^2} = \frac{1+m^2}{m}$$

(2) The function is not continuous.

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(2) $w = f(x,y)$, $u^2 - v = 3x+y$

$u - 2v^2 = x - 2y$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u}$$

$$\frac{\partial x}{\partial u} = -\frac{J\left(\frac{F,G}{u,y}\right)}{J\left(\frac{F,G}{x,y}\right)} = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix}} = -\frac{4u+1}{-7}$$

$$\frac{\partial y}{\partial u} = -\frac{J\left(\frac{F,G}{u,x}\right)}{J\left(\frac{F,G}{y,x}\right)} = -\frac{2u-3}{7}$$

(3) $f(x,y) = e^{2x+3y}$

$f(x,y) =$	$f(0,0) = 1$
$f_x = 2e^{2x+3y}$	$f_x = 2$
$f_y = 3e^{2x+3y}$	$f_y = 3$
$f_{xx} = 4e^{2x+3y}$	$f_{xx} = 4$
$f_{yy} = 9e^{2x+3y}$	$f_{yy} = 9$
$f_{xy} = 6e^{2x+3y}$	$f_{xy} = 6$

$$f(x,y) = f(0,0) + xf_x + yf_y + \frac{1}{2!}(x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy}) + \cdots$$

$$= 1 + 2x + 3y + \frac{4}{2}x^2 + 6xy + \frac{9}{2}y^2 + \cdots$$

(4) $f(x,y) = x^2 + 2y^2 + 6x + 1$

$f_x = 2x + 6 = 0$ $\qquad$ $f_y = 4y = 0$

$x = -3$ $\qquad$ $y = 0$

$f_{xx} = 2$ $\qquad$ $f_{yy} = 4$

$f_{xy} = 0$

$\Delta = f_{xx}f_{yy} - f_{xy}^2 = 8 > 0$

$f_{xx} > 0$

$\therefore (-3, 0)$ min.

min value is $(9 + 0 + (6 \times -3) + 1) = -8$

$\boxed{Q.2}$ ① proof

② $z = \sec^{-1} \dfrac{x^3+y^3+xy^2}{x+y}$

$z = \sec^{-1} u \Rightarrow u = \sec z$

$u = \dfrac{x^3+y^3+xy^2}{x+y}$

$u \to z \to x, y$

$x u_x + y u_y = 2u$

$u_x = \dfrac{du}{dz} \cdot \dfrac{\partial z}{\partial x}$, $u_y = \dfrac{du}{dz} \dfrac{\partial z}{\partial y}$

$u_x = (\sec z \tan z)\, z_x$, $u_y = (\sec z \tan z)\, z_y$

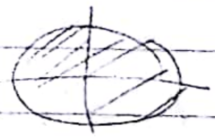
$x z_x + y z_y = \dfrac{2\sec z}{\sec z \tan z} = 2\cot z$

③ $I = \oint_c (x+2y)dx + (y+3x)dy$

$4x^2 + 9y^2 = 36$

$\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$I = \iint (3-2)\, dx\, dy$



$x = 3r\cos\theta$, $y = 2r\sin\theta$

$dx\,dy = 6r\,dr\,d\theta$

$I = \iint 6r\,dr\,d\theta = 3r^2\Big|_0^1 \theta\Big|_0^{2\pi} = 3(2\pi)$
$= 6\pi$

$\boxed{3}$

$$\oint_C (x+2y)\,dx + (y+3x)\,dy$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$x = 3\cos\theta$ $\qquad$ $y = 2.\sin\theta$

$dx = -3\sin\theta\, d\theta$ $\qquad$ $dy = 2\cos\theta\, d\theta$

$$I = \int (3\cos\theta + 4.\sin\theta)(-3\sin\theta\, d\theta)$$

$$+ (2\sin\theta + 9\cos\theta)(2\cos\theta\, d\theta)$$

$$= \int \big(-9\cos\theta\sin\theta - 12.\sin^2\theta + 4\sin\theta\cos\theta$$
$$+ 18\cos^2\theta \big)\, d\theta$$

$$= \int \left(-5\cos\theta\sin\theta - \frac{12}{2}(1-\cos 2\theta) + \frac{18}{2}(1+\cos 2\theta)\right) d\theta$$

$$= \int 3\, d\theta = 3\theta \Big|_0^{2\pi} = 6\pi$$

$\cos 2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

Q3 :- a) اثبات !

B) $y' = \sqrt{1-(\frac{y}{x})^2} + (\frac{y}{x})$

let $\frac{y}{x} = v$, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\cancel{v} + x\frac{dv}{dx} = \sqrt{1-v^2} + \cancel{v}$$

$$\int \frac{dv}{\sqrt{1-v^2}} = \int \frac{dx}{x} = \ln|x| + c$$

$v = \sin a$ $\quad dv = \cos a \, da$

$$\int \frac{\cos a \, da}{\sqrt{1-\sin^2 a}} = \sin^{-1} v = \ln|x| + c$$

b) $y\sin(xy)\,dx + (1 + x\sin xy)\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = xy\cos xy + \sin xy$$

$$y - \cos xy = c$$

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(3) $\mu = x + y$

$$(3xy + y^2)\,dx + (3xy + x^2)\,dy = 0$$

$$(x+y)(3xy + y^2)\,dx + (3xy + x^2)(x+y)\,dy = 0$$

$$\underbrace{(3x^2y + \cancel{xy^2} + 4xy^2 + y^3)}_{M}dx + \underbrace{(4x^2y + x^3 + 3xy^2 + \cancel{xy})}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = (3x^2 + \cancel{2xy} + 8xy + 3y^2)$$

$$\frac{\partial N}{\partial x} = 8xy + 3x^2 + 3y^2$$

$$\int (3x^2y + 4xy^2 + y^3)\,dx + \int (4x^2y + x^3 + 3xy^2)\,dy = c$$

$$x^3y + 2x^2y^2 + y^3x + \overset{*}{[}2x^2y^2 + x^3y + xy^3] = c$$

$$x^3y + 2x^2y^2 + y^3x = c$$

(4) $r = c \sin\theta$

$$\ln r = \ln c + \ln \sin\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} \longrightarrow -r\frac{d\theta}{dr}$$

$$-r\frac{d\theta}{dr} = \frac{\cos\theta}{\sin\theta}$$

$$+\int \frac{dr}{r} = \int \frac{-\sin\theta}{\cos\theta}\,d\theta$$

$$+\ln r = \ln\cos\theta + k$$

$$r = k\cos\theta$$

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Q.4 : $D^2(D^4-16)y = e^{x} + \sin x$

$\lambda^2(\lambda^4-16)=0$

$\lambda^2(\lambda^2-4)(\lambda^2+4)=0$

$\lambda^2(\lambda-2)(\lambda+2)(\lambda^2+4)=0$

$\lambda = 0, 0, -2, 2, \pm 2i$

$y_h = c_1 + c_2 x + c_3 e^{-2x} + c_4 e^{2x} + c_5 \cos 2x + c_6 \sin 2x$

$y_p = \frac{1}{D^2(D^4-16)}[e^x] + \frac{1}{D^2(D^4-16)} \sin x$

$= \frac{1}{D^2(D^2+4)(D+2)(D-2)}(e^x) + \frac{1}{-1(+1-16)} \sin x$

$= \frac{1}{1(5)(3)(D-1)} e^x + \frac{1}{17} \sin x$

$= \frac{1}{-15} e^x \cancel{\frac{1}{(D+1-1)}(1)} + \frac{1}{15} \sin x$

$= \frac{-1}{15} e^x + \frac{1}{17} \sin x$

$y = y_h + y_p$

$\boxed{7}$

② $(D^3+3D^2-D-3)y = 2+\cos x$

1	3	-1	-3	1
	1	2	1	
1	2	1	$\boxed{0}$	

$\lambda^3+3\lambda^2-\lambda-3=0$

$(\lambda+1)(\lambda^2+2\lambda+1)=0$

$(\lambda+1)(\lambda-1)(\lambda+3)=0$

$(\lambda+1)(\lambda+1)(\lambda+1)=0$

$\lambda = -1 \quad , +1 \quad , -3$

$y_h = c_1 e^{-x} + c_2 x e^{+x} + c_3 x e^{-3x}$

$y_p = \dfrac{1}{D^3+3D^2-D-3}(2+\cos x)$

$= \dfrac{2}{-3} + \dfrac{1}{-D-3-D-3}\cos x$

$= \dfrac{-2}{3} - \dfrac{2}{D+3}\cos x \qquad \dfrac{D-3}{D-3}$

$= \dfrac{-2}{3} - \dfrac{2(D-3)}{D^2-9}\cos x$

$= \dfrac{-2}{3} - \dfrac{2}{-10}(-\sin x - 3\cos x)$

3) $y'' + 9y = \operatorname{cosec} 3x$

$\lambda^2 + 9 = 0 \quad \rightarrow \lambda = \pm 3i$

$y_h = c_1 \cos 3x + c_2 \sin 3x$

$y_p = c_1(x) \cos 3x + c_2(x) \sin 3x$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3\cos^2 3x + 3\sin^2 3x$$
$= 3$

$$W_1 = \begin{vmatrix} 0 & \sin 3x \\ \operatorname{cosec} 3x & 3\cos 3x \end{vmatrix} = -1$$

$$W_2 = \begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \operatorname{cosec} 3x \end{vmatrix} = \frac{\cos 3x}{\sin 3x}$$

$$c_1(x) = \int \frac{W_1}{W} dx = \int \frac{-1}{3} dx = \frac{-1}{3} x$$

$$c_2(x) = \int \frac{W_2}{W} dx = \int \frac{1}{9} \frac{3\cos 3x}{\sin 3x} dx = \frac{1}{9} \ln|\sin 3x|$$

$$\therefore y_p = \frac{-1}{3} x \cos 3x + \frac{1}{9} \ln|\sin 3x| \sin 3x$$

$y_G = $

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(4) $(X^2 D^2 + 9XD + 25)y = 10 \ln|X|$

$$X = e^t$$

$$XD = \theta$$

$$X^2 D^2 = \theta(\theta - 1)$$

$$(\theta(\theta-1) - 9\theta + 25)y = 10t$$

$$\cancel{\theta^2 + 8\theta + 25} \quad (\theta^2 - 10\theta + 25)y = 10t$$

$$\lambda^2 - 10\lambda + 25 = 0$$

$$(\lambda - 5)^2 = 0$$

$$y_h = c_1 e^{5t} + c_2 t e^{5t}$$

$$= c_1 x^5 + c_2 x^5 \ln|x|$$

$$y_p = \frac{1}{\theta^2 - 10\theta + 25}(10t)$$

$$= \frac{1}{25}\left[\frac{1}{1 + \left(\frac{\theta^2 - 10\theta}{25}\right)}(10t)\right]$$

$$= \frac{1}{25}\left[1 - \left(\frac{\theta^2 - 10\theta}{25}\right) + \cdots\right]10t$$

$$= \frac{1}{25}\left[10t - \frac{1}{25}\left(-10 \times 10\right)\right]$$

$\boxed{10}$

(Q.5) a) $L\{\int_0^t \sin 5u \sin u \; \cancel{\cos} \; e^{2u} \, du\}$

$= \frac{1}{s} L(\sin 2t \sin t \; \cancel{\cos} \; e^{2t}]$

$= \frac{1}{4s} L\{(e^{2t} \cancel{+e^{2t}})[\cos 3t - \cos t]$

$= \frac{1}{4s}\left[\frac{s-2}{(s-2)^2+9} + \cancel{\frac{s+2}{(s+2)^2+9}} - \cancel{\frac{s+2}{(s+2)^2+1}}\right.$

$\left. - \frac{s-2}{(s-2)^2+1}\right]$

b) $L(t^2 e^{2t} u(t-1)]$

$= L((t-1+1)^2 e^{2(t-1+1)} u(t-1))$

$= L[[(t-1)^2 + 2(t-1) + 1] e^{2(t-1)} e^{2} u(t-1)]$

$= e^{2} e^{-s} L(t^2 + 2t + 1) e^{2t}$

$= e^{-s+2}\left[\frac{2}{(s-2)^3} + \frac{2}{(s-2)^2} + \frac{1}{(s-2)}\right]$

(3) $L^{-1} \ln\left(\frac{s+3}{s-2}\right) = L^{-1}[\ln(s+3) - \ln(s-2)]$

$t\, f(t) = L^{-1} - \frac{d}{ds}[\ln(s+3) - \ln(s-2)]$

$= -L^{-1}\left[\frac{1}{s+3} - \frac{1}{s-2}\right]$

$= -e^{-3t} + e^{2t}$

$\therefore f(t) = \frac{1}{t}\left(e^{2t} - e^{-3t}\right)$

(4) $F(s) = \frac{e^{-2s}[6-s]}{s^2-4s+3}$

$\frac{6-s}{s^2-4s+3} = \frac{A}{(s-3)} + \frac{B}{(s-1)} = \frac{A(s-1)+B(s-3)}{}$

$s=3 \qquad 3 = 2A \rightarrow A = \frac{2}{3}$

$s=1 \qquad 5 = -2B \rightarrow B = -5/2$

$\therefore F(s) = \left[\frac{\frac{3}{2}}{s-3} - \frac{5/2}{s-1}\right] e^{-2s}$

$f(t) = \frac{2}{3} u(t-2)\left[e^{3(t-2)} - \frac{5}{2} e^{(t-2)}\right]$

$\boxed{5}$ $y'' + y' - 4y = 2e^{t}$, $y(0) = y'(0) = 0$

$$s^2 Y + sY - 4Y = \frac{2}{s-1}$$

$$Y = \frac{2}{(s-1)(s^2+s-4)} = \frac{-1}{s-1} + \frac{s+2}{s^2+s-4}$$

$$= \frac{-1}{s-1} + \frac{s+\frac{1}{2}-\frac{1}{2}+2}{(s+\frac{1}{2})^2 - 4.5}$$

$$= \frac{-1}{s-1} + \frac{s+\frac{1}{2}}{(s+1/2)^2 - 4.5} + \frac{1.5}{(s+\frac{1}{2})^2 - 4.5}$$

$$Y(t) = -e^{t} + e^{-\frac{1}{2}t}\left(\cosh\sqrt{4.5}\,t + \frac{1.5}{\sqrt{4.5}}\sinh\sqrt{4.5}\,t\right)$$