Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: 2nd year Elec. Eng.

Subject: Engineering Mathematics (3)

Date: 1/6/2017
Time allowed: 3 hours
Full mark: 90
Final Term Exam:

Question 1 (30 Marks)

1. A) Evaluate $\int_0^{\infty} e^{-2t}\sqrt{t^5}\,dt$

   B) Prove that $\beta(n,n+1)=\dfrac{\Gamma^2(n)}{2\Gamma(2n)}$.

   C) Prove that $\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$.

2. Find the series solution of $4(x-1)y''+2y'+y=0$, about $x=1$.

Question 2: (30 Marks)

1. Find the value of $y(0.2)$ with $h=0.1$ for the initial value problem:

   $y'=x^2-y \quad , y(0)=1$

   using

   A) Taylor method. B) Runge-Kutta method of order 4.

2. Find Newton's forward difference interpolating polynomial for the following table of values:

x	0.1	0.2	0.3	0.4	0.5
y	1.4	1.56	1.76	2	2.28

Question 3: (30 Marks)

1. Find the general solution of :

   $$X'=\begin{pmatrix}4 & 1\\ -6 & -1\end{pmatrix}X$$

   and find the fundamental matrix solution.

2. Use L-U decomposition method to solve the system of linear equations:

   $$2x+3y+z=9$$
   $$x+2y+3z=6$$
   $$3x+y+2z=8$$

3. Fit the equation $y=ae^{bx}$ to the following readings:

x	1.2	2.8	4.3	5.4	6.8	7.9
y	7.5	16.1	38.9	67	146.6	266.2

With my best wishes

Dr. Samah El-Kholy

نموذج إجابة

Engineering Mathematic(3)

ثانية كهرباء

2016-2017

تاريخ الامتحان 1/6/2017

ترم ثاني

2 nd year Elec. eng.

Q1 :- A)

$$I = \int_0^{\infty} e^{-2t} \sqrt{t^5}\ dt$$

let $\theta = 2t$

$\therefore d\theta = 2dt$

$$I = \frac{1}{2^{9/2}} \Gamma\left(\frac{9}{2}\right) = \frac{1}{\sqrt{2^9}} \left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{1}{\sqrt{2^9}} \left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) \sqrt{\pi}$$

B) $\beta(n, n+1) = \frac{\Gamma(n)\Gamma(n+1)}{\Gamma(2n+1)}$

$$= \frac{n\Gamma^2(n)}{2n\Gamma(2n)} = \frac{\Gamma^2(n)}{2\Gamma(2n)}$$

C) $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$

$$= \frac{\Gamma(m+1)\Gamma(n)}{\Gamma(m+n+1)} + \frac{\Gamma(m)\Gamma(n+1)}{\Gamma(m+n+1)}$$

$$= \frac{m\Gamma(m)\Gamma(n) + n\Gamma(m)\Gamma(n)}{(m+n)\Gamma(m+n)}$$

$$= \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} = \beta(m, n)$$

$\boxed{2}$ $4(x-1)y'' + 2y' + y = 0$, about $x = 1$

let $x - 1 = u$

$\therefore y(u) = u^r \sum_{n=0}^{\infty} a_n u^n$

$y'(u) = \sum_{n=0}^{\infty} (n+r) a_n u^{n+r-1}$

$y''(u) = \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n u^{n+r-2}$

$4u y'' + 2y' + y = 0 \rightarrow (1)$

$4 \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n u^{n+r-1} + 2 \sum_{n=0}^{\infty} (n+r) a_n u^{n+r-1}$

$+ \sum_{n=0}^{\infty} a_n u^{n+r} = 0$

$\therefore 2a_0(2r^2 - r) = 0 \Rightarrow (2r^2 - r) = 0$

$r(2r-1) = 0 \Rightarrow r = 0$ or $r = \frac{1}{2}$

$\therefore y_1 = u^{r_1} \sum_{n=0}^{\infty} a_n u^n = \sum_{n=0}^{\infty} a_n u^n \rightarrow (2)$

$y_2 = u^{r_2} \sum_{n=0}^{\infty} a_n u^n = \sum_{n=0}^{\infty} a_n u^{n+1/2} \rightarrow (3)$

From (2) into (1)

$\therefore 4 \sum_{n=0}^{\infty} n(n-1) a_n u^{n-1} + 2 \sum_{n=0}^{\infty} n a_n u^{n-1} + \sum_{n=0}^{\infty} a_n u^n = 0$

$\therefore \; 2n(2n-1)\, a_n = -a_{n-1}$

$\therefore \; a_n = \dfrac{-a_{n-1}}{2n(2n-1)}$

let $a_0 = 1$

$a_1 = \frac{-1}{2}$, $a_2 = \frac{-a_1}{3.4}$, a_3 ....

$\therefore y_1 = \sum_{n=0}^{\infty} a_n u^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} u^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (X-1)^n$ ⨳

and from ③ into ① and by the same way, we find:-

$y_2(X) = (X-1)^{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (X-1)^n$ ⨳

(2 Q2:-

$\boxed{1}$ $y' = x^2 - y \quad , y(0) = 1$

a) Taylor series:-

$$y_{n+1} = y_n + h y'_n + \frac{h^2}{2!} y''_n + O(h^3)$$

$n = 0$

$$y_1 = y_0 + h y'_0 + \frac{h^2}{2!} y''_0$$

$$= 1 + 0.1 * (0 - 1) + \frac{(0.1)^2}{2} (2(0) - y'_0) + \cdots$$

$$\simeq 0.905$$

$n = 1$

$$y_2 = y_1 + h y'_1 + \frac{h^2}{2!} y''_1 \simeq 0.82098$$

b) Runga-Kutta method:-

$n = 0$

$$K_1 = h f(x_0, y_0) = -0.1$$

$$K_2 = h f(x_0 + \tfrac{h}{2}, y_0 + \tfrac{K_1}{2}) = -0.09475$$

$$K_3 = h f(x_0 + \tfrac{h}{2}, y_0 + \tfrac{K_2}{2}) = -0.09501$$

$$K_4 = h f(x_0 + h, y_0 + K_3) = -0.0895$$

$$y_1 = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.905163$$

at $n = 1$

$$K_1 = h f(X_1, y_1)$$

$$K_2 = h f\left(X_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$$

$$K_3 = h f\left(X_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$$

$$K_3 = h f(X_1 + h, y_1 + K_3)$$

$$\therefore y_2 = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \cong 0.821269$$

$\boxed{2}$

X	y	Δ	Δ^2	Δ^3	Δ^4
0.1	1.4				
0.2	1.56	0.16	0.04		
0.3	1.76	0.2	0.04	0	0
0.4	2	0.24	0.04	0	
0.5	2.28	0.28			

$$y = y_0 + S\Delta y + \frac{S(S-1)}{2}\Delta^2 y$$

$$S = \frac{X - X_0}{h} = \frac{X - 0.1}{0.1} = 10X - 1$$

$$\therefore y = 2X^2 + X + 1.28$$

(c) Q 3:-

$\boxed{1}$ $X' = \begin{pmatrix} 4 & 1 \\ -6 & -1 \end{pmatrix} X$

$|A - \lambda I| = 0$

$\begin{vmatrix} 4-\lambda & 1 \\ -6 & -1-\lambda \end{vmatrix} = 0$

$\lambda^2 - 3\lambda + 6 = 0 \Rightarrow \lambda = 2$ or 1

for $\lambda = 1$ $\quad V_2 = -3V_1 \Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

for $\lambda = 2$ $\quad V_2 = -2V_1 \Rightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\therefore$ The fundamental matrix solution is

$$\overline{X} = \begin{pmatrix} e^{t} & e^{2t} \\ -3e^{t} & -2e^{2t} \end{pmatrix}$$

Scanned by CamScanner

(2

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0.5 & 2.5 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} X \\ y \\ Z \end{bmatrix} = \begin{bmatrix} 9 \\ 1.5 \\ 5 \end{bmatrix}$$

$$\therefore X = \frac{35}{18}, \quad y = \frac{29}{18}, \quad Z = \frac{5}{18}$$

$\boxed{3}$ $y = a e^{bX}$

$$\ln y = \ln b + aX = B + aX$$

$\therefore$

	X	y	$\ln y = Y$	X^2	XY
	1.2	7.5			
	2.8	16.1			
	4.3	38.9			
	5.4	67			
	6.8	146.6			
	7.9	266.2			
Σ	28.4		23.2314	165.58	126.6782

$\therefore n = 6$

$B = 1.332055 \Rightarrow b = 3.7889, \; a = 0.5366$

$$\therefore y = 3.7889\, e^{0.5366X}$$

$\boxed{8}$