Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Second Year-Electric.

Subject: Engineering Mathematics (3)
Code : PHM2009

Date: 2/1/2020
Time allowed: 3 hours
Full mark: 90
Final Term Exam: 2pages

The following questions measure ILOs a1, b1, b7, c1 and c7

Answer the following questions:

Question 1: (25 marks)

a) Prove that $\frac{d}{dz}z^n = nz^{n-1}$. **(b1,c1)(5 marks)**

b) Is the function $u(x,y) = xy^3 - x^3y$ harmonic? If so, find its conjugate harmonic function $v(x,y)$ and find the analytic function $f(z) = u(x,y) + iv(x,y)$ and $f'(z)$. **(a1,b1,c1)(10 marks)**

c) Discuss the analyticity of the following functions and which of them is entire: **(b1,c1)(10 marks)**

i. $f(z) = \sin z$.

ii. $f(z) = e^z$.

Question 2: : (25 marks)

a) Prove that $\frac{d}{dz}(z\,\bar{z})$ does not exist anywhere. **(b7,c1)(5 marks)**

b) Prove that $\tan z = \frac{\sin 2x}{\cos 2x + \cosh 2y} + i\,\frac{\sinh 2x}{\cos 2x + \cosh 2y}$. **(b1,c1)(5 marks)**

c) Prove that $\tanh^{-1} z = \frac{1}{2}\ln(\frac{1+z}{1-z})$ and find the value of $\tanh^{-1}(1+2i)$.

(a1,c1)(5 marks)

d) Find $\frac{d}{dz}f(z)$ when exist and find the points at which $f'(z)$ exist for:

(c1,c7)(10 marks)

i. $f(z) = \ln z$.

ii. $f(z) = \text{sech}^{-1} z$.

Question 3: : (25 marks)

a) Prove that if $f(z)$ is analytic function in a simply connected domain D and $f'(z)$ is continuous at each point within and on a closed contour C ,then $\oint_C f(z)dz = 0$. **(a1,b1,c1)(5 marks)**

b) Evaluate the following integrals around the contour $C:|z|=3$: **(b7,c1)(10 marks)**

i. $\oint e^z dz$.

ii. $\oint \frac{z}{(z^2+4)(z+1)(z+5)}dz$.

iii. $\oint \frac{e^{z^2}}{(z-i)^4}dz$.

c) Find the residue of the following functions using series expansion for the functions about the indicated singularity: **(b1,c1,c7)(10 marks)**

i. $f(z)=\frac{e^{2z}}{(z-1)}$, about $z=1$.

ii. $f(z)=\frac{1}{z^2(z-3)^2}$, about $z=3$.

Question 4: : (15 marks)

a) Using the residue theorem evaluate the integrals: **(a1,b1,c7)(10 marks)**

i. $\int_0^{2\pi} \frac{d\theta}{1+3\cos^2\theta}$,

ii. $\int_0^{\infty} \frac{\cos x}{(x^2+1)(x^2+4)}dx$.

b) Find the bilinear mapping that maps the points $(z_1,z_2,z_3)=(0,1,\infty)$ into the points $(\omega_1,\omega_2,\omega_3)=(-1,-i,1)$ and then find its fixed points.
(a1,b1,c7)(5 marks)

With my best wishes
D. Samah El-Kholy