



Question [1]: (30 marks) [ILOs: a1,c1]

Marks

- a) Derive the formula of Fourier series coefficients a_0 , a_n , and b_n .
b) Find **Fourier series** for the function defined by:

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$$f(x) = \begin{cases} \pi - x & , 0 < x \leq \pi \\ 0 & , \pi \leq x < 2\pi \end{cases}$$

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and sketch the graph of $f(x)$, and show that: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

- c) Find the **Fourier series** of: $f(x) = |x|$ where $-L < x < L$,
and sketch the graph of $f(x)$.

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Question [2]: (30 marks) [ILOs: a1,a2,b1]

- a) Expand $f(x) = e^{\frac{\pi x}{4}}$, $-4 \leq x \leq 4$ in a complex Fourier series.

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- b) Solve the following IVP by using D'Alembert's formula:

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$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad u(x, 0) = 0, \quad u_t(x, 0) = 2 \sin 4\pi x, \quad \forall x$$

- c) Using method of separation of variables (MSV), solve the following heat equation: $u_{xx} = u_t$, $0 \leq x \leq 2$, $t \geq 0$, $u(0, t) = u(2, t) = 0$, $u(x, 0) = x$

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Question [3]: (30 marks) [ILOs: b2]

- a) Fit the curve $y = \frac{1}{a + b \cos \theta}$ to the following data and find RMSE:

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θ_i	1	2	3
θ_i	30	45	60
y_i	0.225	0.27	0.32

- b) Use the bisection method to find the required root of: $3x - e^{-x} = 0$
in the interval $[0.25, 0.27]$, correct to 3-decimal places. Then find absolute error.

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- c) Evaluate by using Gamma and Beta functions:

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i) $I = \int_0^{\infty} \frac{x^2}{2^x} dx$,

ii) $I = \int_0^4 y^2 \sqrt{4-y} dy$

- d) Prove that:

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i) $\beta(n, n+1) = \frac{[\Gamma(n)]^2}{2\Gamma(2n)}$, ii) $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$

Good luck >>>> <<<< Dr. Manal El-Sayed

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