



Marks

Question [1]: (30 marks) [ILOs: a1,b7]

- a) Derive the formula of Fourier series coefficients a_0 , a_n , and b_n . 10
 b) Find **Fourier series** for the function defined by: 10

$$f(x) = \begin{cases} \pi - x & , 0 < x \leq \pi \\ 0 & , \pi \leq x < 2\pi \end{cases}$$

and sketch the graph of $f(x)$, and show that: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

- c) Find the **Fourier series** of: $f(x) = |x|$ where $-L < x < L$, and sketch the graph of $f(x)$. 10

Question [2]: (30 marks) [ILOs: a1,b1,c1]

- a) Expand $f(x) = e^{\frac{\pi x}{4}}$, $-4 \leq x \leq 4$ in a complex Fourier series. 10

- b) Solve the following IVP by using D'Alembert's formula: 10

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad u(x, 0) = 0, \quad u_t(x, 0) = 2 \sin 4\pi x, \quad \forall x$$

- c) Using method of separation of variables (MSV), solve the following heat equation: $u_{xx} = u_t$, $0 \leq x \leq 2$, $t \geq 0$, $u(0, t) = u(2, t) = 0$, $u(x, 0) = x$ 10

Question [3]: (40 marks) [ILOs: c7]

- a) Find Fourier transform of $t e^{-t^2}$ by using: 10

$$F[f'(t)] = i\omega \hat{f}(\omega), \quad \text{where} \quad \int_0^{\infty} e^{-u} du = \frac{\sqrt{\pi}}{2}$$

- b) Solve using Fourier transform: $\frac{dy(t)}{dt} + 0.5y(t) = \sin 3t$ 10

- c) Determine whether vector field $\vec{A} = (2x + y^3)\underline{i} + (3xy^2 + 4)\underline{j}$ is conservative: 10

- i) By computing $\text{curl } \vec{A}$.
 ii) By finding the scalar potential of \vec{A} .

- d) Find the area of parallelogram with vertices (2,2,0), (9,2,0), (10,3,0), and (3,3,0). 10

Good luck >>>>===== <<<<< Dr. Manal El-Sayed

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