Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Prep. Subject: Engineering Mathematics (1)

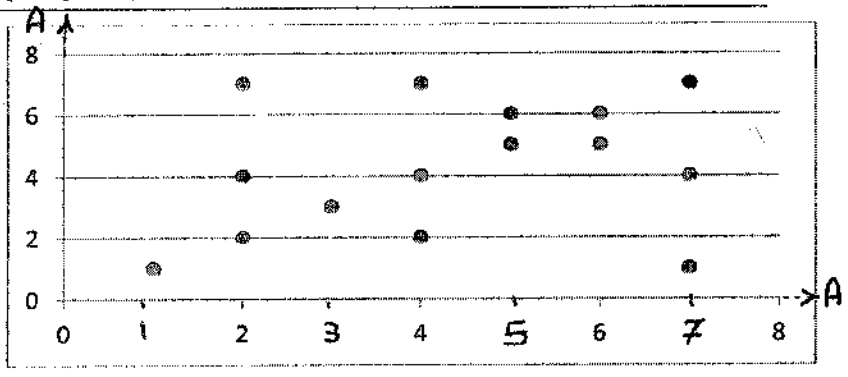
Date: 21/1/2020
Time allowed: 3 hours
Full Marks: 110
Final-Term Exam: 4 pages

[Q1] Choose the correct answer from the following: [55 marks][ILOs: a1,b1]

1. If $\frac{x^2+20}{(x-2)^2(x+4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4}$, then A, B, C are
   (a) (4,0,1) (b) (0,4,1) (c) (1, 0,4) (d) (0,-4,-1)

2. The following system of equations
   x-y+z=0, x+y+kz=0, -x-y+z=1 has a solution if k is
   (a) $k \in \mathcal{R} - \{1\}$ (b) $k \in \mathcal{R} - \{-1\}$ (c) $k \in \mathcal{R}$ (d) None of these

3. The relation R which defined on $A = \{1,2,3,4,5,6,7\}$ as shown in the following figure is
   (a) Reflexive only
   (b) Symmetric only
   (c) Transitive only
   (d) Equivalence relation



4. The solution of the equation $z^2 - 2iz - 5 = 0, z \in C$, is
   (a) $z_1 = 1 - 2i, z_2 = 1 + 2i$
   (b) $z_1 = 2 - i, z_2 = -2 + i$
   (c) $z_1 = 2 + i, z_2 = -2 + i$
   (d) None of these.

5. The three currents i_1, i_2, i_3 of a circuit which are related by:
   $2i_1 + i_2 - i_3 = 8$, $i_1 - i_2 + i_3 = -5$, $3i_1 + 2i_2 = 9$ are
   (a) (1,2,3) (b) (−1,3,3) (c) (−1,−3,3) (d) (1,3,−3)

6. For any two statements p, q, the relation $(p \to q) * (q \to p) \equiv (p \leftrightarrow q)$ is
   (a) (valid if *= ∨) (b) (valid if *= ∧) (c) (valid if *= →) (d) None of these

7. If $z = 1 + \sqrt{3}i$, then $z^6$is
   (a) 64 i (b) 64 (c) -64 i (d) None of these

8. If $z = 2(cos15 + i\, sin15)$,then $z^6$is
   (a) 64 (b) -64 (c) 64 i (d) None of these

9. The expression $((A \cap B) \cup A^c)^c =$
   (a) $A \cap B$ (b) $A \cup B$ (c) $A - B$ (d) $B - A$

10. By using Mathematical induction, the relation 1+3+5+……+(2n-1)= 3n-2 is
    (a) valid for all $n \in N$ (b) not valid for all $n \in N$
    (c) valid at n=1 only (d) None of these.

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Q[2] Choose the correct answer from the following: [55 marks] [ILOs: a1,b7,c1]
أجب عن هذا السؤال بالورقة المخصصة لذلك والموجودة في نهاية ورقة الإجابة

(1) If $y=\cos(\sin^{-1}\sqrt{x})-\sqrt{1-x}+\tan^{-1}(e^x)$, then $y'=$.....

$(a)\ \frac{1}{e^x+1}$ $(b)\ \frac{e^x}{e^{2x}+1}$ $(c)\ \frac{2e^x}{e^{2x}+1}$ (d) None of these

(2) If $y=\ln(\sin x)$, find $e^{-2y}=$.....

$(a)\ \sec^2 x$ $(b)\ -\cos ec^2 x+1$ $(c)\ \cos ec^2 x$ (d) None of these

(3) If $x=t+\frac{1}{t}$, $y=t^2+\frac{1}{t^2}$ then $\frac{d^2y}{dx^2}=$.....

$(a)\ 2$ $(b)\ 2(t+\frac{1}{t})$ $(c)\ 1$ (d) None of these

(4) If $x=2(1-\sin t)$, $y=\cos t$, then $4y^3y''=$.....

$(a)\ 0$ $(b)\ -1$ $(c)\ 1$ (d) None of these

(5) Find Maclaurin expansion of $f(x)=e^x-1$

$(a)(x-\frac{x^2}{2!}-\frac{x^3}{3!}-.....)$ $(b)(x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....)$ $(c)(x-\frac{x^2}{2}-\frac{x^3}{3}-.....)$ (d) None of these

(6) Expand the function $f(x)=\ln(x)^x$ at $x=1$

$(a)\ (x-1)-\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}-.....$ $(b)\ (x-1)+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{6}+.....$

$(c)\ (x-1)+\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}+.....$ (d) None of these

(7) If $y=\tan(\sec^{-1}\sqrt{x})+\cot(\cos ec^{-1}\sqrt{x})$, Find $y'=$.....

$(a)\frac{1}{\sqrt{x-1}}$ $(b)\frac{1}{2\sqrt{x-1}}$ (c)None of these $(d)\frac{-1}{\sqrt{x-1}}$

(8) $\lim_{x\to\infty}[x\ \ln(1+\frac{4}{x})]=$..... $(a)8$ (b)None of these $(c)1$ $(d)4$

(9) $\lim_{t\to 0}\frac{\sqrt{1+\tan t}-\sqrt{1-\tan t}}{t}=$..... $(a)0$ $(b)1$ $c)\sec^2 t$ (d)Not exist

(10) If $f(t)=\frac{1}{t^2+1}$ and $g(t)=\sqrt{t}$, then the derivative of $f(g(t))=$.....

(a)None of these $(b)\ -(t+1)^{-2}$ $(c)\ \frac{-2}{(t^2+1)^2}$ $(d)\ \frac{1}{(t+1)^2}$

(11) Suppose that the function g(x) satisfies the following inequality
$1+x\le g(x)\le e^x$, find the value of $\lim_{x\to 0} g(x)$

$(a)2$ $(b)3$ $c)1$ (d) *None of these*

(12) $Lim_{x\to 0}(x\ \cos ec\ x)=$..... $(a)-\infty$ $(b)-1$ $(c)0$ $(d)1$

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(13) If $\cos(xy)=x$, then $\frac{dy}{dx}=$ .....

$(a)\ \mathrm{cosec}(xy)$ $(b)\ \frac{\mathrm{cosec}(xy)}{-x}$ $(c)\ \frac{\mathrm{cosec}(xy)+y}{-x}$ (d) *None of these*

(14) $\ln\left(\frac{6}{\sqrt{e^{x}}}\right)=$ ..... $(a)\ \frac{-1}{2}(6x)$ $(b)\ \ln 6-\sqrt{x}$ $(c)\ \ln 6-\frac{x}{2}$ (d) *None of these*

(15) *If* $y=\cosh^{-1}(\sec x)$, *then* $y''=$ .....

$(a)\ 2\sec x$ $(b)\ \sec x$ $(c)\ \sec x\tan x$ (d) *Non of these*

(16) The coefficient of $(x-1)^2$ in Taylor expansion of $\sqrt{x}$ *is* $(a)\ \frac{1}{2}$ $(b)\ \frac{-1}{4}$ $(c)\ \frac{1}{8}$ $(d)\ \frac{-1}{8}$

(17) The first three non zero terms of Maclaurin expansion for $\frac{1}{(\cosh x+\sinh x)^2}$ *is*

$(a)\ 1+2x+2x^2+....$ $(b)\ 1+x+\frac{x^2}{2}+....$ (c) *None of these* $(d)\ 1-2x+2x^2+....$

(18) If $y=x\sin x$, find $y^{(n)}$

$(a)\ x\sin(x+\frac{n\pi}{2})$ $(b)\ x\sin(x+\frac{n\pi}{2})+n\sin(x+\frac{(n-1)\pi}{2})$ (c) *None of these* $(d)\ \sin(x+\frac{(n-1)\pi}{2})$

(19) $\sinh^{-1}x=$ .....

$(a)\ \ln(x-\sqrt{x^2+1})$ $(b)\ \ln(x+\sqrt{x^2+1})$ (c) *None of these* $(d)\ \ln(x+\sqrt{x^2-1})$

(20) If $y=\ln(ax+b)$, find $y^{(n)}$

$(a)\ (-1)^{n-1}(n-1)!\,a^n(ax+b)^{-n}$ $(b)\ (-1)^{n-1}(n-1)!\,a^n(ax+b)^{-n-1}$

$(c)\ (-1)^n(n-1)!\,a^n(ax+b)^{-n}$ (d) None of these

(21) If $f(x)=e^x g(x)$ and $g(0)=1$, $g'(0)=-2$ then $f'(0)=$..... $(a)0$ $(b)1$ $(c)2$ $(d)-1$

(22) *If* $y=e^{x\log_5 5^x}$, *then* $y'=$ ..... $(a)e^{x^2}$ $(b)2x\,e^{x^2}$ $(c)2e^{2x}$ (d) None of these

(23) *If* $y=\frac{x}{x-2}$, *then* $y^{(n)}=$ .....

$(a)(-1)^n n!(x-2)^{-n-1}$ $(b)2[(-1)^n n!(x-2)^{-n-1}]$ (c) None of these $(d)2[(-1)^n(x-2)^{-n-1}]$

(24) $\frac{d}{dx}\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}=$ ..... $(a)\ \frac{-1}{2}$ $(b)\ \frac{1}{2}$ $(c)\ 1$ (d) None of these

(25) *The domain of* $\sqrt{4-\sqrt{x-2}}$ is $(a)R$ $(b)R-\{2\}$ (c) *None of these* $(d)[2,18]$

(26) *Find* D_f *and* R_f *of* $f(x)=\frac{1}{\sqrt{x^2-1}}$

$(a)\ \begin{cases} D_f=R-[-1,1] \\ R_f=]0,\infty[\end{cases}$ $(b)\ \begin{cases} D_f=R-[-1,1] \\ R_f=[0,\infty[\end{cases}$ (c) *None of these* $(d)\ \begin{cases} D_f=R \\ R_f=R \end{cases}$

(27) Find $f^{-1}(x)$ of $f(x)=\frac{x-2}{x+2}$ $(a)1-\frac{2}{x-2}$ $(b)\frac{2+2x}{1-x}$ $(c)\frac{2+x}{2-x}$ (d) *None of these*

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(28) Find the invers of $f(x)=\sqrt{3-\sqrt{x+1}}$ $(a)(3-x)^2-1$ $(b)(3-x^2)^2-1$ $c)$ *None of these* $(d)3-\sqrt{x+1}$

(29) *Find* D_f *and* R_f *of* $f(x)=\tan(4x-\pi)$

$a)\begin{cases} D_f = R-\{(2n+1)\frac{\pi}{8}\} \\ R_f = R \end{cases}$ $(b)\begin{cases} D_f = R \\ R_f = R \end{cases}$ $(c)\begin{cases} D_f = R-\{\frac{\pi}{2}-\frac{\pi}{2}\} \\ R_f = R^+ \end{cases}$ (d)*None of these*

(30) Find D_f and R_f *of* $f(x)=\ln(x-9)+9$

$a)\begin{cases} D_f = [9,\infty[\\ R_f = R^+ \end{cases}$ $(b)\begin{cases} D_f =]9,\infty[\\ R_f = R \end{cases}$ $(c)\begin{cases} D_f = R-\{9\} \\ R_f = R \end{cases}$ (d)*None of these*

(31) If $y=\tan(\sec^{-1}3x)$, then $\frac{y'}{y}=$ ..... $a)\ \sec^2 3x$ $(b)\ \frac{9x}{9x^2-1}$ $(c)\ \frac{9x}{3x^2-1}$ $(d)\ \frac{1}{\cot x^2}$

(32) Find D_f and R_f *of* $f(x)=3\cos(3x+\pi)-1$

$a)\begin{cases} D_f = R \\ R_f = [-4,2] \end{cases}$ $(b)\begin{cases} D_f = R \\ R_f = [-2,2] \end{cases}$ $(c)\begin{cases} D_f = R \\ R_f = R^+ \end{cases}$ (d)*None of these*

(33) $\lim\limits_{x\to\infty}\frac{X^2(3+\cos^2 X)}{X+10}=$..... $(a)\frac{3}{10}$ $(b)3$ $(c)\frac{1}{10}$ (d)*Not exist*

(34) *If* $y=\sin x\cos x$ *, then* $y^{(n)}=$ .....

$(a)[2^n\sin(2x+\frac{n\pi}{2})]$ $(b)\frac{1}{2}[2^n\sin(x+\frac{n\pi}{2})]$ (c) None of these $(d)\frac{1}{2}[2^n\sin(2x+\frac{n\pi}{2})]$

(35) If $y=\ln(\ln\cosh^{-1}x)$, find y'

$(a)\ \frac{1}{\ln\cosh^{-1}x}\ \frac{1}{\cosh^{-1}x}\ \frac{1}{\sqrt{x^2+1}}$ $(b)\ \frac{1}{\ln\cosh^{-1}x}\ \frac{1}{\cosh^{-1}x}$

$(c)\ \frac{1}{\ln\cosh^{-1}x}\ \frac{1}{\cosh^{-1}x}\ \frac{1}{\sqrt{x^2-1}}$ (d) None of these

(36) If $y=\cos(x^x)$, then $y'=$ .....

$(a)-x\sin(x^{x-1})$ $(b)\ x\cos(x^{x-1})$ $(c)-\sin(x^x)[x^x+x^x\ln x]$ (d) None of these

(37) If $y=3^x$, find $y^{(n)}$ $(a)3^x\ln(3)$ $(b)3^x(\ln 3)^n$ $(c)3^x$ (d) None of these

(38) If $f(x)=(x-2)^3$, then $f^{-1}(x)=$ ..... $(a)\sqrt[3]{x}-2$ (b) None of these $(c)\sqrt[3]{x}+2$ $(d)\sqrt{x}+2$

(39) If $y=x\,e^{\frac{-1}{2}x^2}$, find y' $(a)\frac{y}{x}(1-x^2)$ $(b)\frac{y}{x}(1+x^2)$ $(c)\ y(x^2-1)$ (d) None of these

(40) Find Maclaurin expansion of f(x) = sinh x

$(a)(1+\frac{x^2}{2!}+\frac{x^4}{4!}+.....)$ (b) None of these $(c)(x+\frac{x^3}{3}+\frac{x^5}{5}+.....)$ $(d)(x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....)$

انتهت الأسئلة

With my best wishes ➔ *Prof. Dr. Arafa Nasef & Dr. Manal El-said Ali*