Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Prep.

Subject: Engineering Mathematics (1)

Date: 18/1/2016
Time allowed: 3 hours
Full Marks: 110
Final-Term Exam: 2 pages

Answer the following questions:

Question 1: [25 marks]

a) Draw the function $f(x) = \sqrt{x}$ then use it to draw $g(x) = (\sqrt{x-2}) - 3$

b) Determine the domain and range of the following functions:

$i)\ y = \sqrt{x^2 - 4}$, $ii)\ y = \ln(x-5)$

c) Find the following limits if exist:

$i)\ \lim_{x \to 2} \dfrac{\sqrt{x-2}}{\sqrt{x^2-4}}$

$ii)\ \lim_{x \to 0} [1 + \sin(x)]^{\frac{1}{x}}$

$iii)\ \lim_{x \to \infty} \dfrac{3x^3 + x^2 + 7}{5x^3 + 2x^2 + x}$

$iv)\ \lim_{x \to 0} \dfrac{x^2}{1 - \cos x}$

d) Redefine the following function to be continuous at x=3:

$f(x) = \dfrac{x^2 + x - 12}{x^2 - 9}$

Question 2: [30 marks]

a) Find $y'(x)$ of the following functions:

$(i)\ y = \ln[\ln(\sinh^{-1} x)] + (\sin x)^x$

$(ii)\ y = e^{\sqrt{x^2-1}} + 2^{\operatorname{cosec} x}$

$(iii)\ y = \tan(\sec^{-1} 3x) + e^{2\ln x}$

b) Deduce $\dfrac{dy}{dx}$ if $y = [\cosh^{-1}(x)]$

c) If y = a cos(lnx) + b sin(lnx), show that :

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

d) Deduce Maclaurin expansion of : $f(x) = (1-x)^n$

then use it to calculate the following approximately $\dfrac{1}{\sqrt{45}}$.

Question 3: [27 marks]

a) By using Venn diagram, represent each of the following sets:
$A \cup B, \quad A \cap B, \quad A^c, \quad A - B, \quad A \Delta B$

b) If $z_1 = 1+i,\ z_2 = -2+3i$. Prove that: $|z_1 + z_2| < |z_1| + |z_2|$

c) Prove that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation
$x^2 - (a+d)x + (ad-bc)I = 0$, *where* $A = x$, *I is the identity matrix.*

d) By using mathematical induction, prove that:
$(1+x)^n = 1 + C_1^n x + C_2^n x^2 + ... + C_n^n x^n$

Question 4: [28 marks]

a) Write Pascal triangle for $(a+b)^7$ and find the value of $(1.05)^8$.

b) Factorize :
$$f(x) = \frac{4+8x^{-1}}{x^3 - 2x} .$$

c) Put in polar form:
$$z = \frac{1}{(2+i)^2} - \frac{1}{(2-i)^2} .$$

d) Calculate the three currents i_1, i_2, i_3 of a circuit which are related by:
$$2i_1 - i_3 + i_2 = 8$$
$$i_1 - i_2 + 5 + i_3 = 0$$
$$3i_1 + 2i_2 = 9$$