نموذج إجابة

رياضيات هندسيه (١- ب)

Engineering Mathematic (1-B)

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Engineering Mathematics (1)

30/5/2016

Answer The following Questions :-

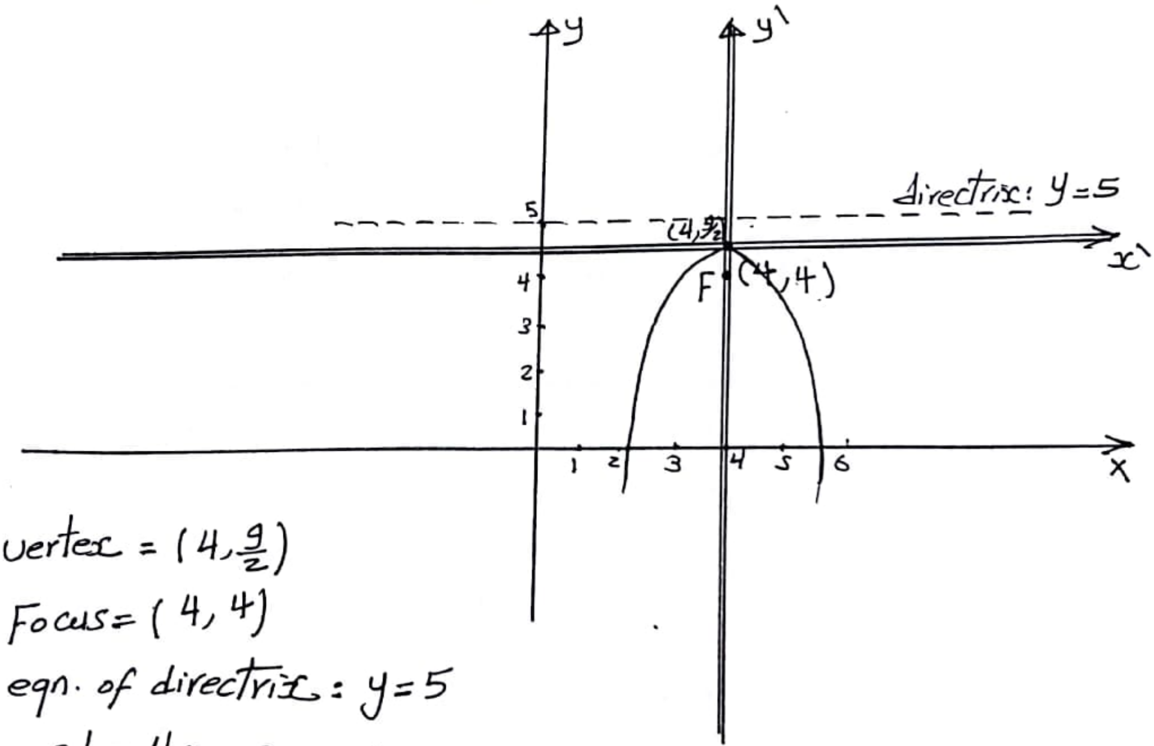
Question [1]

a) Describe the parabola : $x^2+2y=8x-7$

solution

$$x^2+2y=8x-7$$
$$x^2-8x=-2y-7$$
$$(x-4)^2-(4)^2=-2y-7$$
$$(x-4)^2=-2y+9$$
$$\boxed{(x-4)^2=-2\left(y-\frac{9}{2}\right)}$$

$\alpha=4 \qquad \beta=\frac{9}{2} \qquad 4a=2 \longrightarrow a=\frac{1}{2}$



vertex $=(4,\frac{9}{2})$

Focus $=(4,4)$

eqn. of directrix : $y=5$

$2L=4a=2$

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b) Find the equation of ellipse whose vertices are $(-2,1)$, $(0,1)$, $(-1,-1)$, $(-1,3)$. Also discuss and sketch it

solution

$(-2,\underline{1})$, $(0,\underline{1}) \longrightarrow 2b=2 \longrightarrow b=1$

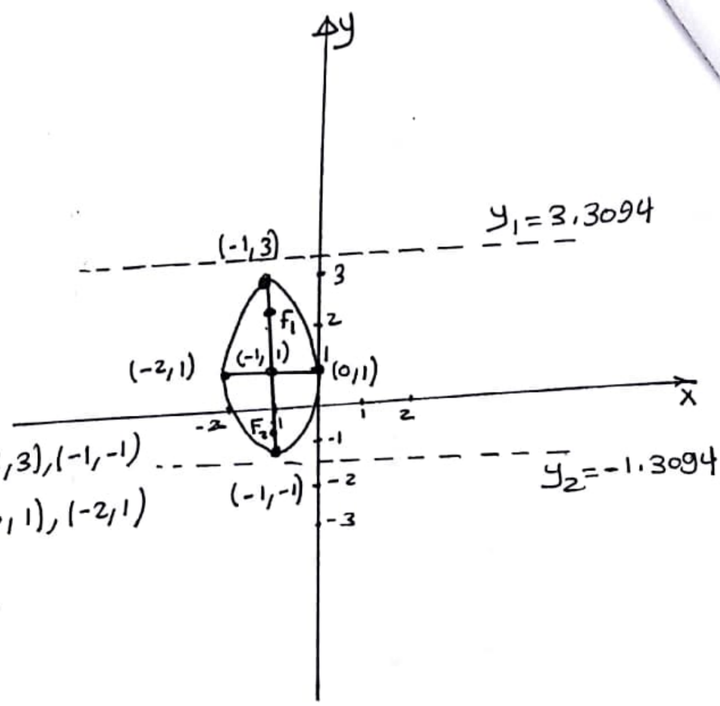
$(\underline{-1},-1)$, $(\underline{-1},3) \longrightarrow 2a=4 \longrightarrow a=2$

$$\frac{(x+1)^2}{1}+\frac{(y-1)^2}{4}=1$$

center $=(-1,1)$

$c=\sqrt{a^2-b^2}=\sqrt{4-1}=\sqrt{3}=1.732$

* Foci $\rightarrow F_1=(-1,1+\sqrt{3})$
  $F_2=(-1,1-\sqrt{3})$



* The ends of major axes are $(-1,3),(-1,-1)$
* The ends of minor axes are $(0,1),(-2,1)$
* Length of major axes $=2a=4$
* length of minor axes $=2b=2$
* $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$
* eqns of directrix : $y=1\pm a/e$

$y_1=1+a/e$ $\qquad$ $y_2=1-a/e$

$y_1=1+\frac{4}{\sqrt{3}}$ $\qquad$ $y_2=1-\frac{4}{\sqrt{3}}=$

$y_1=3.3094$ $\qquad$ $y_2=-1.3094$

(c) Describe the curve $2x^2-y^2-2x-4y=0$

— Solution —

$2x^2-2x-y^2-4y=0$

$2[x^2-x]-[y^2+4y]=0$

$2[(x-\frac{1}{2})^2-(\frac{1}{2})^2]-[(y+2)^2-(2)^2]=0$

$2(x-\frac{1}{2})^2-\frac{1}{2}-(y+2)^2+4=0$

$$\frac{2(x-\frac{1}{2})^2}{-7/2}-\frac{(y+2)^2}{-7/2}=\frac{-\frac{7}{2}}{-7/2}$$

$$\frac{(y+2)^2}{7/2}-\frac{(x-\frac{1}{2})^2}{7/4}=1$$

$a^2 = \frac{7}{4} \rightarrow a = \sqrt{\frac{7}{4}} = 1.3229$

$b^2 = \frac{7}{2} \rightarrow b = \sqrt{\frac{7}{2}} = 1.871$

$c = \sqrt{a^2 + b^2} = \sqrt{\frac{7}{4} + \frac{7}{2}} = 2.2913$

center $= (\frac{1}{2}, -2)$

vertices $= A_1 = (\frac{1}{2}, -2 + 1.871)$

$A_2 = (\frac{1}{2}, -2 - 1.871)$

Foci $\rightarrow F_1 = (\frac{1}{2}, -2 + 2.913)$

$F_2 = \frac{1}{2}(-2 - 2.913)$

eqn. of asympotitic lines =
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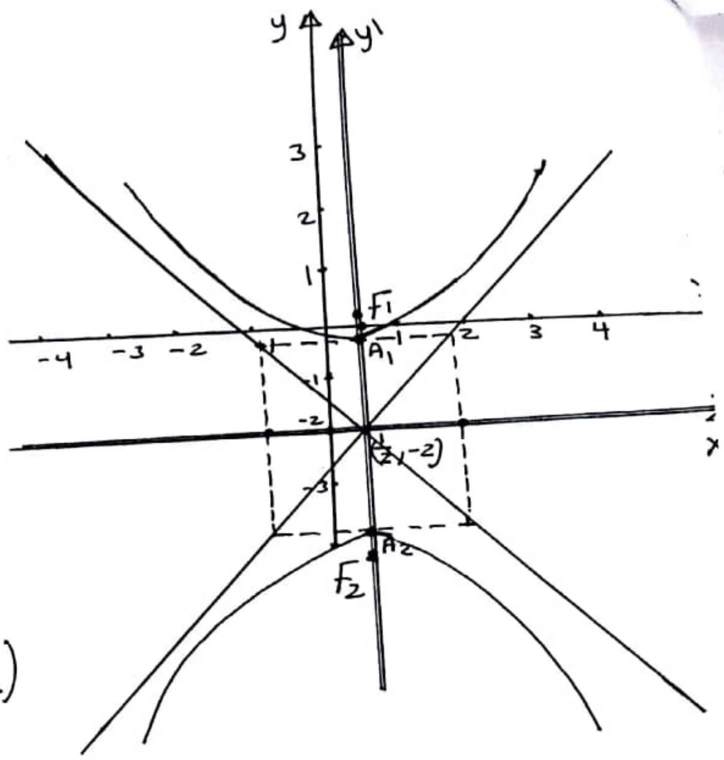
$$(y+2) = \pm \frac{b}{a}(x - \frac{1}{2})$$

$$(y+2) = \pm \sqrt{2}(x - \frac{1}{2})$$



Question [2]

a) prove that $r = a\sin\theta + b\cos\theta$ represent a circle and find each of its center and its radius.

solution

$r = a\sin\theta + b\cos\theta$ (*r)

$r^2 = ar\sin\theta + br\cos\theta$

$r^2 - br\cos\theta - ar\sin\theta = 0$

$x^2 + y^2 - bx - ay = 0 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore$ The eqn represents a circle

center $= (-g, -f) = (\frac{b}{2}, \frac{a}{2})$

radius $= \sqrt{g^2 + f^2 - c} = \sqrt{\frac{b^2}{4} + \frac{a^2}{4}} = \frac{\sqrt{b^2 + a^2}}{2}$

b) Find the value of K to represent the following equation pairs of Lines, also find the point of its intersections and the angle between them.

$$2x^2 + Kxy - 6y^2 + 3x + y + 1 = 0$$

solution

$a = 2 \quad b = -6 \quad h = \frac{K}{2} \quad g = \frac{3}{2} \quad f = \frac{1}{2} \quad c = 1$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Longrightarrow \begin{vmatrix} 2 & \frac{K}{2} & \frac{3}{2} \\ \frac{K}{2} & -6 & \frac{1}{2} \\ \frac{3}{2} & \frac{1}{2} & 1 \end{vmatrix} = 0$$

$$2\left[-6-\frac{1}{4}\right] - \frac{K}{2}\left[\frac{K}{2} - \frac{3}{4}\right] + \frac{3}{2}\left[\frac{K}{4} + 9\right] = 0$$

$$-\frac{25}{2} - \frac{K^2}{4} + \frac{3K}{8} + \frac{3K}{8} + \frac{27}{2} = 0$$

$$-\frac{25}{2} - \frac{K^2}{4} + \frac{3}{4}K + \frac{27}{2} = 0 \qquad (*4)$$

$$-50 - K^2 + 3K + 54 = 0$$

$$K^2 - 3K - 4 = 0$$

$$(K+1)(K-4) = 0 \rightarrow \boxed{K_1 = -1} \quad \boxed{K_2 = 4}$$

<u>at $K = -1$</u>

$$2x^2 - xy - 6y^2 + 3x + y + 1 = 0$$

$$2x^2 - xy - 6y^2 = 0$$

$$(2x + 3y + C_1)(x - 2y + C_2) = 0$$

Coeff. of $x \longrightarrow C_1 + 2C_2 = 3$

Coeff. of $y \longrightarrow -2C_1 + 3C_2 = 1$

$C_1 = 1$

$C_2 = 1$

$\therefore$ Two lines

$$2x + 3y + 1 = 0$$

$$x - 2y + 1 = 0$$

<u>at $K = 4$</u>

$$2x^2 + 4xy - 6y^2 + 3x + y + 1 = 0$$

$$2x^2 + 4xy - 6y^2 = 0$$

$$(2x + 6y + C_1)(x - y + C_2) = 0$$

Coeff. of $x \rightarrow C_1 + 2C_2 = 3$

Coeff. of $y \longrightarrow -C_1 + 6C_2 = 1$

$C_1 = 2$

$C_2 = \frac{1}{2}$

Two lines

$$2x + 6y + 2 = 0$$

$$x - y + \frac{1}{2} = 0$$

Intersection points (α, β)

$$2x+3y+1=0$$
$$x-2y+1=0$$
$$x=-5/7$$
$$y=\frac{1}{7}$$
$$(\alpha,\beta)=\left(-\frac{5}{7},\frac{1}{7}\right)$$
$$\theta=\tan^{-1}\frac{\pm 2\sqrt{h^2-ab}}{a+b}$$
$$\theta=\tan^{-1}\frac{\pm 2\sqrt{\frac{1}{4}+12}}{2-6}$$
$$\theta_1=\tan^{-1}\frac{7}{4}=60.255^\circ$$
$$\theta_2=180-60.255=119.7449^\circ$$

Intersection point (α, β)

$$2x+6y+2=0$$
$$x-y+\frac{1}{2}=0$$
$$x=-5/8$$
$$y=-\frac{1}{8}$$
$$(\alpha,\beta)=(-5/8,-1/8)$$
$$\theta=\tan^{-1}\frac{\pm 2\sqrt{h^2-ab}}{a+b}$$
$$\theta=\tan^{-1}\frac{\pm 2\sqrt{4+12}}{-4}$$
$$\theta_1=\tan^{-1}2=63.435$$
$$\theta_2=180-63.435=116.565^\circ$$

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c) If the origin is translated to the point $(-1,2)$ and axes rotated by +ve angle $\frac{\pi}{4}$, find the new form for eqn

$$4x^2+y^2+8x-4y+7=0$$

solution

$$\alpha=-1 \qquad \beta=2 \qquad \theta=\frac{\pi}{4}$$

$$x=u\cos\theta-v\sin\theta+\alpha$$
$$x=\frac{u}{\sqrt2}-\frac{v}{\sqrt2}-1$$

$$y=u\sin\theta+v\cos\theta+\beta$$
$$y=\frac{u}{\sqrt2}+\frac{v}{\sqrt2}+2$$

$$\because 4x^2+y^2+8x-4y+7=0$$

$$4\left[\left(\frac{u}{\sqrt2}-\frac{v}{\sqrt2}-1\right)\left(\frac{u}{\sqrt2}-\frac{v}{\sqrt2}-1\right)\right]+\left(\frac{u}{\sqrt2}+\frac{v}{\sqrt2}+2\right)\left(\frac{u}{\sqrt2}+\frac{v}{\sqrt2}+2\right)$$
$$+\frac{8u}{\sqrt2}-\frac{8v}{\sqrt2}-8-\frac{4u}{\sqrt2}-\frac{4v}{\sqrt2}-8+7=0$$

$$2u^2-2uv\cancel{-\frac{4u}{\sqrt2}}-2uv+2v^2+\frac{4v}{\sqrt2}\cancel{-\frac{4u}{\sqrt2}}\cancel{+\frac{4v}{\sqrt2}}\cancel{+4}$$
$$+\frac{u^2}{2}+\frac{uv}{2}\cancel{+\frac{2u}{\sqrt2}}+\frac{uv}{2}+\frac{v^2}{2}\cancel{+\frac{2v}{\sqrt2}}\cancel{+\frac{2u}{\sqrt2}}\cancel{+\frac{2v}{\sqrt2}}\cancel{+4}\cancel{+\frac{8u}{\sqrt2}}$$
$$\cancel{-\frac{8v}{\sqrt2}}\cancel{-8}\cancel{-\frac{4u}{\sqrt2}}\cancel{-\frac{4v}{\sqrt2}}-1=0$$

$$5/2\,u^2+5/2\,v^2-3uv-1=0$$

3 a- using true or false in the false case si
the correct answer

(1) $\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = (\checkmark)$ correct answer

(2) $\int (\cos 3x)\, e^{\sin 3x}\,dx = -e^{3\sin x} + c$ (x) false

$\int (\cos 3x)\, e^{\sin 3x}\,dx = \frac{-1}{3}\int -3\cos 3x\, e^{\sin 3x} = \frac{-1}{3} e^{\sin 3x} + c$

3) $\int_a^a f(x)\,dx = 0$ $(\checkmark)$ true

4) $\int \frac{\cos 2x}{[5+4\sin 2x]^2}\,dx = \frac{1}{[5+4\sin 2x]^2} + c$ (x) false

$\int \frac{\cos 2x}{(5+4\sin 2x)^2}\,dx = \frac{-1}{8}\int -8\cos 2x\,(5+4\sin 2x)^{-2}\,dx$

$= \frac{-1}{8}\,\frac{[5+4\sin 2x]^{-1}}{-1} + c = \frac{1}{8}\,\frac{1}{[5+4\sin 2x]} + c$

b) 1) $\int x\tan^3(5x^2)\,dx$

$I = \int x\tan(5x^2)\,\tan^2(5x^2)\,dx$

$= \int x\tan(5x^2)\,(\sec^2(5x^2) - 1)\,dx$

$= \frac{1}{10}\int 10x\,\overset{\text{تفاضل}}{\sec^2(5x^2)}\,\overset{\text{دالة}}{\tan(5x^2)}\,dx - \int x\tan(5x^2)\,dx$

$= \frac{1}{10}\,\frac{\tan^2(5x^2)}{2} - \frac{1}{10}\int 10x\,\frac{\sin(5x^2)}{\cos(5x^2)}$

$= \frac{1}{20}\tan^2(5x^2) - \frac{1}{10}\ln|\cos(5x^2)| + c$

3 b-

(2) $I = \int \frac{x^3}{(2+3x)^4} dx$

let $u = 2+3x \Longrightarrow x = \frac{u-2}{3} \longrightarrow (*)$ $dx = \frac{1}{3} du$

$I = \int \frac{(\frac{u-2}{3})^3}{u^4} \frac{1}{3} du = \int \frac{(1/3)^3 (1/3) (u-2)^3}{u^4} du$

$= (\frac{1}{3})^4 \int \frac{(u-2)(u-2)^2}{u^4} du = \frac{1}{81} \int \frac{(u-2)(u^2-4u+4)}{u^4} du$

$= \frac{1}{81} \int \frac{u^3 - 6u^2 + 12u - 8}{u^4} du$

$= \frac{1}{81} \int [\frac{1}{u} - \frac{6}{u^2} + \frac{12}{u^3} - \frac{8}{u^4}] du$

$= \frac{1}{81} [\ln|u| + \frac{6}{u} - \frac{6}{u^2} + \frac{8}{3u^3}] + c$

$I = \frac{1}{81} [\ln|2+3x| + \frac{6}{2+3x} - \frac{6}{(2+3x)^2} + \frac{8}{3(2+3x)^3}] + c$

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3) $\int \frac{dx}{\sqrt{8+2x-x^2}}$

$I = \int \frac{dx}{\sqrt{-(x^2-2x-8)}} = \int \frac{dx}{\sqrt{-[(x-1)^2-1-8]}}$

$= \int \frac{dx}{\sqrt{-[(x-1)^2-9]}}$

$= \int \frac{dx}{\sqrt{9-(x-1)^2}} =$

$I = \sin^{-1} [\frac{(x-1)}{3}] + c$

another solution

let $\sqrt{a^2-x^2}$ use $x = a\sin\theta$

$\sqrt{9-(x-1)^2}$ use $x-1 = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$I = \int \frac{3\cos\theta}{\sqrt{9-9\sin^2\theta}} d\theta$

$= \int \frac{3\cos\theta}{3\cos\theta} d\theta = \int d\theta$

$I = \theta + c$ $\because \theta = \sin^{-1}(\frac{x-1}{3})$

$I = \sin^{-1}(\frac{x-1}{3}) + c$

3 b)

(4) $I = \int 2^{(1+\cot 5t)} \operatorname{cosec}^2(5t)\, dt$

$$I = \frac{-1}{5}\int 2^{(1+\cot 5t)} \left[-5 \operatorname{cosec}^2(5t)\right] dt$$

$$\int a^{f(x)} f'(x)\, dx = \frac{a^{f(x)}}{\ln a} + c$$

$$I = \frac{-1}{5}\left[\frac{2^{(1+\cot(5t))}}{\ln 2}\right] + c$$

3 c) Find the reduction formula of the integral $I_n = \int x^n \sin(ax)\, dx$ then find $\int x^2 \sin(5x)\, dx$

$$I_n = -\frac{x^n}{a}\cos ax + \frac{n}{a^2} x^{n-1} \sin ax - \frac{n(n-1)}{a^2} \underbrace{\int x^{n-2} \sin ax}_{I_{n-2}}$$

	u		dV
	x^n	$+$	$\sin ax$
	$n\, x^{n-1}$	$-$	$-\frac{\cos ax}{a}$
	$n(n-1)\, x^{n-2}$	$+\int$	$-\frac{\sin ax}{a^2}$

$$I_n = \frac{-x^n}{a}\cos(ax) + \frac{n}{a^2} x^{n-1} \sin ax - \frac{n(n-1)}{a^2} I_{n-2}$$

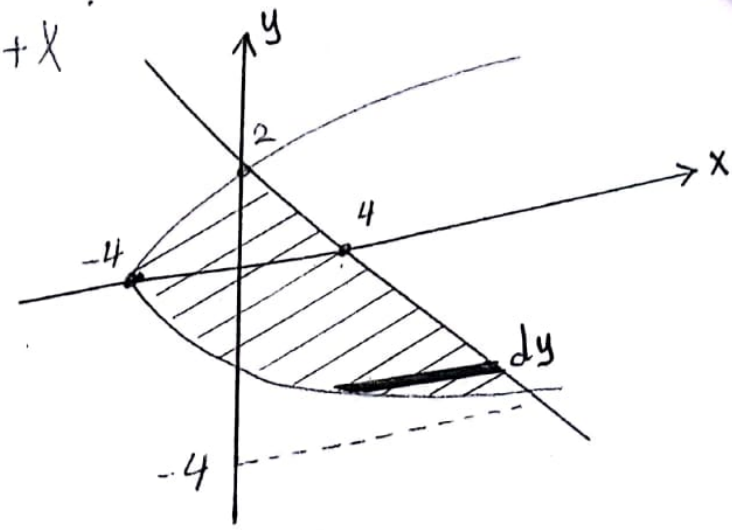
$$I_2 = \frac{-1}{5} x^2 \cos 5x + \frac{2}{25} x \sin 5x - \frac{2}{25} I_0$$

$$I_0 = \int x^0 \sin 5x\, dx = \int \sin 5x\, dx = -\frac{\cos 5x}{5}$$

$$\therefore I_2 = \frac{-1}{5} x^2 \cos 5x + \frac{2}{25} x \sin 5x + \frac{2}{125} \cos 5x + C$$

Q4 a) Find the area between the following curv...

$x + 2y = 4 \quad , \quad y^2 = 4 + x$



solution

① $x + 2y = 4 \longrightarrow$ Line

$x = 0 \longrightarrow y = 2$
$y = 0 \longrightarrow x = 4$

② $y^2 = 4 + x \longrightarrow x = y^2 - 4$

* Find intersection points

$y^2 = 4 + x$
$y^2 = 4 + 4 - 2y \Longrightarrow y^2 + 2y - 8 = 0 \Longrightarrow (y+4)(y-2) = 0$

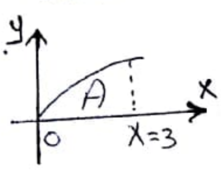
$\boxed{y = -4} \quad , \quad \boxed{y = 2}$

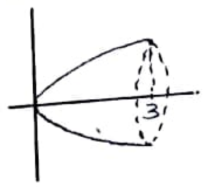
$$A = \int_{-4}^{2} \left[(4 - 2y) - (y^2 - 4) \right] dy$$

$$= \int_{-4}^{2} \left[-y^2 - 2y + 8 \right] dy = \left[\frac{-y^3}{3} - y^2 + 8y \right]_{-4}^{2}$$

$$= \left[\frac{-8}{3} - 4 + 16 \right] - \left[\frac{64}{3} - 16 - 32 \right] = 36 \text{ unit of area}$$

b) volume from the rotation of closed area between curves $y^2 = 8x$, $x = 3$ and x-axis





(1) rotation around x-axis

$$V = \pi \int_a^b [f(x)]^2 dx = \pi \int_0^3 8x \, dx$$

$$= 8\pi \left[\frac{x^2}{2} \right]_0^3 = 4\pi [9]$$

$V = 36\pi$ unit of volume

(2) rotation around y-axis

$$V = 2\pi \int_a^b x f(x) \, dx = 2\pi \int_0^3 x (\sqrt{8x})$$

$$= 2\sqrt{8} \pi \int_0^3 x \sqrt{x} \, dx$$

$$= 2\sqrt{8} \pi \int_0^3 x^{3/2} \, dx$$

$$= 2\sqrt{8} \pi \left[\frac{x^{5/2}}{5/2} \right]_0^3$$

$$= \frac{4}{5} \sqrt{8} \pi \left[3^{5/2} - 0 \right]$$

$V = 110.8123$ unit of volume

Use simpson's rule and trapezoidal rule [illegible]
steplength $h = 0.4$ to estimate

$$I = \int_{2}^{6} \frac{\ln(2+3\sqrt{x})}{1+x^2}\,dx$$ and compare your

result with result $I = 0.596545$

x	2	2.4	2.8	3.2	3.6	4	4.4	4.8	5.2	5.6	6
y	0.3663	0.2802	0.2204	0.1777	0.1461	0.1223	0.1039	0.0894	0.0777	0.0682	0.0604
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

[1] Trapezoidal method

$$I = \frac{h}{2}\left[y_0 + y_{10} + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)\right]$$

$$\boxed{I = 0.59997}$$

$$error = 0.596545 - 0.59997 = \boxed{-3.155 * 10^{-3}}$$

[2] simpson's rule

$$I = \frac{h}{3}\left[y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$

$$\boxed{I = 0.5965467}$$

$$error = 0.596545 - 0.5965467$$

$$error = \boxed{-1.7 * 10^{-6}}$$