نموذج إجابة

رياضيات هندسية (١-م)

Engineering Mathematics (1-A)

(٢٠١٥-٢٠١٦) اعدادى ترم أول (تفاضل + جبر)

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(2015-2016)

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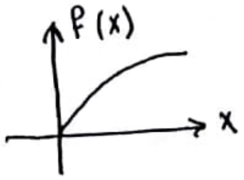
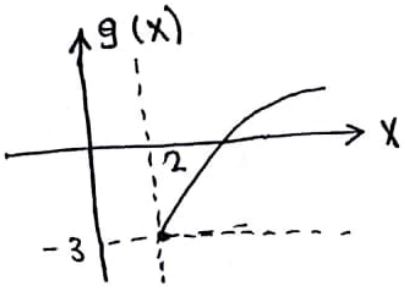
Engineering Mathematics (1)

18/11/2016

Q1 a) Draw the function $f(x)=\sqrt{x}$ then use it to draw $g(x)=(\sqrt{x-2})-3$



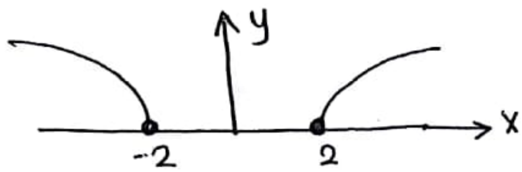


b) i) $y=\sqrt{x^2-4}$

$x^2-4 \geqslant 0 \rightarrow x^2 \geqslant 4$

$\pm x \geqslant 2 \rightarrow x \geqslant 2 \quad -x \geqslant 2$

$x \geqslant 2 \quad x \leqslant -2$

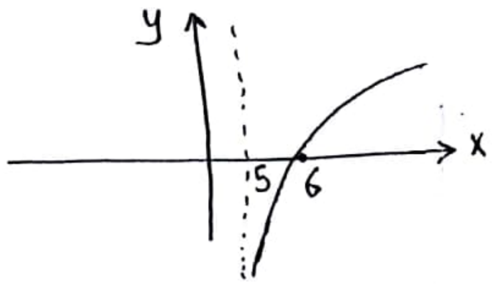


$D_f = R -]-2,2[$

$R_f = [0,\infty[$

ii) $y=\ln(x-5)$

$x-5>0 \longrightarrow x>5$



$D_f =]5,\infty[$

$R_f =]-\infty,\infty[$

c) Find the limits if exist:

i) $\lim\limits_{x\to 2}\dfrac{\sqrt{x-2}}{\sqrt{x^2-4}} = \lim\limits_{x\to 2}\dfrac{\sqrt{x-2}}{\sqrt{x-2}\sqrt{x+2}} = \lim\limits_{x\to 2}\dfrac{1}{\sqrt{x+2}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$

ii) $\lim\limits_{x\to 0}[1+\sin(x)]^{\frac{1}{x}} =$

1) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \sin x = 0$

2) $\lim\limits_{x\to 0}\dfrac{1}{x} = \infty$

3) $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1 = m$

$\therefore \lim\limits_{x\to 0}(1+\sin(x)]^{1/x} = e^{m} = e$

iii) $\lim\limits_{x\to\infty} \dfrac{3x^3+x^2+7}{5x^3+2x^2+x} = \dfrac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{3+1/x+7/x^3}{5+2/x+1/x^2} = \dfrac{3}{5}$ exist

iv) $\lim\limits_{x\to 0} \dfrac{x^2}{1-\cos x} = \dfrac{0}{0}$ using لوبيتال

$\lim\limits_{x\to 0} \dfrac{2x}{\sin x} = \dfrac{0}{0} \Longrightarrow \lim\limits_{x\to 0} \dfrac{2}{\cos x} = 2$ exist

d) Redefine function to be continuous at $x=3$

$f(x) = \dfrac{x^2+x-12}{x^2-9}$

$\lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-4)}{\cancel{(x-3)}(x+3)} = \lim\limits_{x\to 3} \dfrac{x+4}{x+3} = \dfrac{7}{6} = f(3)$

$\therefore f(x) = \begin{cases} \dfrac{x^2+x-12}{x^2-9} & , x \neq 3 \\ 7/6 & , x = 3 \end{cases}$

$\boxed{2}$ a) Find $y'(x)$ of the functions:

i) $y = \ln[\ln(\sinh^{-1}x)] + (\sin x)^x$

$y' = \dfrac{1}{\ln \sinh^{-1}x} \cdot \dfrac{1}{\sinh^{-1}x} \cdot \dfrac{1}{\sqrt{1+x^2}} + (\sin x)^x \left[\dfrac{x}{\sin x}\cos x + \ln \sin x\right.$

ii) $y = e^{\sqrt{x^2-1}} + 2^{\operatorname{cosec} x}$

$y' = e^{\sqrt{x^2-1}} \dfrac{2x}{2\sqrt{x^2-1}} + 2^{\operatorname{cosec} x} \cdot \ln 2 \,(-\operatorname{cosec} x \cdot \cot x)$

iii) $y = \tan(\sec^{-1} 3x) + e^{2\ln x}$

$y' = \sec^2(\sec^{-1}3x) \cdot \dfrac{3}{3x\sqrt{(3x)^2-1}} + 2x$

b) deduce $\frac{dy}{dx}$ if $y = [\cosh^{-1}(x)]$

$\cosh y = x \longrightarrow \sinh y \cdot y' = 1$

$y' = \frac{1}{\sinh y}$

$\cosh^2 y - \sinh^2 y = 1 \Longrightarrow \sinh y = \sqrt{\cosh^2 - 1} = \sqrt{x^2 - 1}$

$\therefore y' = \frac{1}{\sinh y} = \frac{1}{\sqrt{x^2-1}}$

c) if $y = a\cos(\ln x) + b\sin(\ln x)$ show that:

$x^2 y^{(n+2)} + (2n+1)\, x\, y^{(n+1)} + (n^2+1)\, y^{(n)} = 0$

$y^{(1)} = -a\,\frac{\sin(\ln x)}{x} + b\,\frac{\cos(\ln x)}{x}$

$x\, y^{(1)} = -a \sin(\ln x) + b\cos(\ln x)$

$x\, y^{(2)} + y^{(1)} = -a\,\frac{\cos(\ln x)}{x} - b\,\frac{\sin(\ln x)}{x}$

$x^2 y^{(2)} + x\, y^{(1)} = -y$

$x^2 y^{(2)} + x\, y^{(1)} + y = 0$

$x^2 y^{(n+2)} + \frac{n}{1!}\, 2x\, y^{(n+1)} + \frac{n(n-1)}{2!}\, 2\, y^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1)\, x\, y^{(n+1)} + (n^2+1)\, y^{(n)} = 0$ #

d) Deduce Maclaurine expansion: $f(x) = (1-x)^n$ then use it to calculate the following approximately $\frac{1}{\sqrt{45}}$.

$f(x) = (1-x)^n$

$f'(x) = -n(1-x)^{n-1}$

$f''(x) = n(n-1)(1-x)^{n-2}$

$f'''(x) = -n(n-1)(n-2)(1-x)^{n-3}$

$f(0) = 1$

$f'(0) = -n$

$f''(0) = n(n-1)$

$f'''(0) = -n(n-1)(n-2)$

$$(1-x)^n = 1 - \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3$$

$$\frac{1}{\sqrt{45}} = \frac{1}{\sqrt{49-4}} = \frac{1}{7\sqrt{1-4/49}} = \frac{1}{7}(1-0.0816)^{-1/2}$$

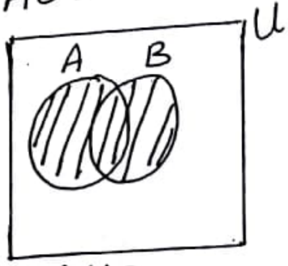
$$\frac{1}{\sqrt{45}} = \frac{1}{7}\left[1 - \frac{(-1/2)}{1!}(0.0816) + \frac{(-1/2)(-1/2-1)}{2!}(0.0816 + \cdots\right] \simeq 0.1491$$

Engineering Mathematics (1)
18/1/2016

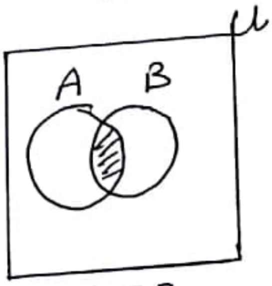
Q.3

a) By using venn diagram represent each of the following sets

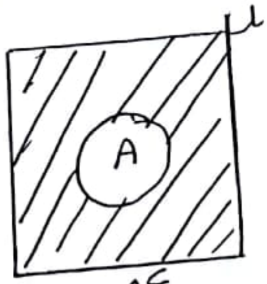
$A \cup B$, $A \cap B$, A^c , $A - B$, $A \Delta B$



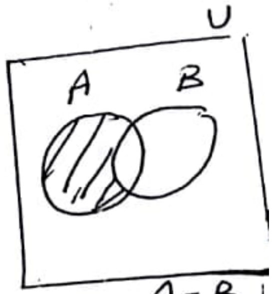
$A \cup B$



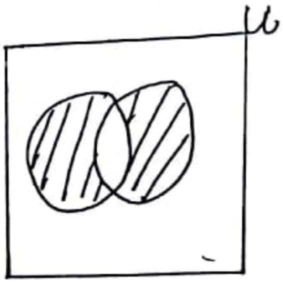
$A \cap B$



A^c



$A - B$



$A \Delta B$

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b) If $z_1 = 1+i$, $z_2 = -2+3i$, prove that $|z_1+z_2| < |z_1|+|z$

* $z_1 + z_2 = -1+4i$
  $|z_1+z_2| = \sqrt{1+16} = \sqrt{17} = 4.123 \longrightarrow (1)$
* $|z_1| = \sqrt{1+1} = \sqrt{2}$
* $|z_2| = \sqrt{4+9} = \sqrt{13}$
* $|z_1| + |z_2| = \sqrt{2} + \sqrt{13} = 5.01976 \longrightarrow (2)$

from (1) , (2)

$\therefore |z_1+z_2| < |z_1|+|z_2|$

b) factorize..

$$f(x) = \frac{4+8x^{-1}}{x^3-2x}$$

$$f(x) = \frac{4x+8}{x^4-2x^2} = \frac{4x+8}{x^2(x^2-2)} = \frac{4x+8}{x^2(x-\sqrt{2})(x+\sqrt{2})}$$

$$f(x) = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-\sqrt{2}} + \frac{D}{x+\sqrt{2}}$$

$$\frac{4x+8}{x^2(x-\sqrt{2})(x+\sqrt{2})} = \frac{Ax(x-\sqrt{2})(x+\sqrt{2})+B(x-\sqrt{2})(x+\sqrt{2})+Cx^2(x+\sqrt{2})}{x^2(x-\sqrt{2})(x+\sqrt{2})}$$

$$+ Dx^2(x-\sqrt{2})$$

$\therefore Ax(x-\sqrt{2})(x+\sqrt{2}) + B(x-\sqrt{2})(x+\sqrt{2}) + Cx^2(x+\sqrt{2}) + Dx^2(x-\sqrt{2}) = 4x+8$

let $x = \sqrt{2} \longrightarrow C(\sqrt{2})^2(2\sqrt{2}) = 4\sqrt{2}+8 \longrightarrow \boxed{C = 1+\sqrt{2}}$

Let $x = -\sqrt{2} \longrightarrow D(-\sqrt{2})^2(-2\sqrt{2}) = -4\sqrt{2}+8 \longrightarrow \boxed{D = 1-\sqrt{2}}$

Let $x=0 \longrightarrow -2B = 8 \longrightarrow \boxed{B = -4}$

Let $x=1 \longrightarrow A(1-\sqrt{2})(1+\sqrt{2}) + B(1-\sqrt{2})(1+\sqrt{2}) + C(1+\sqrt{2}) + D(1-\sqrt{2}) =$

$-A + B + (1+\sqrt{2})C + D(1-\sqrt{2}) = 12$

$-A - 4 + 3 + \cancel{2\sqrt{2}} + 3 - \cancel{2\sqrt{2}} = 12$

$\boxed{A = -10}$

$$f(x) = \frac{4+8x^{-1}}{x^3-2x} = \frac{-10}{x} + \frac{-4}{x^2} + \frac{1+\sqrt{2}}{x-\sqrt{2}} + \frac{1-\sqrt{2}}{x+\sqrt{2}}$$

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c) put in polar form

$$Z = \frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$$

$$= \frac{1}{4+4i-1} - \frac{1}{4-4i-1} = \frac{1}{3+4i} - \frac{1}{3-4i}$$

$$Z = \frac{\cancel{3}-4i - \cancel{3}-4i}{(3+4i)(3-4i)} = \frac{-8i}{9+16} = \frac{-8}{25}i$$

$x=0 \qquad y = \frac{-8}{25}$

$r = \sqrt{x^2+y^2} = \frac{8}{25}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{8/25}{0} = \frac{\pi}{2}$

$\theta_1 = \frac{3\pi}{2}$

$Z = \frac{8}{25}\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$

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d) Calculate the three currents i_1, i_2, i_3 of a circuit which are related by :-

$2i_1 - i_3 + i_2 = 8$

$i_1 - i_2 + 5 + i_3 = 0$

$3i_1 + 2i_2 = 9$

solution

$2i_1 + i_2 - i_3 = 8$

$i_1 - i_2 + i_3 = -5$

$3i_1 + 2i_2 + 0i_3 = 9$

$$\begin{array}{cccc} & + & - & + \\ \Delta = & \multicolumn{3}{c}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 2 & 0 \end{vmatrix}} \end{array} \begin{aligned} &= 2[-2] - 1[-3] - 1[2+3] \\ &= -4 + 3 - 5 = -6 \end{aligned}$$

$$\begin{array}{cccc} & + & - & + \\ \Delta i_1 = & \multicolumn{3}{c}{\begin{vmatrix} 8 & 1 & -1 \\ -5 & -1 & 1 \\ 9 & 2 & 0 \end{vmatrix}} \end{array} \begin{aligned} &= 8[-2] - 1[-9] - 1[-10+9] \\ &= -16 + 9 + 1 = -6 \end{aligned}$$

$$\begin{array}{cccc} & + & - & + \\ \Delta i_2 = & \multicolumn{3}{c}{\begin{vmatrix} 2 & 8 & -1 \\ 1 & -5 & 1 \\ 3 & 9 & 0 \end{vmatrix}} \end{array} \begin{aligned} &= 2[-9] - 8[-3] - 1[9+15] \\ &= -18 + 24 - 24 = -18 \end{aligned}$$

$$\begin{array}{cccc} & + & - & + \\ \Delta i_3 = & \multicolumn{3}{c}{\begin{vmatrix} 2 & 1 & 8 \\ 1 & -1 & -5 \\ 3 & 2 & 9 \end{vmatrix}} \end{array} \begin{aligned} &= 2[-9+10] - 1[9+15] + 8[2+3] \\ &= 2 - 24 + 40 = 18 \end{aligned}$$

$$i_1 = \frac{\Delta i_1}{\Delta} = \frac{-6}{-6} = 1$$

$$i_2 = \frac{\Delta i_2}{\Delta} = \frac{-18}{-6} = 3$$

$$i_3 = \frac{\Delta i_3}{\Delta} = \frac{18}{-6} = -3$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

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