نموذج إجابة

رياضيات هندسية ٢ ـــ ١

٢٠١٧ / ٢٠١٨

أولى كهرباء + مدني + ميكانيكا

Q1:

(1) $f(x,y) = xe^y + \sin\left(\frac{x}{y}\right) + \ x^3y^2$

$$f_y = xe^y + \frac{x}{y^2}\cos\left(\frac{x}{y}\right) + \ 2x^3y$$

$$f_x = e^y + \frac{1}{y}\cos\left(\frac{x}{y}\right) + \ 2x^3y^2$$

$$f_{xx} = \frac{1}{y^2}\sin\left(\frac{x}{y}\right) + \ 6xy^2$$

$$f_{yy} = xe^y + \frac{x^2}{y^3}\sin\left(\frac{x}{y}\right) - \ \frac{2x}{y^3}\cos\left(\frac{x}{y}\right) + \ 2x^3$$

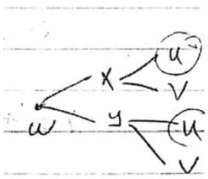
$$f_{xy} = e^y + \frac{x}{y^3}\sin\left(\frac{x}{y}\right) - \ \frac{1}{y^2}\cos\left(\frac{x}{y}\right) + \ 6x^2y = f_{yx}$$

(2) $w = f(x,y)$, $u^2 - \sin V = 3x + y$, $\cos u - 2V^2 = 2x - y$

$$5x = \ u^2 - \sin V \ + \ \cos u - 2V^2$$

$$y = \ u^2 - \sin V - 3x = u^2 - \sin V - \frac{3}{5}u^2 + \frac{3}{5}\sin V - \frac{3}{5}\cos V + \ \frac{6}{5}V$$

$$= \quad \tfrac{2}{5}u^2 - \tfrac{2}{5}\sin V - \tfrac{3}{5}\cos V + \ \tfrac{6}{5}V^2$$



$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} * \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} * \frac{\partial y}{\partial u}$$

$$= w_x\left[\frac{2}{5}u - \ \frac{1}{5}\sin u\right] + \ w_y\left[\frac{4}{5}u + \ \frac{3}{5}\sin u\right]$$

(3) $f(x,y) = \ x^2\ y + 3y - \ 2$

$f_x = 2\ x\ y$

$f_y = \ x^2 + 3$

$f(1,2) = \ -10$

$f_x = -4$

$f_y = 4$

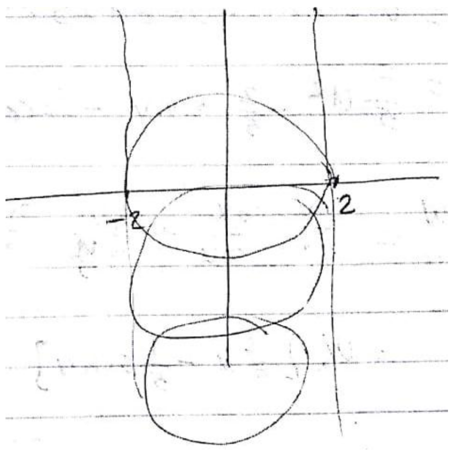
$f_{xx} = 2\,y$	$f_{yx} = -4$
$f_{yy} = 0$	$f_{yy} = 0$
$f_{xy} = 2\,y$	$f_{xy} = 2$

$$f(x,y) = f(a,b) + (x-a)f_x + (y-b)f_y + \frac{1}{2}\,[\,(x-a)^2 f_{xx}$$

$$+ \left(y-b\right)^2 f_{yy} + 2(x-a)(y-b)f_{xy}\big] + \ldots\ldots$$

$$f(x,y) = -10 - 4(x-1) + 4(y-2) - 2(x-1)^2$$
$$+ 2(x-1)(y+2) + \ldots.$$

(4) $x^2 + (y-a)^2 = 4$ ➔ (1)



Differentiate w.r.t a

2 (y – a) = 0 ➔ $a = y$

From (1)

$$x^2 = 4$$

$$x = \pm 2$$

(5) For homogenous function

$$xZ_x + yZ_y = kZ$$

Differentiate w.r.t x and y we can prove that

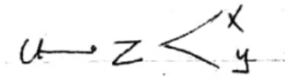
$$x^2 Z_{xx} + 2xyZ_{xy} + y^2 Z_{yy} = k(k-1)Z$$

$$For\ Z = \cos^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$$

$$Let\ u = \left(\frac{x^3 + y^3}{x - y}\right)$$

U w homage of degree 2

$\therefore\ xu_x + yu_y = 2u$



$$u = \cos z$$

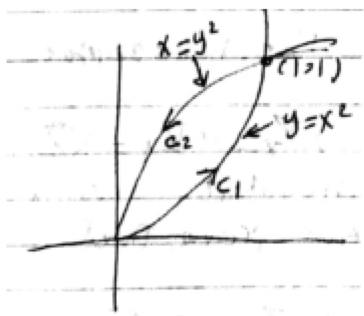
$$u_x = u_z z_x \quad , \quad u_y = u_z z_y$$

$$u_x = z_x(-\sin z) \quad , \quad u_y = z_y(-\sin z)$$

$$\therefore\ x z_x(-\sin z) + y z_y(-\sin z) = 2\cos z$$

$$\therefore\ x z_x + y z_y = 2\cot z$$

(6) $\oint_c\ xy\,dx + x^2\,dy$



$C:\quad x = y^2 \quad , \quad y = x^2$

$C_1:\ -(0,0) \quad \to \quad (1,1)$

$y = x^2$

$dy = 2x\,dx$

$$\therefore\ I_1 = \int_0^1 x^3\,dx + x^2\,(2x\,dx) = \int_0^1 3x^2\,dx = \frac{3}{4}$$

$On\ C_2:\ -(1,1) \quad \to \quad (0,0)$

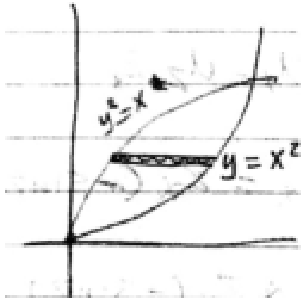
$x = y^2$

$dx = 2y\,dy$

$$\therefore\ I_2 = \int_1^0 y^3\,(2y\,dy) + y^4\,dy = \int_1^0 3y^4\,dy = -\frac{3}{5}$$

$$I = I_1 + I_2 = \frac{3}{20}$$

$$I = \int_0^1 \int_{y^2}^{\sqrt{y}} x \, dx \, dy$$



$$I = \int_0^1 \frac{x^2}{2} \int_{y^2}^{\sqrt{y}} dy$$

$$= \frac{1}{2} \int_0^1 [y - y^4] dy = \frac{1}{2} \left[\frac{y^2}{2} - \frac{y^5}{5}\right]_0^1$$

$$= \frac{1}{2} \left[\frac{1}{2} - \frac{1}{5}\right] = \frac{3}{20}$$

(7) $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{36} = 1$

Spherical coordinates

$$\frac{x}{3} = r \sin\theta \cos\varphi$$

$$\frac{y}{2} = r \sin\theta \sin\varphi$$

$$\frac{z}{6} = r \cos\theta$$

$$dx\, dy\, dz = abc\, r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= 36\, r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$r : 0 \rightarrow 1$$

$$\theta : 0 \rightarrow \pi$$

$$\varphi : 0 \rightarrow \pi$$

$$Volume = \int\int\int dx\, dy\, dz$$

$$= \int_{0}^{} \int_{0}^{1} \int_{0}^{\pi} abc\ r^2 \sin\theta\ d\theta\ dr\ d\varphi$$

$$= abc\ \frac{r^3}{3}\Big|_0^1\ [-\cos\theta]_0^\pi\ \ \varphi|_0^\pi$$

$$= abc\ [\pi]\ \ [2] = \frac{2}{3}\ \pi\ abc$$

$$= \frac{2}{3}\ (36)\pi = 24\,\pi$$

Q: 2

(1) $x^2 + \ y^2 = \ a^2$

$$2x + 2y\, y^{`} = 0$$

$$\frac{dy}{dx} = \ -\frac{x}{y}$$

(2) $M(x,y)dx + N(x,y)dy = 0$

$$\mu\ M\ dx + \mu\ N\ dy = 0$$

$$\frac{\partial\ \mu\ M}{\partial\ x} = \ \frac{\partial\ \mu\ N}{\partial\ y}$$

$$\mu\ \frac{\partial\ M}{\partial\ x} + M\ \frac{\partial\ \mu}{\partial\ x} = \ \mu\ \frac{\partial\ N}{\partial\ y} + N\ \frac{\partial\ \mu}{\partial\ y}$$

Let μ is a function of x only ➔ $\frac{\partial\ \mu}{\partial\ y} = 0$

$$\mu\ \frac{\partial\ M}{\partial\ x} + M\ \frac{\partial\ \mu}{\partial\ x} = \ \mu\ \frac{\partial\ N}{\partial\ y}$$

$$M\ \frac{\partial\ \mu}{\partial\ x} = \ \mu\left[\frac{\partial\ N}{\partial\ y} - \ \frac{\partial\ M}{\partial\ x}\right]$$

$$\frac{\partial\ \mu}{\mu} = \ \frac{1}{M}\left[\frac{\partial\ N}{\partial\ y} - \ \frac{\partial\ M}{\partial\ x}\right]$$

$$\therefore\ \ \mu = \ e^{\int \frac{1}{M}\left[\frac{\partial\ N}{\partial\ y} - \ \frac{\partial\ M}{\partial\ x}\right]dx}$$

3) $y^{`}\ = \ \frac{3x-4y-2}{6x-8y-5}$

$$\frac{3}{6} = \ + \ \frac{4}{8}$$

$\therefore$ let $U\ = \ 3x - 4y$

$$\frac{du}{dx} = 3 - 4\frac{dy}{dx}$$

$$\therefore\ \ \frac{3}{4} - \frac{1}{4}\frac{du}{dx} = \ \frac{U - 2}{2U - 5}$$

$$\frac{du}{dx} = \frac{2U - 7}{2U - 5}$$

$$\int\left[\frac{2U - 5}{2U - 7}\right]\ du = \ \int dx$$

$$\int\left[1 + \ \frac{2}{2U - 7}\right]\ du = x + c$$

$$U + \ \ln(2U - 7) = x + c$$

$$(3x - 4y) + \ \ln(2\ (3x - 4y) - 7) = x + c$$

8)

$$x\ \frac{dy}{dx} = y - x\ \ \coth^{2}\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{y}{x} - \coth^2\left(\frac{y}{x}\right)$$

Homogeny d.e.

Let $V = \frac{y}{x}$ $\quad\rightarrow\quad$ $\frac{dy}{dx} = V + x\,\frac{dV}{dx}$

$$V + x\,\frac{dV}{dx} = V - \coth^2 V$$

$$\int \tanh^2 V\, dV = \int -\frac{dx}{x}$$

$$\int [1 - \operatorname{sech}^2 V]\, dV = -\ln x + c$$

$$V - \tanh V = -\ln x + c$$

$$\left(\frac{y}{x}\right) - \tanh\left(\frac{y}{x}\right) = -\ln x + c$$

c) $2xy \ln y\, dx + \left(x^2 + y^3\sqrt{1 - y^2}\right)dy = 0$

$$\frac{\partial M}{\partial y} = 2x \ln y + 2x \neq \frac{\partial N}{\partial x} = 2x$$

$$\frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = \frac{-2\,x \ln y}{2\,x\,y \ln y} = \frac{-1}{y}$$

$$\therefore\ \mu = e^{\int -\frac{1}{y} dy} = e^{-\ln y} = \frac{1}{y}$$

$$2x \ln y\ dx + \left(\frac{x^2}{y} + y^2\sqrt{1 - y^2}\right) dy = 0$$

$$\int 2x \ln y\ dx + \int \left(\frac{x^2}{y} + y^2\sqrt{1 - y^2}\right) dy = C$$

$$x^2 \ln y + \int y^2 \sqrt{1-y^2}\ dy = C \quad \to (1)$$

$$I = \int y^2 \sqrt{1-y^2}\ dy$$

$$let\ \ y = \sin\theta \quad \to \quad dy = \cos\theta\ d\theta$$

$$I = \int \sin^2\theta\ \sqrt{1-\sin^2\theta}\cos\theta\ d\theta$$

$$I = \int \sin^2\theta * \cos^2\theta\ d\theta = \int (\sin\theta\cos\theta)^2\ d\theta$$

$$= \frac{1}{2}\int \sin^2 2\theta\ d\theta = \frac{1}{4}\int [1-\cos 4\theta]\ d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right] \qquad \to\ in\ (1)$$

(4) $r = c\ (cosec\ \theta + \cot\theta)$

$$\ln r = \ln(cosec\ \theta + \cot\theta) + \ln c$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{cosec\theta\cot a + \ cosec^2\theta}{cosec\theta + \cot\theta} = \ -cosec\ \theta$$

$$\frac{1}{r}\frac{dr}{d\theta} \quad \to \quad -r\frac{d\theta}{dr}$$

$$\int \frac{dr}{r} = \int \sin\theta\ d\theta$$

$$\ln r = \ -\cos\theta + k$$

λ

(5) $D^2\ (D^2+4)(D+1)y = 2^x + \sin x$

$$\lambda^2\left(\lambda^2+4\right)(\lambda+1) = 0$$

$$\lambda = 0, 0 \quad , \quad \lambda = \pm 2i \qquad , \lambda = -1$$

$$y_h = c_1 + c_2\, x + c_3 \sin^2 x + c_4 \cos 2x + c_5\, e^{-x}$$

$$y_p = y_{p_1} + y_{p_2}$$

$$y_{p_1} = \frac{1}{D^2(D^2+4)(D+1)y}\; 2^x = \frac{1}{D^2(D^2+4)(D+1)y}\; e^{x \ln 2}$$

$$= \frac{1}{(\ln 2)^2(\ln 2)^2(\ln 2 + 1)}\; 2^x$$

$$y_{p_2} = \frac{1}{D^2(D^2+4)(D+1)} \sin x$$

$$= \frac{1}{-(-1+4)(D+1)}\,\frac{D-1}{D-1} \sin x$$

$$= \frac{1}{-3}\,\frac{(D-1)}{(D^2-1)} \sin x = \frac{1\,(D-1)}{-3\,(-2)}[-\sin x]$$

$$= \frac{1}{6}\;[\cos x - \sin x]$$

$$y_G = y_p + y_h$$

c) $(D^2+1)y = \sin x$

$$\lambda^2 - 1 = 0 \rightarrow \lambda = \pm i$$

$$y_h = c_1 \cos x + c_2 \sin x$$

$$y_p = \frac{1}{D^2+1} \sin x = I_m\; \frac{1}{D^2+1}\; e^{+ix}$$

$$= I_m\; \frac{1}{(D+i)(D-i)}\; e^{ix}$$

$$= I_m\; \frac{1}{2i\,(D-i)}\; (e^{ix}.1)$$

$$= I_m \left[\frac{e^{ix}}{2i} \cdot \frac{1}{D + i - i} (1)\right] = I_m \left[\frac{x\, e^{ix}}{2i}\right]$$

$$= -\frac{x}{2} \left[I_m\, (i \cos x - \sin x)\right] = -\frac{x}{2} \cos x$$

$$\therefore\ y_G = y_h + y_p$$

$$= c_1 \cos x + c_2 \sin x - \frac{x}{2} \cos x$$

B) $(D^6 - 1)y = \cos^2 c \cos 2x$

$(\lambda^6 - 1) = 0 \rightarrow (\lambda^3 - 1)(\lambda^3 + 1) = 0$

$(\lambda - 1)(\lambda^2 - \lambda + 1)(\lambda + 1)(\lambda^2 + \lambda + 1) = 0$

$\lambda = 1, -1, \frac{1}{2} \pm i\, \frac{\sqrt{3}}{2}, \frac{-1}{2} \pm i\, \frac{\sqrt{3}}{2}$

$$y_h = c_1 e^x + c_2 e^{-x} + e^{\frac{1}{2}x} \left[c_3 \cos \frac{\sqrt{3}}{2} x + c_4 \sin \frac{\sqrt{3}}{2} x\right]$$

$$+ e^{-\frac{1}{2}x} \left[c_5 \sin \frac{\sqrt{3}}{2} x + c_6 \cos \frac{\sqrt{3}}{2} x\right]$$

$$y_p = \frac{1}{D^6 - 1} \left[\cos^2 x \cos 2x\right]$$

$$y_p = \frac{1}{D^6 - 1} \left[\frac{1}{2}(1 + cos\ 2x) \cos 2x\right]$$

$$y_p = \frac{1}{D^6 - 1} \left[\frac{1}{2} cos\ 2x + \frac{1}{2} \cos^2 2x\right]$$

$$y_p = \frac{1}{D^6 - 1} \left[\frac{1}{2} cos\ 2x + \frac{1}{4}(1 + cos\, 4x)\right]$$

$$y_p = \frac{1}{D^6 - 1}\left[\frac{1}{2} cos\ 2x + \frac{1}{4}\ e^o + \frac{1}{4} cos\ 4x)\right]$$

$$= \frac{1}{2} \cdot \frac{1}{(-2)^3 - 1} \cos 2x + \frac{1}{4x - 1} + \frac{1}{4(-4)^3 - 1} \cos 4x$$

c) $y^{``} - 4y^{`} + 4y = (x+1)e^{2x}$

$$(D^2 - 4D + 4)y = (x1)e^{2x}$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

$$y_h = c_1 e^{2x} + c_2\ xe^{2x}$$

$$W\ (e^{2x}, x\ e^{2x}) = \begin{vmatrix} e^{2x} & x\ e^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & x\ e^{2x} \\ (x+1)e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = -\ (x^2 + x)\ e^{4x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & (x+1)e^{2x} \end{vmatrix} = (x+1)\ e^{4x}$$

$$c_1(x) = \int \frac{w_1}{w}\ dx = \frac{-x^3}{3} - \frac{x^2}{2}$$

$$c_2(x) = \int \frac{w_2}{w}\ dx = \frac{x^2}{2} + x$$

$$y_p = c_1(x)\ e^{2x} + c_2(x)\ xe^{2x}$$

$$\therefore\ y_G = y_h + y_p$$

Q3: (1) L.T

$$f(t) = sinht\ (t + \cosh^2 t)$$

$$= \left(t + \frac{1}{4}\ (e^{t} +\ e^{-t})^2\right) \sinh t$$

$$= \left(t + \frac{1}{4}\ (e^{2t} +\ e^{-2t} + 2)\right) \sinh t$$

$$L\ (\sinh t) = \frac{1}{s^2 - 1}$$

$$L[(\sinh t\ \left(\frac{1}{4} e^{2t} + \frac{1}{4} e^{-2t} + \frac{1}{2} + t\right)] =$$

$$= \frac{1}{4}\left[\frac{1}{(s-2)^2 - 1}\right] + \frac{1}{4}.\frac{1}{(s+2)^2 - 1} + \frac{1}{2}\ \frac{1}{s^2 - 1} - \frac{d}{ds}\frac{1}{s^2 - 1}$$

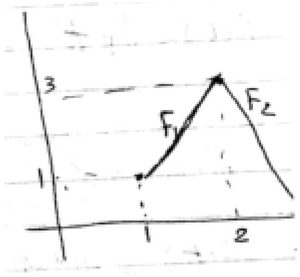
$$F(s) = \frac{1}{4}.\ \frac{1}{(s-2)^2 - 1} +\ \frac{1}{4}.\frac{1}{(s+2)^2 - 1} + \frac{1}{2}.\frac{1}{s^2 - 1} -\ \frac{2s}{(s^2 - 1)^{-2}}$$

$$B = f(t) = \int_0^t e^{-2u}\frac{\sin u}{u}\ du$$

$$L\left(\frac{\sin t}{t}\right) = \int_s^\infty \frac{1}{u^2 + 1}\ du = \tan^{-1} u\Big|_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1} s$$

$$f(s) = \frac{1}{s}\left[\frac{\pi}{2} - \tan^{-1}(s + 2)\right]$$



$f_1(t) = 2\ (t - 1) + 1$

$f_2(t) = -3\ (t - 3)$

$\therefore f(t) =\ [2\ (t - 1) + 1][u(t - 1) - u(t - 2)]$

$\quad\quad + [-3\ (t - 3)][u(t - 2) - u(t - 3)]$

$$= [2(t-1)+1]\,u(t-1) + 3(t-3)u(t-3) - 5(t-2)u(t-2)$$

$$f(s) = e^{-s}\left[\frac{2}{s^2} + \frac{1}{s}\right] + \frac{3e^{-3s}}{s^2} - \frac{5\,e^{-2s}}{s^2}$$

For elec. And mech. Eng.

Q3- b)

$$f(s) = \frac{1}{s^5} + \frac{s}{s^2+2s+5}$$

$$= \frac{4!}{4!\,s^5} + \frac{(s+1)-1}{(s+1)^2+4}$$

$$f(t) = \frac{1}{4!}t^4 + e^{-t}\left[\cos 2t - \frac{1}{2}\sin 2t\right]$$

$$f(s) = \frac{e^{-\pi s}}{s^2-16}$$

$$f(t) = u(t-\pi)L^{-1}\frac{1}{s^2-16}$$

$$f(t) = u(t-\pi)\,.\frac{1}{4}\sinh 4(t-\pi)$$

c) $y^{``} - y = 1$ $\quad ,y(0) = 1$ $\quad ,y^{`}(0) = -2$

$$s^2Y - s\,y(0) - y^{`}(0) - Y = \frac{1}{s}$$

$$(s^2-1)Y - s + 2 = \frac{1}{s}$$

$$Y = \frac{1}{s\,(s^2-1)} + \frac{s-2}{(s^2-1)}$$

$$= \frac{1 + s^2 - 2s}{s\ (s^2 - 1)} = \frac{(s-1)^2}{s\ (s-1)(s+1)}$$

$$\frac{(s-1)}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

A = -1 , B = 2

$$y(t) = L^{-1}\left[-\frac{1}{s} + \frac{2}{s+1}\right]$$

$= -1 + 2e^{-t}$