

Kafrelsheikh University

Faculty of Engineering

Physics & Engineering Mathematics Dept.

Year: Prep.



Subject: Engineering Mathematics (1)

Date: 15/1/2017

Time allowed: 3 hours

Full Marks: 110

Final-Term Exam: 2 pages

Question 3: [20 marks]

Complete the following sentences:

- 1) If $A = \{x : 0 \leq x \leq 2, x \in \mathbb{Z}\}$, then $P(A) = \dots\dots\dots$
- 2) $A \Delta B = \dots\dots\dots$
- 3) $R \subset A \times A$ is an equivalence relation if R is,,
- 4) $P = \{A_1, A_2, \dots, A_n\}$ is a partition on A if,,
- 5) A is orthogonal matrix if and A is hermitian if
- 6) If $A = \{\dots, -4, -3, 6, 7, 8, \dots\}$ then $A^c = \dots\dots\dots$
- 7) $|e^{i\theta}| = \dots\dots\dots$
- 8) $x \notin A \cap B \Leftrightarrow \dots\dots\dots$
- 9) If $A = [-5, 3)$, $B = (1, \infty)$, then $A \cap B = \dots\dots\dots, A \cup B = \dots\dots\dots$
- 10) The Algebraic system $(B, +, \cdot)$ is called Bool system if it satisfies:

Question 4: [35 marks]

a) By using Mathematical Induction, prove that:

$$2^{(n+1)} > n^2, \quad \forall n \in \mathbb{N}.$$

b) Let $X = \{2, 4, 6, 7\}$, and $R: X \rightarrow X$ given by :

$$R = \{(x, y) : x, y \in X \text{ and } x + y \text{ is an even integer}\}.$$

Prove that R is an Equivalence relation.

c) Analyze $I = \frac{x^4 + 2x^3 + 6x^2 + 2x - 5}{x^3 - x}$

d) Use De Moivre to find $(\sqrt{3} + i)^5$

e) If $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 5 & 2 \\ 0 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$

Find the matrix X which satisfies the equation $AX=B$

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**Answer the following questions:****Question 1: [25 marks]**

a) Draw the curves of $y = \cos^{-1} x$, $y = \cos x$ and determine the domain & range for both of them.

b) Find the following limits if exist:

$$i) \lim_{x \rightarrow \infty} \left(\frac{x+5}{x+1} \right)^{x+3}, \quad ii) \lim_{x \rightarrow 0} (x \cdot \ln x^3)$$

$$iii) \lim_{x \rightarrow 0} \frac{\sin x}{\ln(1+8x)}$$

c) Determine the point of discontinuity of $f(x)$, then obtain if you can

$$\text{redefine it to be continuous at this point } f(x) = \frac{x - |x|}{x}$$

d) Deduce $\frac{dy}{dx}$ if $y = \sec^{-1} x$

Question 2: [30 marks]

a) Find $y'(x)$ of the following functions:

$$(i) y = 5^{\ln(\operatorname{cosec} x + \tan x)} + e^{\coth^{-1} x} + \sin^{-1}(\tan x^2)$$

$$(ii) y = \cos^2(8^x) + (\tanh x)^{\sin x}$$

$$(iii) y = \cos^3 t, \quad x = \sqrt{t}$$

b) Deduce the n^{th} derivative of the following function:

$$y = e^{ax} \sin bx, \quad \text{then find } y = e^{\sqrt{2}x} \sin \sqrt{2}x$$

c) Deduce Maclaurin expansion of: $f(x) = \sinh x$

d) If $y = \ln \sqrt{\sin x}$, show that

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} \operatorname{cosec}(2x) = 0$$