



**Answer the following questions:**

**Question 1: [30 marks]**

- a) Fit the curve  $y = \frac{1}{a+b \cos \theta}$  to the following data, and find the root mean square error.

i	1	2	3
$\theta$	30	45	60
$y_i$	0.225	0.27	0.32

- b) By using function  $f(x) = \frac{\sin x}{1+x^2}$  in interval  $0 \leq x \leq \pi$ .

Make a table of data between  $x$  &  $f(x)$  which has 5 points, then by using Newton's interpolation find  $f(0.1)$ ,  $f(3.1)$  according to the reading of table.

- c) Find the root of the equation by Secant method for  $f(x) = x - e^{-x}$  correct to 2 decimal places.

**Question 2: [30 marks]**

- a) Find eigenvalues and the corresponding eigenvectors of the matrix A,

where  $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

- b) Use Runge Kutta method to estimate  $y(0.4)$  if  $y' = 2x + y$ ,  $y(0) = 1$   
 c) Approximate the following integral by using Trapezoidal and Simpson's rules

$\int_0^{0.6} \frac{1}{\sqrt{4-x^2}} dx$ ,  $n=6$  and then compare the two results with exact solution

(Note:  $\int \frac{1}{\sqrt{4-x^2}} dx = \sin^{-1}\left(\frac{x}{2}\right) + c$ )

**Question 3: [30 marks]**

- a) Use Gauss elimination method to find the solution of the system:

$$5x - 5y + 10z = -25$$

$$2x + 8z = 6$$

$$x + y + z = 9$$

- b) Prove that  $\Gamma(n) = (n-1)\Gamma(n-1)$

- c) Evaluate  $I = \int_0^5 \sqrt{\frac{x}{(125-x^3)}} dx$