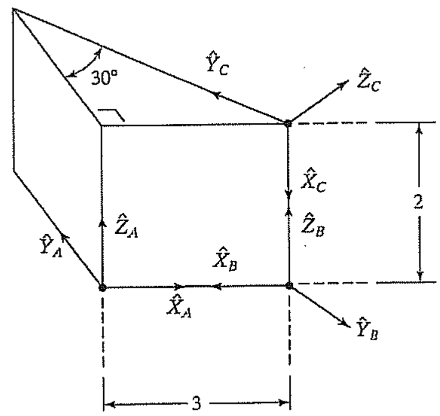
Question 1 (12 Marks):

a) Define the following: Forward kinematics, inverse kinematics, workspace, and manipulator singularities.

b) Define actuator space, joint space, and Cartesian space. Explain the mapping between these kinematic descriptions.

c) Referring to shown figure, give the value of ${}^{B}_{C}T$.



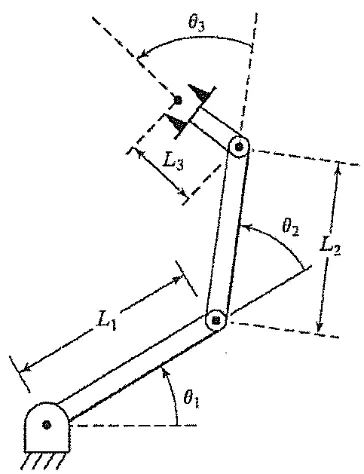
Question 2 (12 Marks):

Solve the inverse kinematics for the manipulator shown in the figure.
Given the following kinematic equation.

$$ {}^{0}_{3}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1c_1 + l_2c_{12} \\ s_{123} & c_{123} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

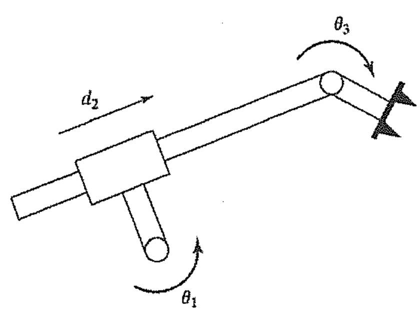
And

$$ {}^{0}_{3}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



Question 3 (12 Marks):

The arm with three degrees of freedom shown in the figure. The positive sense is of the joint angle is indicated. Assign link frames {0} to {3} for this arm – that is sketch the arm showing the attachment of the frames. Drive link parameters and the transformation matrices ${}^{0}_{1}T$, ${}^{1}_{2}T$, and ${}^{2}_{3}T$.



Question 4 (10 Marks):

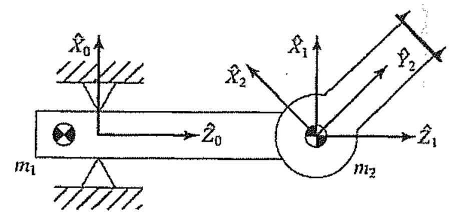
The position of the origin of link 2 for an RP manipulator is given by

$$ {}^{0}P_{2ORG} = \begin{bmatrix} a_1c_1 - d_2s_1 \\ a_1s_1 + d_2c_1 \\ 0 \end{bmatrix} $$

Find the 2 x 2 Jacobian that relates the two joint rates to the linear velocity of the origin of frame {2}. Give a value of the joint variables where the device is at a singularity. Ignoring gravity, what are the joint forces or torques required in order that the manipulator will apply static force vector ${}^{0}F = 5\hat{X}_0$.

Question 5 (14 Marks):

Derive the equations of motion for the PR manipulator shown in the figure. Neglect friction, but include gravity. (Here, X_0 is upward.)



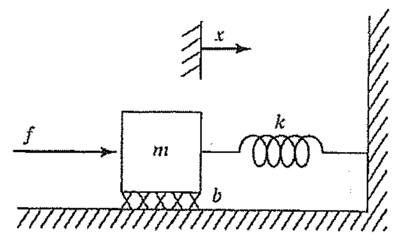
The inertia tensors of the links are diagonal, with moments $I_{xx1}, I_{yy1}, I_{zz1}$, and $I_{xx2}, I_{yy2}, I_{zz2}$.

The centers of mass for the links are given by:

$$
{}^{1}P_{C1} = \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix} \text{ and } {}^{2}P_{C2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

Question 6 (16 Marks):

a. Consider the system of shown in the figure with the parameter values in $m = 1$, $b = 4$, and $k = 5$. The system is also known to possess an unmodeled resonance at $\omega_{res} = 6.0$ radians/second. Determine the gains k_v and k_v that will critically damp the system with as high a stiffness as is reasonable.



b. Give the nonlinear control equations for an α and β- partitioned controller for the following system:

$$
\tau = \left(2\sqrt{\theta} + 1\right)\ddot{\theta} + 3\dot{\theta}^2 - \sin\theta
$$

Choose gains so that this system is always critically damped with $k_{cL} = 10$.

End of Questions

Best wishes

Useful relations

$a_i =$ *the distance from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured along* $\hat{X}_i$;

$\alpha_i =$ *the angle from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured about* $\hat{X}_i$;

$d_i =$ *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

$\theta_i =$ *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{i+1}\omega_{i+1} = {}^{i+1}_{\ \ i}R\, {}^{i}\omega_i + \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}.
$$

$$
{}^{i+1}v_{i+1} = {}^{i+1}_{\ \ i}R({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1})
$$