Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Second Year-Electric.

Date: 26/1/2015
Time allowed: 3 hours
Full mark: 90

Subject: Engineering Mathematics (3)
Mid-Term Exam: 1 page

Answer the following questions:

Question 1: [30 marks]

a) Let $z_1 = 1+i$, $z_2 = 1+\sqrt{3}i$ then find $\mathrm{Re}(z_1 z_2)$, $Im(z_1/z_2)$.

b) Find the real part u and the imaginary part v for the function $f(z) = u + iv$ if $(i)\ f(z) = ze^{-z}$, $(ii)\ f(z) = \ln z$.

c) Find the following limits if exist: $(i)\ \lim\limits_{z\to 0} \frac{3x^2 y}{x^2 + y^2}$ $(ii)\ \lim\limits_{z\to 0} \frac{\bar{z}}{z}$

d) Discuss the continuity of the following function: $f(z) = \begin{cases} \frac{\bar{z}}{z} & , \quad z \neq 0 \\ 3 & , \quad z = 0 \end{cases}$

Question 2: [30 marks]

a) Prove that $\frac{d}{dz}(\ln z) = \frac{1}{z}$

b) Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane, then find a conjugate harmonic v of u.

c) Prove that $\ln \frac{(x+iy)}{(x-iy)} = 2i \tan^{-1}\left(\frac{y}{x}\right)$

d) Evaluate the following integral: $\int_{1+i}^{2+4i} z^2\, dz$ along the parabola x=t, y=t^2, 1<t<2

Question 3: [30 marks]

a) Evaluate the following integrals around the contour $C: |z| = 2$:

$(i)\ \oint_c \frac{z^2 - 3}{z^2 + 4z + 3} dz$ $(ii)\ \oint_c \frac{e^z}{(z-1)^2} dz$ $(iii)\ \oint_c \frac{\sin z}{(z+3)z^2} dz$

b) If $f(z) = \ln(1+z)$, find Maclaurin series, then find Maclaurin series of $\tanh^{-1} z = \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$

c) Determine the order of the poles for the following function and find the residue at each pole: $f(z) = \frac{z+1}{z^2(z-2)}$

d) By using Cauchy's Residue theorem, compute the integral: $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$

With my best wishes . ============== *Dr. Manal El-said Ali*