



The following questions measure ILOs a1, a2, b1, b2 and c1.

Question 1: (a1,b2) (35 Marks)

1. Find the domain of definition of the following function :

$$f(x, y) = \ln(|x| + |y| - 2).$$

2. Can $f(0,0)$ be defined so that $f(x,y)$ is continuous at $(0,0)$ when

$$f(x, y) = \frac{\sin(x^2 + y)}{x + y}.$$

3. Let $w = f(x, y)$, $\tan u - \sin v = 3x + 2y$ and $\cos u - 2 \ln v = 3x - y$ then find $\frac{\partial w}{\partial u}$.

4. Expand in Maclaurin series the function, $f(x, y) = \cos 3(x + y)$.

5. Prove that if $z = f(x, y)$ is homogeneous of degree k , then $xz_x + yz_y = kz$, and If

$$z = \ln[(x^3 + 2xy^2 + y^3)/(x - y)] \text{ then find the value of } xz_x + yz_y.$$

6. Find the shortest distance from the point $z_0 = (1, -2, -1)$ to the straight line

$$x = y = z.$$

7. Verify Green's theorem for the integral $I = \oint_C y dx + 2x dy$, where C is the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

Question 2: (a1,b2, b1) (40 Marks)

1. Find the differential equation of the family of circles centered at $(C,0)$ and with radius $r=C$.

2. Find the orthogonal trajectories of $r = c(\sec \theta + \tan \theta)$.

3. Solve the D. Eqs. :

A) $(\cos 2x \sin x - xy^2)dx - y(x^2 - 1)dy = 0$

B) $xy' + y = xy^2 \ln x$

C) $xy^2 dy + (x^2 + 1)(y + 1)dx = 0$

4. Solve the D. Eqs. :

A) $D^2(D^2 + 4)(D + 1)^2 y = 4^x + \cos x$.

B) $(D^6 + 5D^4 + 4D^2)y = 2e^x + \sin^2 x$.

C) $y'' + y = \ln(\sin x)$.

D) $x^2 y'' - 2xy' + 2y = 5 + 2x^2 \ln x$.

Question 3: (b1, c1) (15 Marks)

1. Evaluate Laplace transform for:

A) $f(t) = (t^3 + t \cos 2t) \cosh t$.

B) $f(t) = e^{2t} \int_0^t \frac{\sin u}{u} du$.

2. Find inverse Laplace for:

A) $L^{-1}\left(\frac{s+2}{s(s^2+1)}\right)$

B) $L^{-1}\left(\ln \frac{s+5}{s-1}\right)$

3. Solve the following differential equation using Laplace transform :

$$y'' - y = e^{2t}, y(0) = y'(0) = 0.$$

Good Luck
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