

نموذج أجابة

مبانيات هندسية (أ - ٢)

الفرقة الإعدادية

2018/2019

الترم الأول

1/1/2019

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Answer of Q₁

choose the suitable answer:

1) If $\frac{x^3-1}{(x+1)(x+2)} = (x+\alpha) + \frac{\beta}{x+1} + \frac{\gamma}{x+2}$ then the value of α, β, γ

(a) $(-3, -2, 9)$

2) If $Z = \left| \frac{2i}{3+i} \right|$, then $|Z|$ equal to

(d) $\frac{\sqrt{10}}{5}$

3) For the system: $3i_1 - 2i_2 + 4i_3 = 2$
 $i_1 + 3i_2 - 6i_3 = 8$, the value of i_3 is
 $2i_1 - i_2 - 2i_3 = 0$

(d) $\frac{1}{2}$

4) If $2 \begin{bmatrix} a & -b \\ 3c & -2d \end{bmatrix} = \begin{bmatrix} 3a & 1 \\ c & 3+2d \end{bmatrix} - 3 \begin{bmatrix} -1 & a-b \\ a+b & 2c \end{bmatrix}$ then a, b, c, d are

(a) $(-3, -2, 3, 5/2)$

Answer of Q₂

① (✓)

② (X)

③ (X)

④ (X)

⑤ (✓)

⑥ (✓)

⑦ (X)

⑧ (✓)

⑨ (X)

⑩ (✓)

[Q₃] Complete(1) $P = \{A_1, A_2, \dots, A_n\}$ is a partition on A if

$$\textcircled{1} A_i \neq \phi, A_i \in P \quad \textcircled{2} A_i \cap A_j = \phi, i \neq j \quad \textcircled{3} \bigcup_{i=1}^n A_i = A$$

$$(2) \{a, b\} \Delta \{b, c\} = \{a, c\}$$

$$(3) A \text{ is symmetric matrix if } A = A^T$$

$$(4) (A \setminus B) \cap B = \phi$$

$$(5) A \text{ is orthogonal matrix if } A A^T = I$$

$$(6) |e^{i\theta}| = |\cos\theta + i\sin\theta| = 1$$

$$(7) (-2 + \sqrt{-16})(4 - \sqrt{-9}) = 4 + 22i$$

$$(8) \text{Im} [\overline{(2+i)(3+i)}] = -5$$

$$(9) \left| 2 + \frac{3}{2}i \right| = \frac{5}{2}$$

$$(10) A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$$

Answer of Q4

a) Find y' if $x^y = 1 + \ln(\sinh^{-1} x)^x$

$$e^{y \ln x} = 1 + x \ln(\sinh^{-1} x)$$

$$y e^{y \ln x} = 1 + x \ln(\sinh^{-1} x)$$

$$e^{y \ln x} \left[y \frac{1}{x} + y' \ln x \right] = x \frac{1}{\sinh^{-1} x} + \ln(\sinh^{-1} x)$$

$$y' \left[\frac{y e^{y \ln x}}{e^{y \ln x}} \cdot \ln x \right] = \left\{ x \frac{1}{\sinh^{-1} x} + \ln(\sinh^{-1} x) - \frac{y}{x} e^{y \ln x} \right\}$$

$$\therefore y' = \frac{\left\{ x \frac{1}{\sinh^{-1} x} + \ln(\sinh^{-1} x) - \frac{y}{x} e^{y \ln x} \right\}}{\left[\frac{y e^{y \ln x}}{e^{y \ln x}} \cdot \ln x \right]}$$

b) If $y = \frac{1}{x} + \sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)$, show that $x^5 y'' + 2x^4 y' + xy = 1$

$$y' = \frac{-1}{x^2} + \cos\left(\frac{1}{x}\right) \left(\frac{-1}{x^2}\right) - \sin\left(\frac{1}{x}\right) \left(\frac{-1}{x^2}\right) \quad * x^2$$

$$x^2 y' = -1 - \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$$

$$x^2 y'' + 2x y' = \sin\left(\frac{1}{x}\right) \left(\frac{-1}{x^2}\right) + \cos\left(\frac{1}{x}\right) \left(\frac{-1}{x^2}\right) \quad * x^2$$

$$x^4 y'' + 2x^3 y' = \underbrace{-\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)}$$

$$x^4 y'' + 2x^3 y' = -y + \frac{1}{x}$$

$$x^5 y'' + 2x^4 y' = -xy + 1$$

$$x^5 y'' + 2x^4 y' + xy = 1 \quad \neq$$

c) Deduce Maclaurin expansion of $f(x) = \ln(1+x)$ then
 Find Maclaurin expansion of $g(x) = e^x \ln(1+x)$
 up to third degree.

$$f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$$f''(x) = -(1+x)^{-2}$$

$$f'''(x) = 2(1+x)^{-3}$$

$$f^{(4)}(x) = -6(1+x)^{-4}$$

$$f(0) = \ln 1 = 0$$

$$f'(0) = 1$$

$$f''(0) = -1$$

$$f'''(0) = 2$$

$$f^{(4)}(0) = -6$$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^x \ln(1+x) = \left(1 + x + \frac{x^2}{2!} + \dots\right) \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \dots\right)$$

$$e^x \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + x^2 - \frac{x^3}{2} + \frac{x^3}{2} + \dots$$

$$e^x \ln(1+x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

d) Deduce $\frac{dy}{dx}$ if $y = \cosh^{-1}x$

$$y = \cosh^{-1}x \iff x = \cosh y$$

$$1 = y' \sinh y \implies y' = \frac{1}{\sinh y}$$

$$\cosh^2 y - \sinh^2 y = 1 \implies \sinh y = \sqrt{\cosh^2 y - 1} = \sqrt{x^2 - 1}$$

$$y' = \frac{1}{\sqrt{x^2 - 1}}$$

$$\therefore \frac{d}{dx} \cosh^{-1}x = \frac{1}{\sqrt{x^2 - 1}}$$

تفاضل ②

e) Deduce $y^{(n)}$ of $y = \cos(ax+b)$, then find $y^{(n)}$ if

$$y = \cos^4 x$$

$$y = \cos(ax+b)$$

$$y^{(1)} = -a \sin(ax+b) = a \cos(ax+b + \pi/2)$$

$$y^{(2)} = -a^2 \sin(ax+b + \pi/2) = a^2 \cos(ax+b + 2\pi/2)$$

$$y^{(3)} = -a^3 \sin(ax+b + \pi/2) = a^3 \cos(ax+b + 3\pi/2)$$

⋮

$$y^{(n)} = a^n \cos(ax+b + \frac{n\pi}{2})$$

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

$$\cos^4 x = \left[\frac{1}{2} (1 + \cos 2x) \right]^2 = \frac{1}{4} (1 + \cos 2x)^2$$

$$\approx = \frac{1}{4} [1 + 2 \cos 2x + \cos^2 2x]$$

$$\approx = \frac{1}{4} \left[1 + 2 \cos 2x + \frac{1}{2} (1 + \cos 4x) \right]$$

$$\approx = \frac{1}{4} + \frac{1}{2} \cos 2x + \frac{1}{8} + \frac{1}{8} \cos 4x$$

$$\cos^4 x = y = \frac{3}{8} + \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$$

$$y^{(n)} = \frac{1}{2} 2^n \cos(2x + \frac{n\pi}{2}) + \frac{1}{8} 4^n \cos(4x + \frac{n\pi}{2})$$

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(Answer Sheet)

Answer Q5

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| 1 | <input type="radio"/> A | <input type="radio"/> B | <input checked="" type="radio"/> C | <input type="radio"/> D | 16 | <input type="radio"/> A | <input type="radio"/> B | <input checked="" type="radio"/> C | <input type="radio"/> D |
| 2 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 17 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 3 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D | 18 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D |
| 4 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 19 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
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| 6 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 21 | <input type="radio"/> A | <input type="radio"/> B | <input checked="" type="radio"/> C | <input type="radio"/> D |
| 7 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 22 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 8 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D | 23 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D |
| 9 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 24 | <input type="radio"/> A | <input type="radio"/> B | <input checked="" type="radio"/> C | <input type="radio"/> D |
| 10 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 25 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 11 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 26 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 12 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D | 27 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 13 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 28 | <input type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input checked="" type="radio"/> D |
| 14 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D | 29 | <input type="radio"/> A | <input checked="" type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |
| 15 | <input type="radio"/> A | <input type="radio"/> B | <input checked="" type="radio"/> C | <input type="radio"/> D | 30 | <input checked="" type="radio"/> A | <input type="radio"/> B | <input type="radio"/> C | <input type="radio"/> D |