



الاجابة  
1  
من  
27/5/16

**Answer the following questions:**

**Question 1: [28 marks]**

- a) Describe the parabola :  $x^2 + 2y = 8x - 7$
- b) Find the equation of an ellipse whose vertices are  $(-2,1)$ ,  $(0,1)$ ,  $(-1,-1)$ ,  $(-1,3)$ . Also, discuss and sketch it.
- c) Describe the curve:  $2x^2 - y^2 - 2x - 4y = 0$

**Question 2: [27 marks]**

- a) Prove that:  $r = a \sin \theta + b \cos \theta$  represent a circle and find each of its center and its radius.
- b) Find the value of  $k$  to represent the following equation pairs of lines, also find the point of its intersections and the angle between them.
- $2x^2 + kxy - 6y^2 + 3x + y + 1 = 0$
- c) If the origin is translated to the point  $(-1,2)$  and the axes rotated by +ve angle  $\pi / 4$  , find the new form for the equation:

$$4x^2 + y^2 + 8x - 4y + 7 = 0$$



**Question 3: [28 marks]**

a) Answer the following using “true” or “false”, in the false case state the

correct answer:

$$(1) \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \sin^{-1}[f(x)] + c \quad (2) \int (\cos 3x) e^{\sin 3x} dx = -e^{3\sin x} + c$$

$$(3) \int_a^a f(x) dx = 0 \quad (4) \int \frac{\cos 2x}{[5+4\sin 2x]^2} dx = \frac{1}{[5+4\sin 2x]^2} + c$$

b) Find the following integral:

$$(1) \int x \tan^3(5x^2) dx \quad (2) \int \frac{x^3}{(2+3x)^4} dx$$

$$(3) \int \frac{dx}{\sqrt{8+2x-x^2}} \quad (4) \int 2^{(1+\cot 5t)} \operatorname{cosec}^2(5t) dt$$

c) Find the reduction formula of the integral:

$$I_n = \int x^n \sin(ax) dx, \text{ then find } \int x^2 \sin(5x) dx$$

**Question 4: [27 marks]**

a) Find the area between the following curves:  $x + 2y = 4$ ,  $y^2 = 4 + x$

b) Find the output volume from the rotation of the closed area between the curves:  $y^2 = 8x$ ,  $x = 3$  and  $x$ -axis

(1) The rotation around x-axis

(2) The rotation around y-axis

c) Use the Simpson's and trapezoidal rules with step length  $h=0.4$

$$\text{to estimate } I = \int_2^6 \frac{\ln(2+3\sqrt{x})}{1+x^2} dx \text{ and compare your result with exact}$$

solution  $I=0.596545$