

2.17/1/16 ex 60 - 100 - 100 - 100



All questions are equal marks

Question [1]:

- a) Find the domain for a function $f(x, y) = (x^2 - y^2)^{\frac{-3}{2}}$
 b) Discuss the continuity for a function:

$$f(x, y) = \begin{cases} \frac{\sin(x+y)}{x+y} & , (x, y) \neq (0, 0) \\ 1 & , (x, y) = (0, 0) \end{cases}$$

- c) If $f = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$,
 show that $(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 = (\frac{\partial f}{\partial r})^2 + \frac{1}{r^2} (\frac{\partial f}{\partial \theta})^2$

- d) Prove that $I = \int_0^2 \int_0^{2-y} \frac{dx dy}{(x+y)^2} = \ln(2) - \frac{1}{2}$

Question [2]:

- a) Find Maclaurin expansion for the function $f(x, y) = e^{ax+by}$, a, b constants.
 b) Prove that, the expression $(3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy$ represent total differential for $f=f(x,y)$ and find the function.
 c) Find the maximum and minimum values (if exist) for the function:

$$f(x, y) = \frac{x^2}{2a} + \frac{y^2}{2b}, \quad a, b > 0$$

- d) Find $V = \int_0^a \int_0^{a-x} \int_0^{a-x-y} dz dy dx$

Question [3]:

- a) Find the solution of the following ordinary differential equations:

i) $y' = \frac{1+2xy}{x^2}$

ii) $y dx = (x + \sqrt{y^2 - x^2}) dy$

iii) $\cosh x \sin y dy = \sinh x \cos y dx$

- b) Find the family of curves, which be orthogonal on $y = -x$ (Explain by graph)
 c) Find the general solution of:

i) $(D^5 - 32)y = 0$

ii) $(D + 2)^3(D - 1)(D^2 + 1)y = 0$

Question [4]:

- a) Deduce Laplace Transform of : $f(t) = \sin at$
 b) Find the following:

i) $L[e^{3t} \sinh 5t]$,

ii) $L[\cos^3 t]$,

iii) $L^{-1} \left[\frac{1}{4s^2 - 1} \right]$

- c) By using Laplace Transform solve the following differential equation:

$$\frac{dy}{dx} + ay = x, \quad y(0) = 0$$