نموذج إجابة

الرياضيات الهندسية ١ - أ

للفصل الدراسي الأول

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Question [1] :-

a) use Mathematical Induction to prove that :

$$1+r+r^2+\cdots+r^n=\frac{1-r^{n+1}}{1-r}\ ,\ \forall n\in N\ ,\ r\neq 1$$

solution

[1] For $n=1$

$L.H.S = 1+r$ $\qquad R.H.S=\frac{1-r^2}{1-r}=\frac{(1-r)(1+r)}{1-r}=1+r$

$\therefore$ العلاقة صحيحة عند $n=1$

[2] for $n=K$

$$1+r+r^2+\cdots+r^K=\frac{1-r^{K+1}}{1-r}$$

نفرض ان هذه العلاقة صحيحة

[3] For $n=K+1$

$$1+r+r^2+\cdots+r^{K+1}=\frac{1-r^{K+2}}{1-r}$$

المطلوب إثبات صحة هذه العلاقة

$$L.H.S=\underbrace{1+r+r^2+\cdots}+r^{K+1}$$

$$=\frac{1-r^{K+1}}{1-r}+r^{K+1}$$

$$=\frac{1-r^{K+1}+(1-r)r^{K+1}}{1-r}=\frac{1-r^{K+1}+r^{K+1}-r^{K+2}}{1-r}$$

$$=\frac{1-r^{K+2}}{1-r}=R.H.S$$

$\therefore$ العلاقة صحيحة لكل $n\in N$

(b) Resolve $\dfrac{x^2-x}{x^3+x^2-17x+15}$

— solution —

$$\frac{x^2-x}{x^3+x^2-17x+15} = \frac{x(x-1)}{(x+5)(x-3)(x-1)}$$

$$\frac{x}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$= \frac{A(x-3)+B(x+5)}{(x+5)(x-3)}$$

$$A(x-3)+B(x+5) = x$$

Let $x=-5 \longrightarrow -8A=-5 \longrightarrow \boxed{A=\frac{5}{8}}$

Let $x=3 \longrightarrow 8B=3 \longrightarrow \boxed{B=\frac{3}{8}}$

$$\frac{x^2-x}{x^3+x^2-17x+15} = \frac{5/8}{x+5} + \frac{3/8}{x-3}$$

c) prove that :-

(i) $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

— solution —

A	B	$A-B$	$B-A$	$(A-B)\cup(B-A)$ ↓	$A\cup B$	$A\cap B$	$(A\cup B)-(A\cap B)$ ↓
$\in$	$\in$	$\notin$	$\notin$	$\notin$	$\in$	$\in$	$\notin$
$\in$	$\notin$	$\in$	$\notin$	$\in$	$\in$	$\notin$	$\in$
$\notin$	$\in$	$\notin$	$\in$	$\in$	$\in$	$\notin$	$\in$
$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$

$(A-B)\cup(B-A) = (A\cup B)-(A\cap B)$ من العمود الخامس والثامن ينتج أن

ii) $A \to B \equiv \sim(A \wedge \sim B)$

A	B	$A \to B$	$\sim B$	$A \wedge \sim B$	$\sim(A \wedge \sim B)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	T

من العمود الثالث والسادس ينتج أن

$A \to B \equiv \sim(A \wedge \sim B)$

Question [2]

a) Let $X = \{2, 4, 6, 7\}$ and R be a relation from X to X, defined by :

$R = \{(x, y) : x, y \in X \text{ and } x+y \text{ is an even integer}\}$

Discuss R ?

(solution)

$R = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6), (7,7)\}$

* R is Reflexive Relation $\forall x \in X$, xRx
* R is symmetric Relation xRy , yRx
* R is Translating Relation $xRy \wedge yRz \to xRz$
* R is Equivalence Relation

b) find the value of $\left[\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^{10}$

— Solution —

$$Z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} * \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{1+2\sqrt{3}i-3}{1+3} = -\frac{2}{4}+\frac{2\sqrt{3}}{4}i$$

$$Z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}/2}{1/2} = 60^\circ$$

$$\theta_1 = 180^\circ - 60^\circ = 120^\circ$$

$$Z = r\left[\cos\theta_1 + i\sin\theta_1\right] = 1\left[\cos 120 + i\sin 120\right]$$

$$Z^{10} = \left(\cos 120 + i\sin 120\right)^{10} = \cos 10*120 + i\sin 10*120$$

$$Z^{10} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(c) use the inverse matrix method to calculate the currents I_1, I_2, I_3 of a circuit in the following system:

$$\begin{bmatrix} 4I_1 - 3I_2 + I_3 & 2I_1 + I_2 - 4I_3 \\ I_1 + 2I_2 - 2I_3 & 0 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ 1 & 0 \end{bmatrix}$$

— Solution —

$$4I_1 - 3I_2 + I_3 = 11$$

$$2I_1 + I_2 - 4I_3 = -1$$

$$I_1 + 2I_2 - 2I_3 = 1$$

$$\begin{bmatrix} 4 & -3 & 1 \\ 2 & 1 & -4 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \\ 1 \end{bmatrix}$$

$$A \quad . \quad X \quad = \quad B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{1}{|A|} \, adj\,(A)$$

$$|A| = \begin{vmatrix} \oplus 4 & \ominus -3 & \oplus 1 \\ 2 & 1 & -4 \\ 1 & 2 & -2 \end{vmatrix} = 4(-2+8) + 3(-4+4) + 1(4-1)$$

$$= 27$$

$$adj\,(A) = \begin{bmatrix} +(6) & -(0) & +(3) \\ -(4) & +(-9) & -(11) \\ +(11) & -(-18) & +(10) \end{bmatrix}^T = \begin{bmatrix} 6 & -4 & 11 \\ 0 & -9 & 18 \\ 3 & -11 & 10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{6}{27} & \frac{-4}{27} & \frac{11}{27} \\ 0 & \frac{-9}{27} & \frac{18}{27} \\ \frac{3}{27} & \frac{-11}{27} & \frac{10}{27} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{6}{27} & \frac{-4}{27} & \frac{11}{27} \\ 0 & \frac{-9}{27} & \frac{18}{27} \\ \frac{3}{27} & \frac{-11}{27} & \frac{10}{27} \end{bmatrix} \begin{bmatrix} 11 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$I_1 = 3 \qquad I_2 = 1 \qquad I_3 = 2$

Q3 a) i- Find dy/dx

$$y=\ln\left[\frac{e^{\tan^{-1}x^2}\sqrt{x^2+\operatorname{cosec} e^x}}{\ln \sec x}\right]$$

$$y=\ln\left(e^{\tan^{-1}x^2}\sqrt{x^2+\operatorname{cosec} e^x}\right)-\ln(\ln \sec x)$$

$$y=\ln e^{\tan^{-1}x^2}+\ln\sqrt{x^2+\operatorname{cosec} e^x}-\ln(\ln \sec x)$$

$$y=\tan^{-1}x^2+\frac{1}{2}(x^2+\operatorname{cosec} e^x)-\ln(\ln \sec x)$$

$$\therefore y'=\frac{2x}{1+x^4}+\frac{1}{2}\,\frac{2x-\operatorname{cosec} e^x\cot e^x\cdot e^x}{x^2+\operatorname{cosec} e^x}-\frac{\tan x}{\ln \sec x}$$

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ii) $y=3^{\ln\cot x^2}+\cos^2 e^{\sqrt{x^2-1}}+\sinh^{-1}2x$

$$\therefore y'=3^{\ln\cot x^2}\cdot\ln 3\cdot(-\operatorname{cosec}^2 x^2\cdot 2x)+2\cos e^{\sqrt{x^2-1}}$$

$$\left(-\sin e^{\sqrt{e^x-1}}\cdot e^{\sqrt{x^2-1}}\,\frac{2x}{2\sqrt{x^2-1}}\right)+\frac{2}{\sqrt{1+4x^2}}$$

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b) if $y=\frac{1}{\sqrt{1-x^2}}\sin^{-1}x$ show that

i) $(1-x^2)\,y^{(1)}=1+xy$

ii) $(1-x^2)\,y^{(n+1)}-(2n+1)x\,y^{(n)}-n^2\,y^{(n-1)}=0$

$$\sqrt{1-x^2}\;y=\sin^{-1}x$$

$$\sqrt{1-x^2}\;y^{(1)}+y\,\frac{-2x}{2\sqrt{1-x^2}}=\frac{1}{\sqrt{1-x^2}}\qquad *\sqrt{1-x^2}$$

$$(1-x^2)\,y^{(1)}-xy=1$$

$$(1-x^2)\,y^{(1)}=1+xy$$

$$(1-x^2)\,y^{(n+1)} + n(-2x)\,y^{(n)} + \frac{n(n-1)}{2}(-2)\,y^{(n-1)}$$
$$= x\,y^{(n)} + n\,y^{(n-1)}$$
$$\therefore (1-x^2)\,y^{(n+1)} - (2n+1)x\,y^{(n)} - n^2\,y^{(n-1)} = 0$$

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c) Expand the function $f(x) = \sin x$ with the power of $(x - \pi/6) \equiv (x - x_0)$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \cdots\cdots$$

$f(x) = \sin x$	$f(\pi/6) = \sin \pi/6 = 1/2$
$f'(x) = \cos x$	$f'(\pi/6) = \cos \pi/6 = \sqrt{3}/2$
$f''(x) = -\sin x$	$f''(\pi/6) = -\sin \pi/6 = -1/2$
$f'''(x) = -\cos x$	$f'''(\pi/6) = -\cos \pi/6 = -\sqrt{3}/2$

$$\sin x = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \pi/6) - \frac{1/2}{2!}(x - \pi/6)^2$$
$$+ \frac{\sqrt{3}/2}{3!}(x - \pi/6)^3 + \cdots\cdots$$

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(Answer Sheet)

Q4

1	D	16	A
2	C	17	B
3	B	18	C
4	A	19	D
5	B	20	C
6	C	21	B
7	D	22	A
8	D	23	A
9	D	24	A
10	C	25	B
11	B	26	C
12	A	27	D
13	B	28	C
14	C	29	B
15	D	30	A