Model Answer

Kafrelsheikh University
Faculty of Engineering
Dept. Mech. Engineering
Year: Preparatory Year
Subject: Mechanics I _ Statics (MEP0001)

Semester: 1st Semester
Final Examination
Date: Jan. 10th, 2018
Time allowed: 3 hours
Full Mark: 70

(a) This exam measures ILOs no.: a1, b2, b3, b11, c1, and d3
(b) No. of pages: 9 - No. of questions: 7. Pages no 6/9 and 9/9 are empty
(c) This is a close book exam.
(d) Clear, systematic answers and solutions are required. In general, marks will not be assigned for answers and solutions that require unreasonable (in the opinion of the instructor) effort to decipher.
(e) Ask for clarification if any question statement is not clear to you.
(f) Attempts in all questions.
(g) The weight of each problem is indicated.
(h) The exam will be marked out of 70. There are 7 marks bonus.

1. Choose the correct answer. Justify your answer with calculations or sketch, or both. Marks will not be given to answers require justification (8 Marks)

1- The following can be considered a rigid body : **(1 Mark)**
   a. A 300mm diameter and 1m long axial rod of mild steel subjected to 1N force.
   b. water
   c. Air
   d. An elastic rubber band

2- The unit of force in SI is **(1 Mark)**
   a. lb
   b. lb.ft
   c. kg m sec^{-2}
   d. kg m sec^{2}

3- If the masses of the two particle and distance between them is doubled, the force of gravitational attraction will **(2 Marks)**
   a. double
   b. remain same
   c. halve
   d. become one fourth

4- Two forces of same magnitude P are having an angle of 60^0 between them, the magnitude of their resultant will be **(2 Marks)**
   a. P
   b. 2P
   c. $\sqrt{3}P$
   d. $\frac{\sqrt{3}}{2}P$

5- Forces $3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$ N and $-7\mathbf{i}-\mathbf{k}$ N are acting at a point. The resultant force will be of magnitude **(2 Marks)**
   a. 3 N
   b. 4 N
   c. 5 N
   d. $4\sqrt{2}$ N

With our best wishes

Associt. Prof. Maher Abou Al-Sood M. M. Abou Al-Sood
Assist. Prof. Magda El-Fakharany M. El-Fakharany

2. The beam, show in the Fig. 1, is to be hoisted using two chains. If the resultant force is to be 60(N directed along the positive y- axis and F_B lies in the second quarter (-x, +y), determine graphically or analytically the magnitudes of forces F_A and F_B acting on each chain and the angle θ of F_B for the following cases: be **(5 Marks)**

   a. F_B is minimum be **(3 Marks)**
   b. F_B is maximum be **(2 Marks)**

Solution

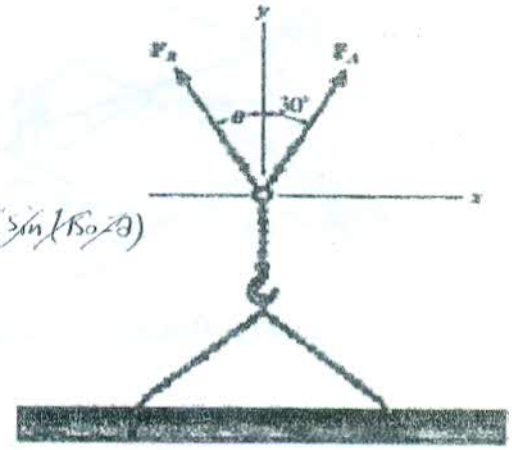


Fig. 1 Schematic Diagram of Prob. #2

$$R^2 = F_A^2 + F_B^2 - 2F_A F_B \cos(150 - \theta)$$

$$0 = 0 + 2F_B - 2F_A \cos(150 - \theta) + 2F_A \sin(150 - \theta)$$

$$F_B = F_A \cos(150 - \theta)$$

$$\frac{dF_B}{d\theta} = 0 \quad \text{for min and max}$$

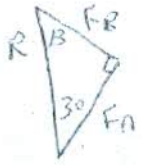
$$\cos(150 - \theta) = 0$$

$$150 - \theta = 90 \quad \text{or} \quad 150 - \theta = 270$$

$$\theta = 60 \quad \text{or} \quad \theta = 240 \quad \text{but } F_B \text{ lies in the second quarter}$$

$$\therefore \theta = 90 \quad \text{max}$$

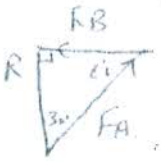
For min $\theta = 60$



$$\frac{600}{\sin 90} = \frac{F_B}{\sin 30} = \frac{F_A}{\sin 60}$$

$$F_B = 300 \text{ N} \qquad F_A = 519.62 \text{ N}$$

For max $\theta = 90$



$$\frac{600}{\sin 60} = \frac{F_B}{\sin 30} = \frac{F_A}{\sin 90}$$

$$F_B = 346.4 \text{ N} \qquad F_A = 692.82 \text{ N}$$

3. Determine the magnitude and coordinate direction angles of the resultant force of the two forces 600 lb and 750 lb acting in in AB and AC directions, respectively. **(15 Marks)**

Solution

$A(0,0,4)$ $B(3,-3,2.5)$ $C(2,4,0)$

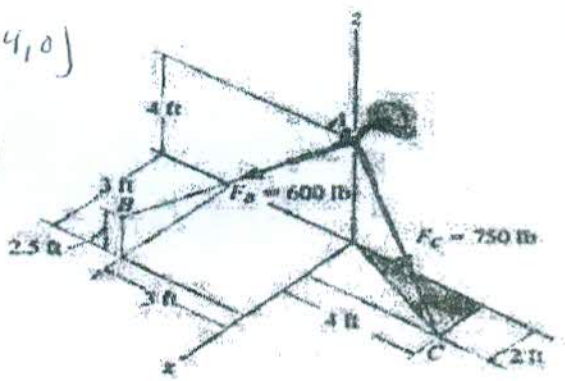


Fig. 2 Schematic Diagram of Prob. #3

$\vec{AC} = (2,4,0) - (0,0,4) = (2,4,-4)$

$AC = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$

$U_{\vec{AC}} = \frac{\vec{AC}}{AC} = \frac{2}{6}\underline{i} + \frac{4}{6}\underline{j} - \frac{4}{6}\underline{k}$

$\vec{F}_{AC} = F_C \cdot U_{\vec{AC}} = 750\left(\frac{2}{6}\underline{i} + \frac{4}{6}\underline{j} - \frac{4}{6}\underline{k}\right)$

$= 250\underline{i} + 500\underline{j} - 500\underline{k}$

$\vec{AB} = (3,-3,2.5) - (0,0,4) = (3,-3,-1.5)$

$AB = \sqrt{(3)^2 + (3)^2 + (-1.5)^2} = 4.5$

$U_{\vec{AB}} = \frac{\vec{AB}}{AB} = \frac{3}{4.5}\underline{i} - \frac{3}{4.5}\underline{j} - \frac{1.5}{4.5}\underline{k}$

$F_{AB} = F_B \, U_{\vec{AB}} = 600\left(\frac{3}{4.5}\underline{i} - \frac{3}{4.5}\underline{j} - \frac{1.5}{4.5}\underline{k}\right)$

$= 400\underline{i} - 400\underline{j} - 200\underline{k}$

$\vec{R} = F_{\vec{AC}} + F_{\vec{AB}} = 650\underline{i} + 100\underline{j} - 700\underline{k}$

$R = \sqrt{(650)^2 + (100)^2 + (-700)^2} = 960.4686$

$\cos\alpha = \frac{R_x}{R} = \frac{650}{960.47} \Rightarrow \alpha = 47.4$

$\cos\beta = \frac{R_y}{R} = \frac{100}{960.47} \Rightarrow \beta = 84.02$

$\cos\gamma = \frac{R_z}{R} = \frac{-700}{960.47} \Rightarrow \gamma = 136.78$

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Assist. Prof. **Magda El-Fakharany** M. El-Fakharany

4. The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown. Determine the resultant of forces and moments at A. **(5 Marks)**

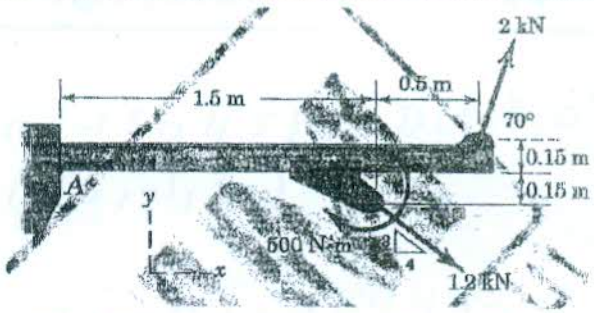

Fig. 3 Schematic Diagram of Prob. # 4

Solution

$* M_A = -0.500$
$-2\cos(70)*0.15 + 2\sin(70)*1.5$
$-1.2(3/5)*1.5 \quad +1.2(4/5)*0.15$
$= 2.22$ kN.m (CCW) ↺

$* \Sigma F_x = 2\cos 70 + 1.2*0.8 \quad = 1.644$ kN

$\Sigma F_y = 2\sin 70 - 1.2*0.6 \quad = 1.159$

$R = (R_x^2 + R_y^2)^{1/2}$
$= (1.644)^2 + (1.159)^2 \quad = 2.01$ kN

$\theta = \tan^{-1}(R_y/R_x) = 35.18^\circ = 35$

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5. The rigid pole and cross-arm assembly are shown in Fig. 5. If the tension in cables AE and GF are 1 KN and 1.5 KN, respectively, determine the following: **(20 Marks)**
   a. Resultant of the three tension, **R**, and their resultant moment $\mathbf{M_R}$ at point O. **(10 Marks)**
   b. Replace this system of **R** and $\mathbf{M_R}$ at O by a wrench and find its force, moment, pitch, and its intersection with x-y plane. **(10 Marks)**

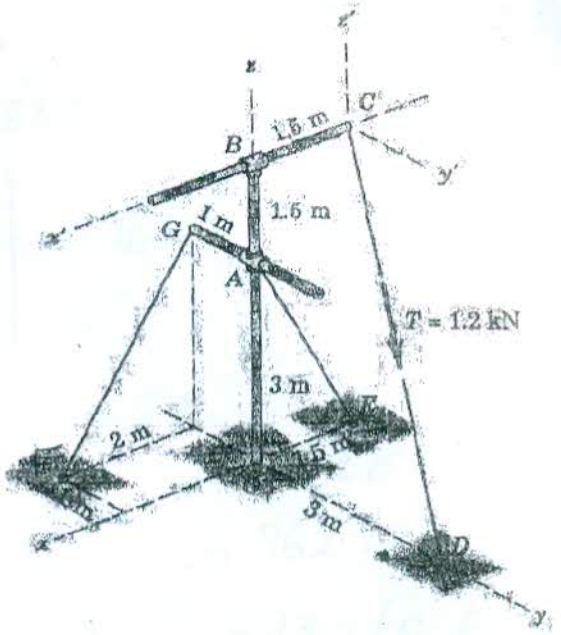

Fig. 4 Schematic Diagram of Prob. # 4

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Solution

3) $O(0,0,0)$ $A(0,0,3)$ $C(-1.5,0,4.5)$ $D(0,3,0)$
$E(-1.5,0,0)$ $F(2,-1,0)$ $G(0,-1,3)$ (2)

$$\bar{U}_{AE} = \frac{-1.5\bar{i}+0\bar{j}-3\bar{k}}{\sqrt{(-1.5)^2+(0)^2+(-3)^2}} = -0.447\bar{i}+0\bar{j}-0.894\bar{k}$$

$$\bar{U}_{GF} = \frac{2\bar{i}+0\bar{j}-3\bar{k}}{\sqrt{4+0+9}} = 0.555\bar{i}+0\bar{j}-0.832\bar{k} \quad (3)$$

$$\bar{U}_{CD} = \frac{1.5\bar{i}+3\bar{j}-4.5\bar{k}}{\sqrt{(1.5)^2+(3)^2+(-4.5)^2}} = 0.267\bar{i}+0.535\bar{j}-0.818\bar{k}$$

$$\bar{T}_{AE} = T_{AE}\bar{U}_{AE} = -0.447\bar{i}+0\bar{j}-0.894\bar{k}$$

$$\bar{T}_{GF} = T_{GF}\bar{U}_{GF} = 0.832\bar{i}+0\bar{j}-1.248\bar{k} \quad (3)$$

$$\bar{T}_{CD} = T_{CD}\bar{U}_{CD} = 0.321\bar{i}+0.641\bar{j}-0.962\bar{k}$$

$$\bar{R} = \bar{T}_{AE}+\bar{T}_{GF}+\bar{T}_{CD} = 0.706\bar{i}+0.641\bar{j}-3.104\bar{k} \quad (1)$$

$$\bar{M}_o = \bar{r}_{OA} \times \bar{T}_{AE} + \bar{r}_{OG} \times \bar{T}_{GF} + \bar{r}_{OD} \bar{T}_{CD}$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & 3 \\ -0.447 & 0 & -0.894 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & -1 & 3 \\ 0.832 & 0 & -1.248 \end{vmatrix}$$

$$+ \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 3 & 0 \\ 0.321 & 0.641 & -0.962 \end{vmatrix} \quad (6)$$

$$= 0\bar{i} - 1.341\bar{j} + 0\bar{k}$$

$$+1.248\bar{i} + 2.496\bar{j} + 0.832\bar{k}$$

$$-2.886\bar{i} + .0\bar{j} - 0.962\bar{k}$$

$$\bar{M}_o = -1.637\bar{i} + 1.155\bar{j} - 0.130\bar{k} \ \text{KN.m} \quad (1)$$

(a)

(b) For wrench

$$\bar{F}_{RN} = \bar{R} = 0.706\bar{i} + 0.641\bar{i} - 3.104\bar{k}$$

$$F_{RN} = R = 3.247 \ \text{KN} \quad (1)$$

$$\bar{U}_R = \bar{R}/R = (0.706\bar{i} + 0.641\bar{j} - 3.104\bar{k})/3.247$$

$$= 0.217\bar{i} + 0.197\bar{j} - 0.956\bar{k} \quad (1)$$

$$M_W = M_{11} = \bar{U}_R \bar{M}_o \quad (1)$$

$$= (0.217\bar{i} + 0.197\bar{j} - 0.956\bar{k}) \cdot (-1.637\bar{i} + 1.155\bar{j} - 0.130\bar{k})$$

$$= -0.0034 \ \text{KN.m}$$

$$\bar{M}_W = \bar{M}_{11} = M_{11}\bar{U}_R = -0.001\bar{i} - 0.001\bar{j} + 0.003\bar{k} \quad (1)$$

$$\rho = M_{11}/R = -0.034/3.247 = 0.001 \ m = 1 \ mm \quad (1)$$

$$\bar{M}_L = \bar{M}_o - M_{11} = -1.636\bar{i} + 1.154\bar{j} - 0.127\bar{k} \quad (1)$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & y & 0 \\ 0.706 & 0.641 & -3.104 \end{vmatrix} \quad (1)$$

(2) $y = 0.527 \ m$

$x = 0.372 \ m$

check $M_z = 0.641 \times -0.706y$

$-0.641 \times 0.372 - 0.706 \times 0.52$

6. In each of the five following examples (i.e. Fig. 5a and 5.b), the body to be isolated is shown in the left-hand diagram, and an *incomplete* free-body diagram (FBD) of the isolated body is shown on the right. Add whatever forces are necessary in each case to form a complete free-body diagram. The weights of the bodies are **negligible** unless otherwise indicated. Dimensions and numerical values are omitted for simplicity. **(10 Marks)**

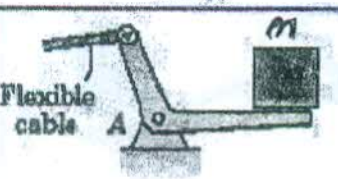
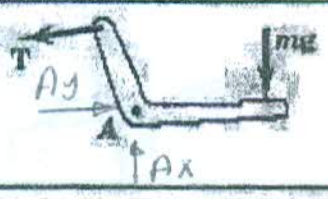
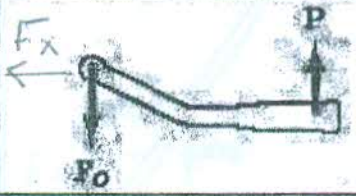
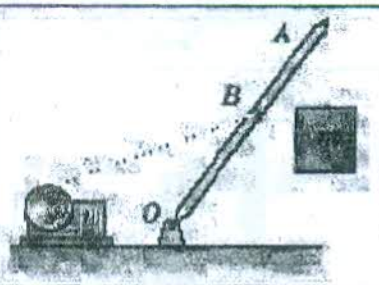
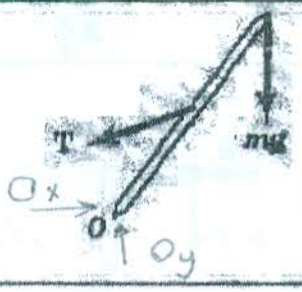
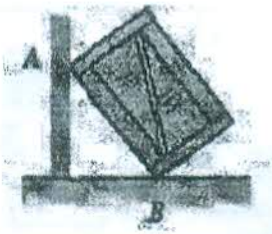
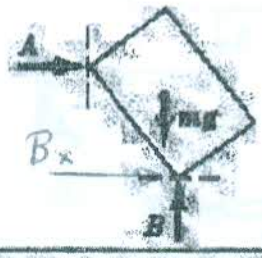
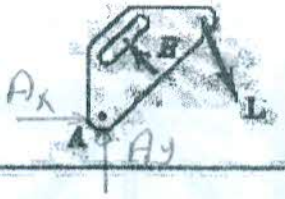
	Body	Incomplete *FBD*
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

Fig. 5a

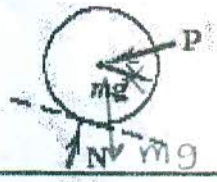
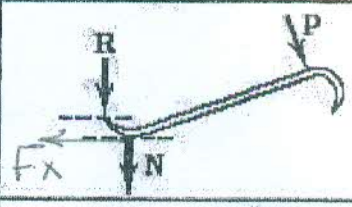
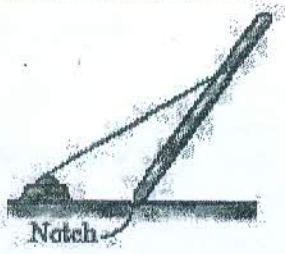
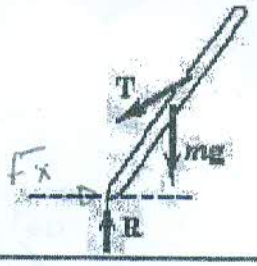
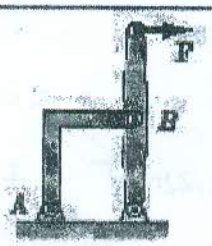
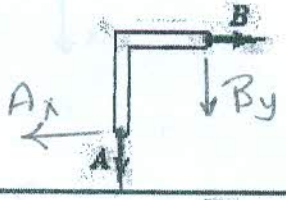
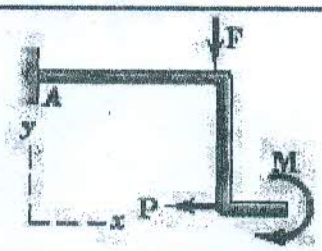
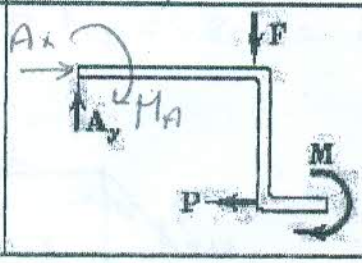
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

Fig. 5b

7. For the loaded truss shown in Fig. 7, Determine the following: **(14 Marks)**
   a. Reaction forces at supports A and G. **(4 Marks)**
   b. Specify zero force members by inspection if exist **(2 Marks)**
   c. forces in members BC , CF, and FE and state if the members are in tension or compression **(9 Marks)**

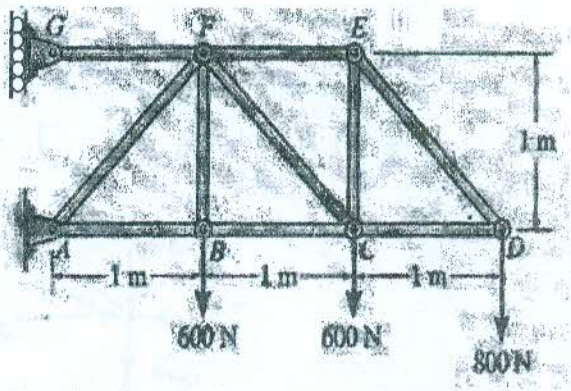


Fig. 5 Schematic Diagram of Prob. # 7

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Assist. Prof. Magda El-Fakharany M. El-Fakharany

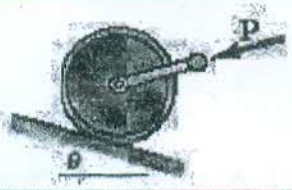
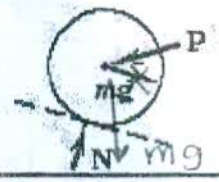
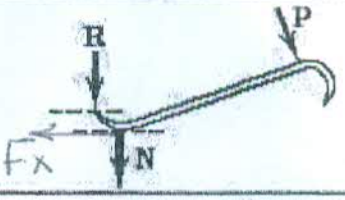
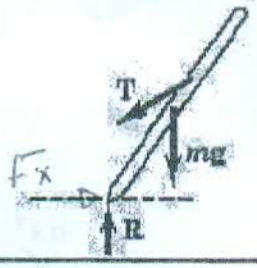
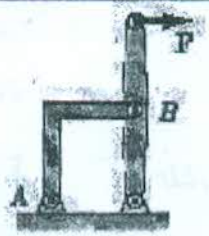
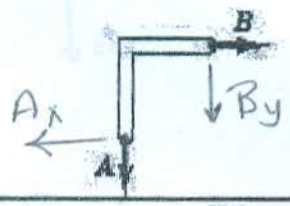
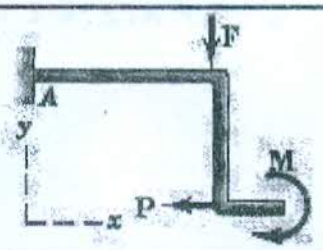
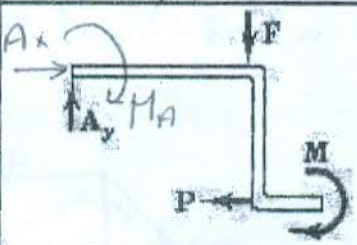
	Body	Wrong or Incomplete *FBD*
1. Lawn roller of mass *m* being pushed up incline *θ*.		
2. Prybar lifting body *A* having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass *m* being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at *A* and subjected to two forces and couple.		

Fig. 5b

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7. For the loaded truss shown in Fig. 7, Determine the following: **(14 Marks)**
   a. Reaction forces at supports A and G. **(4 Marks)**
   b. Specify zero force members by inspection if exist **(2 Marks)**
   c. forces in members BC , CF, and FE and state if the members are in tension or compression **(9 Marks)**

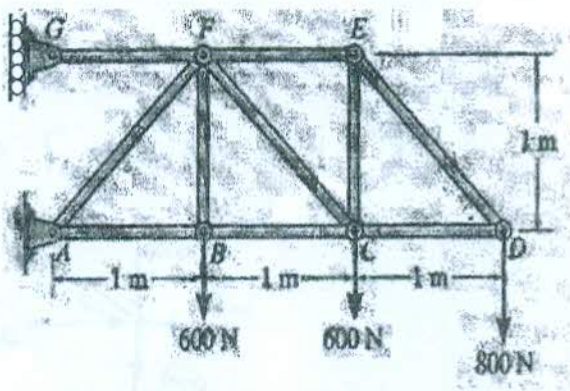


Fig. 5 Schematic Diagram of Prob. # 7

1) $\Sigma F_y = 0$

$A_y - 600 - 600 - 800 = 0 \Rightarrow A_y = 2000\ N \uparrow$

$\Sigma F_x = 0 \quad A_x + G_x = 0$

$\Sigma M_A = 0 \quad -600 \times 1 - 600 \times 2 - 800 \times 3 + G_x \times 1 = 0$

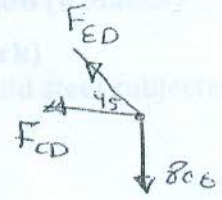
$-600 - 1200 - 2400 = G_x$

$G_x = 4200 \leftarrow$

$A_x = 4200 \rightarrow$

2) There is no zero force members

3) Joint (D)

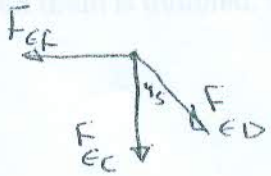


$\Sigma F_y = 0 \quad 800 - F_{ED} \cos 45 = 0$

$F_{ED} = 1131.37$ Tension

$\Sigma F_x = 0 \quad F_{CD} + 1131.37 \sin 45 = 0 \Rightarrow F_{CD} = -800 = 800$ Compression

Joint (E)



$\Sigma F_y = 0$

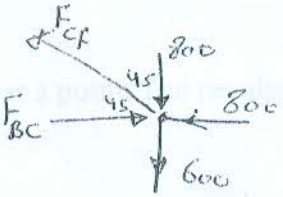
$F_{EC} + 1131.37 \cos 45 = 0$

$F_{EC} = -800 = 800$ Compression

$\Sigma F_x = 0$

$F_{EF} - 1131.37 \sin 45 = 0 \Rightarrow F_{EF} = 800$ Tension

Joint (C)



$\Sigma F_y = 0 \quad 600 + 800 - F_{CF} \cos 45 = 0$

$F_{CF} = 1979.7$ Tension

ΣF_y

$800 - F_{BC} + 1979.7 \sin 45 = 0$

$F_{BC} = 2200$ Tension