Kafrelsheikh University
Faculty of Engineering
Electrical Engineering Department
Model Answer-Final Exam, 2017/2018
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3rd Year (Electrical Engineering)
Automatic Control (2)
Time: 180 minutes
Marks= 90
Date: 31-12-2017

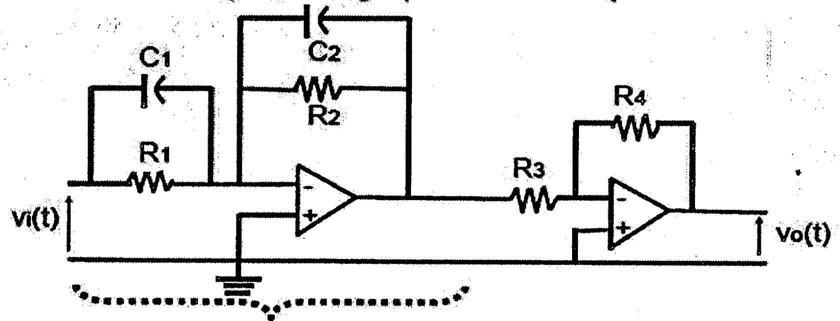
**Answer all the following questions:**

**Intended learning outcomes (ILOs): [a1, a4, a12 , b2, b6, b11, b12, b14, c3,c14]**

**Problem 1: (35 Marks) - - (ILOs):** [a1, a12, b2, b6, c3]

a) Construct the configuration of the electronic lag compensator using operational amplifiers and drive the transfer function of the compensator. (10 Marks)

- **One example, using operational amplifiers**

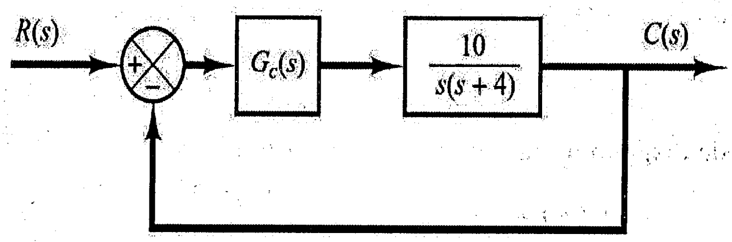



$$\frac{R_2}{R_1} \cdot \frac{R_1C_1s+1}{R_2C_2s+1}$$

- **Transfer function**

$$C(s) = \frac{V_o(s)}{V_i(s)} = \frac{\dfrac{\frac{R_2}{C_2s}}{R_2+\frac{1}{C_2s}}}{\dfrac{\frac{R_1}{C_1s}}{R_1+\frac{1}{C_1s}}} \cdot \frac{R_4}{R_3} = \underbrace{\frac{R_4C_1}{R_3C_2}}_{K} \; \frac{s+\overbrace{\frac{1}{R_1C_1}}^{z}}{s+\underbrace{\frac{1}{R_2C_2}}_{p}}$$

**b)** Design a lag compensator for following unity feedback system such that the static velocity error constant is 50 sec$^{-1}$ without appreciably changing the closed loop poles, which are at $s = -2 \pm j\sqrt{6}$. **(15 Marks)**



**Solution.** Assume that the transfer function of the lag compensator is

$$G_c(s) = \hat{K}_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \qquad (\beta > 1)$$

Since $K_v$ is specified as 50 $\text{sec}^{-1}$, we have

$$K_v = \lim_{s \to 0} sG_c(s)\frac{10}{s(s+4)} = \hat{K}_c\beta 2.5 = 50$$

Thus

$$\hat{K}_c\beta = 20$$

Now choose $\hat{K}_c = 1$. Then

$$\beta = 20$$

Choose $T = 10$. Then the lag compensator can be given by

$$G_c(s) = \frac{s + 0.1}{s + 0.005}$$

The angle contribution of the lag compensator at the closed-loop pole $s = -2 + j\sqrt{6}$ is

$$\left. \underline{/G_c(s)} \right|_{s=-2+j\sqrt{6}} = \tan^{-1}\frac{\sqrt{6}}{-1.9} - \tan^{-1}\frac{\sqrt{6}}{-1.995}$$
$$= -1.3616°$$

which is small. Thus the change in the location of the dominant closed-loop poles is very small. The open-loop transfer function of the system becomes

$$G_c(s)G(s) = \frac{s + 0.1}{s + 0.005}\frac{10}{s(s+4)}$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10s + 1}{s^3 + 4.005s^2 + 10.02s + 1}$$

c) Draw the bode diagram of the following transfer function:

$$G(j\omega) = \frac{5(1 + j0.1\omega)}{j\omega(1 + j0.5\omega)(1 + j0.6\frac{\omega}{50} + \left(\frac{j\omega}{50}\right)^2)}$$

After plotting Bode diagram evaluate the gain margin and the phase margin of the system and Comment on the stability of the system whose Bode diagram. **(10 Marks)**

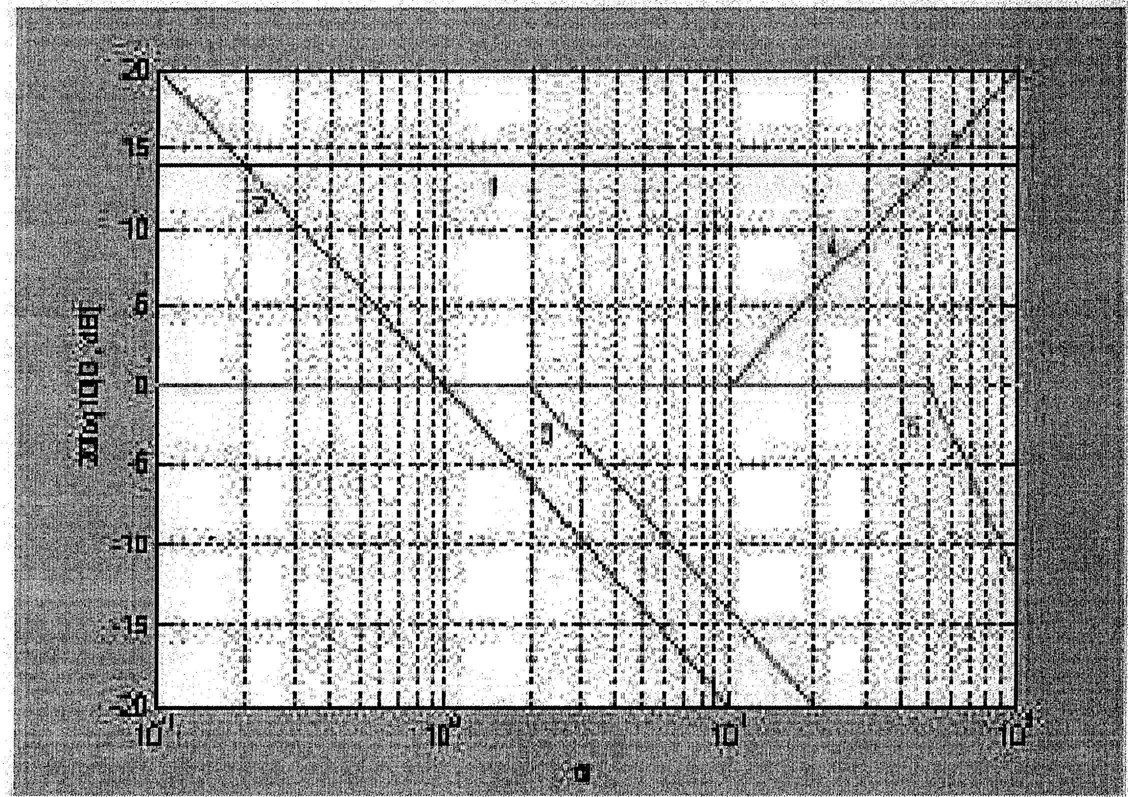
**Solution**

The factors, in order of their occurrence as frequency increases, are as follows:

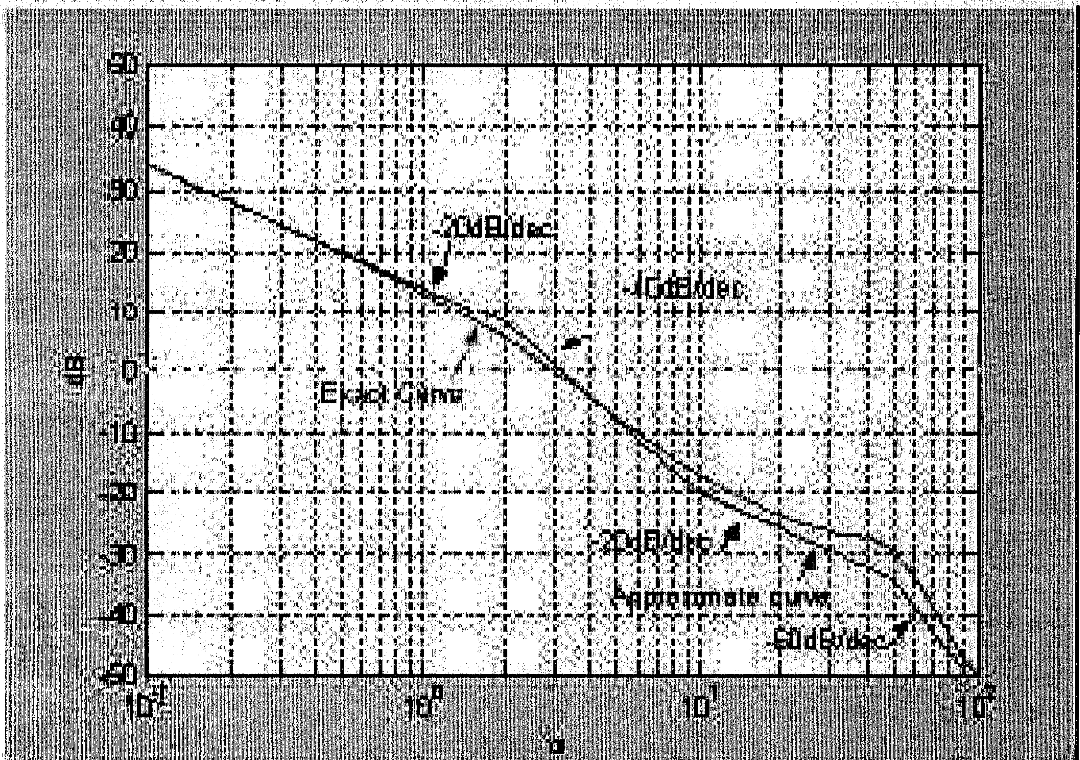
1. A constant gain $K=5$
2. A pole at the origin
3. A pole at $\omega=2$
4. A zero at $\omega=10$
5. A pair of complex poles at $\omega=\omega_n=50$

First we plot the magnitude characteristic for each individual pole and zero factor and the constant gain

1) The constant gain is $20\log_{10}5=14\text{dB}$, as shown in the figure.
2) The magnitude of the pole at the origin extends from zero frequency to infinite frequencies and has a slope of $-20$ dB/decade intersecting the 0–dB line at $\omega=1$, as shown in the figure.
3) The asymptotic approximation of the magnitude of the pole at $\omega=2$ has a slope of –20dB/decade beyond the break frequency at $\omega=2$. The asymptotic magnitude below the break frequency is 0 dB, as shown in the figure.
4) The asymptotic approximation for the zero at $\omega=10$ has a slope of +20dB/decade beyond the break frequency at $\omega=10$. The asymptotic magnitude below the break frequency is 0 dB, as shown in the figure.
5) The asymptotic approximation for the pair of complex poles has a slope of –40dB/decade beyond the break frequency at $\omega=\omega_n=50$. The asymptotic magnitude below the break frequency is 0 dB, as shown in the figure. This approximation must be corrected to the actual magnitude because the damping ratio is $\zeta=0.3$, and the magnitude differs appreciably from the approximation.




The total asymptotic magnitude can be plotted by adding the asymptotes due to each factor, as shown . The exact magnitude curve, obtained using MATLAB, is also shown for comparison.



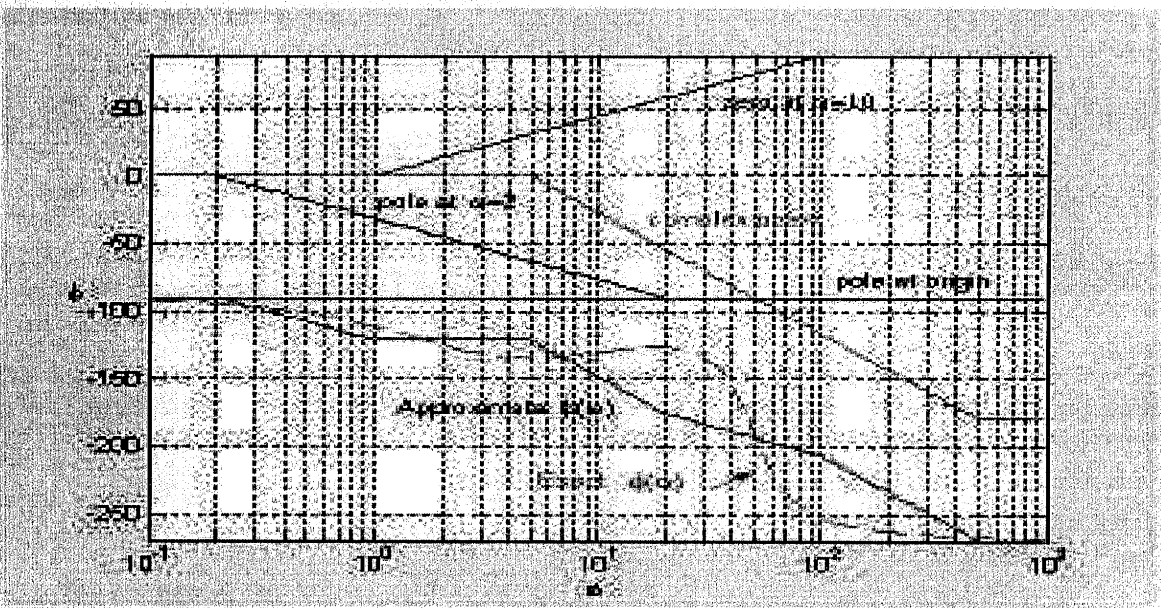
The linear approximations of the phase characteristic of the individual factors are as follows:

1) The phase of the constant gain is 0°.
2) The phase of the pole at the origin is a constant -90°.
3) The linear approximation of the phase of the pole at $\omega = 2$ has a slope of -45°/decade between the frequencies $\omega = \frac{2}{10}$ and $\omega = 10 * 2$.
4) The linear approximation of the phase of the zero at $\omega = 10$ has a slope of +45°/decade between the frequencies $\omega = \frac{10}{10}$ and $\omega = 10 * 10$.
5) The linear approximation of the phase of the complex poles at $\omega_n = 50$ has a slope of -90°/decade between the frequencies $\omega = \frac{50}{10}$ and $\omega = 10 * 50$.

The individual linear approximations of the phase characteristics for the poles and zeros are shown in the figure. The approximate total phase characteristic, $\phi_a(\omega)$, is obtained by adding the phase due to each factor. The exact phase characteristic calculated from

$$\phi(\omega) = -90° - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{2\zeta\frac{\omega}{50}}{1-(\frac{\omega}{50})^2}$$

is also shown for comparison.




The frequency response of $G(j\omega)$ can be calculated and plotted using MATLAB. To do this, rewrite $G(j\omega)$ as:

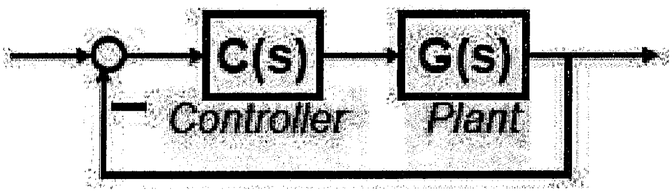
**Problem 2: (30 Marks) - - (ILOs):** [a12, b2,b6, b11]

a) Define gain margin, phase margin and explain graphically. What are the gain margin and the phase margin indicate? **(10 Marks)**

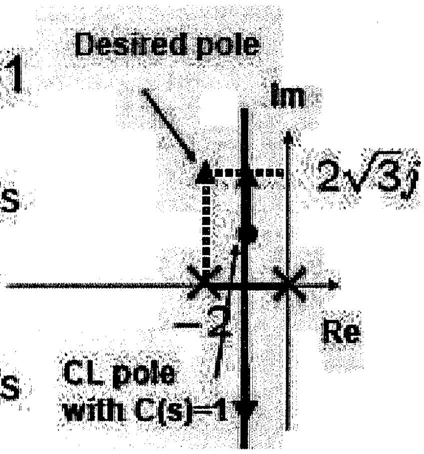
b) How to select pole and zero for the lead compensator to the system: G(s)=4/(s(s+2)) based on root locus method. **(10 Marks)**

- Consider a system

$$G(s) = \frac{4}{s(s+2)}$$




- Analysis of CL system for C(s)=1
  - Damping ratio $\zeta$=0.5
  - Undamped natural freq. $\omega_n$=2 rad/s
- Performance specification
  - Damping ratio $\zeta$=0.5
  - Undamped natural freq. $\omega_n$=4 rad/s




- A point s to be on root locus ←→ it satisfies
  - **Angle condition**

***Odd number***

$$\angle L(s) = 180^{o} \times (2k+1),\ k = 0, \pm1, \pm2, \ldots$$

- For a point on root locus, gain K is obtained by
  - **Magnitude condition**

$$|L(s)| = \frac{1}{K}$$

Evaluate G(s) at the desired pole.

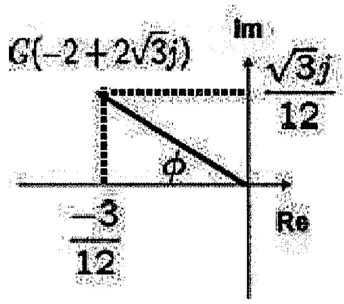
$$G(-2+2\sqrt{3}j) = \frac{4}{(-2+2\sqrt{3}j)2\sqrt{3}j} = \frac{-1}{3+\sqrt{3}j} = \frac{-3+\sqrt{3}j}{12}$$

- o If *angle condition* is satisfied, compute the corresponding K.
- o In this example,

  $$\angle G(-2+2\sqrt{3}j) = -210$$

  Angle condition is not satisfied.

➡ **Angle deficiency** $\phi = 30$



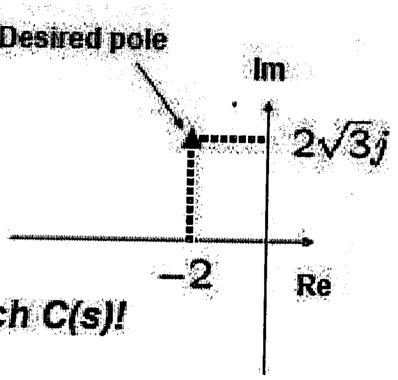
To compensate angle deficiency, design a lead compensator C(s)

$$C(s) = K\frac{s+z}{s+p}$$

satisfying

$$\angle C(-2+2\sqrt{3}j) = 30 (=: \phi)$$

$$[\Rightarrow \angle GC(-2+2\sqrt{3}j) = -180]$$



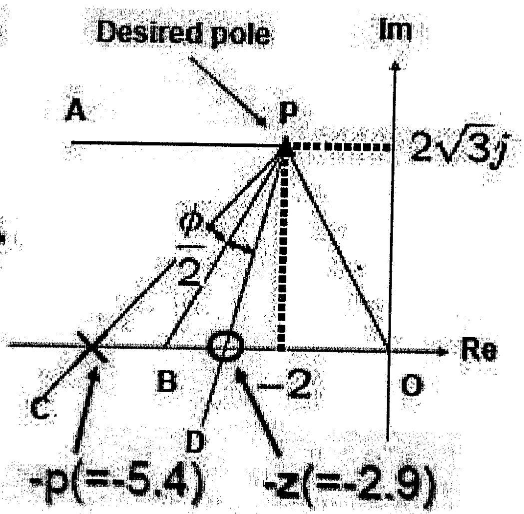

***There are many ways to design such C(s)!***

- Draw horizontal line PA
- Draw line PO
- Draw bisector PB

$$\angle APB = \angle BPO = \frac{1}{2}\angle APO$$

- Draw PC and PD

$$\angle CPB = \angle BPD = \frac{\phi}{2}$$

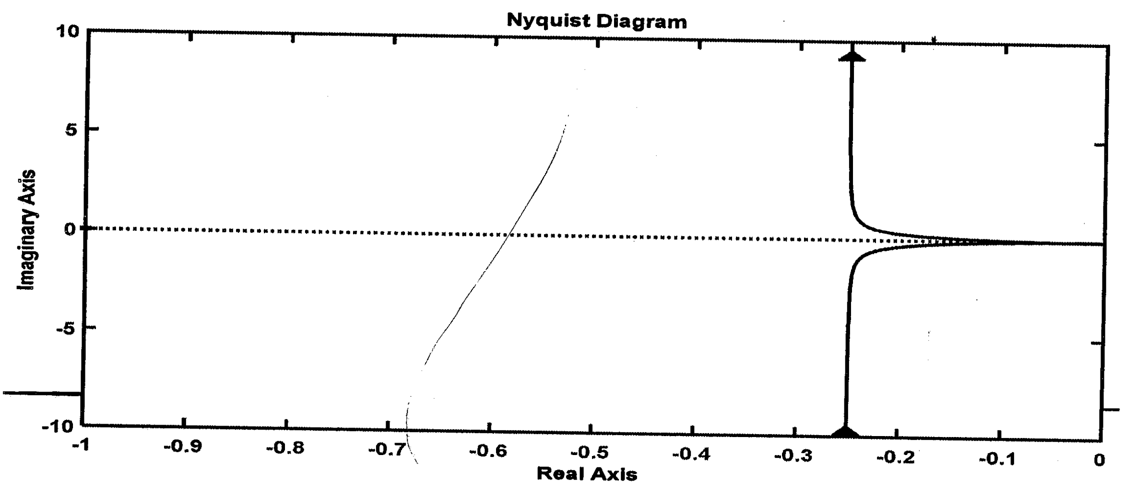



- Pole and zero of C(s) are shown in the figure.

**c)** Consider a unity feedback system having an open-loop transfer function, **(10 Marks)**

$$G(j\omega) = \frac{K}{j\omega(1 + j0.2\omega)(1 + j0.05\omega)}.$$

Draw the polar plot of the system and Comment on the stability of the system.

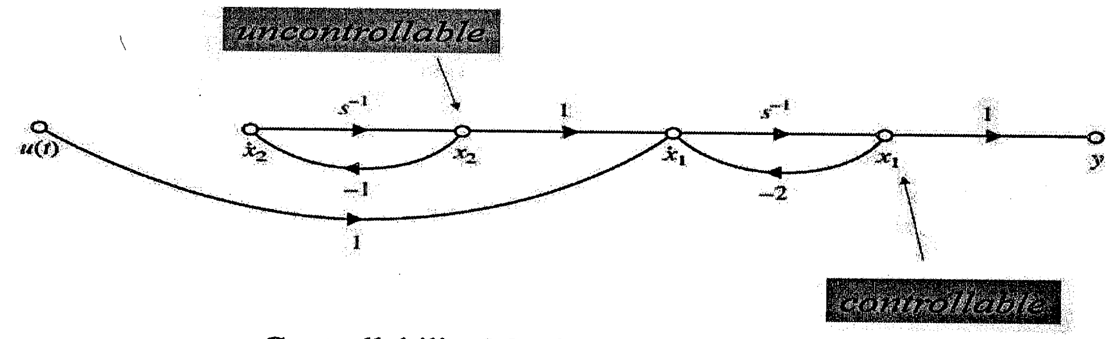


**Problem 3: (25 Marks)** - - **(ILOs):** [a1, b2, b14, c3, c14]

a) What are controllability and observability of control systems? (10 Marks)

# State Controllability

A system is completely controllable if there exists an unconstrained control *u(t)* that can transfer any initial state $x(t_o)$ to any other desired location *x(t)* in a finite time, $t_o \le t \le T$.




Controllability Matrix CM

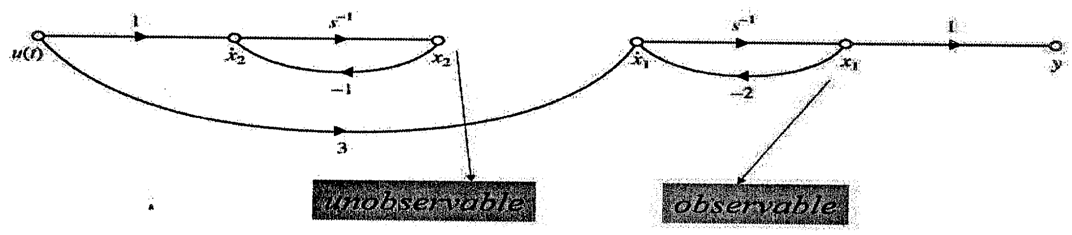
$$CM = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

System is said to be state controllable if

$$rank(CM) = n$$

# State Observability

A system is completely observable if and only if there exists a finite time *T* such that the initial state *x(0)* can be determined from the observation history *y(t)* given the control *u(t)*, $0 \le t \le T$.




Observable Matrix (OM)

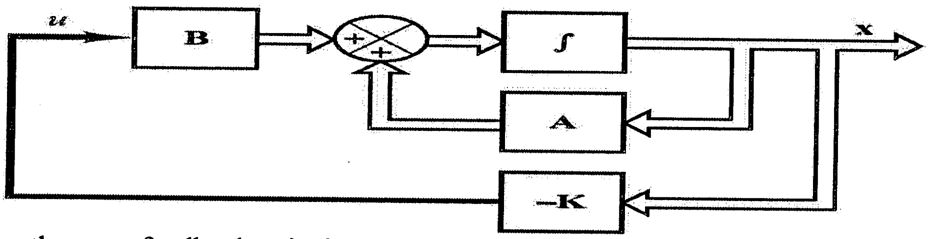
$$\textbf{Observability Matrix } OM = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

The system is said to be completely state observable if

$$rank(OM) = n$$

b) Consider the regulator system shown in following figure. The plant is given by: **(15 Marks)**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$




Determine the state feedback gain for each state variable to place the poles at -1+j, -1-j,-3.

**Solution**

Step-1

- First, we need to check the controllability matrix of the system. Since the controllability matrix **CM** is given by

$$CM = [B \quad AB \quad A^2B] = \begin{bmatrix} 1 & 2 & 10 \\ 0 & 3 & 9 \\ 1 & 2 & 7 \end{bmatrix}$$

- 
- We find that rank(**CM**)=3. Thus, the system is completely state controllable and arbitrary pole placement is possible.

Step-2

- Let K be
- $$K = [k_1 \quad k_2 \quad k_3]$$

$$|sI - A + BK| = \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} [k_1 \quad k_2 \quad k_3] \right|$$

- Desired characteristic polynomial is obtained as

$$(s+1-j)(s+1+j)\,(s+3) = s^3 + 5s^2 + 7s + 3$$

- Comparing the coefficients of powers of s

K= [3.0 5.6667 5.0]

**Using matlab code:**

```
>> A=[1 2 1;0 1 3;1 1 1];B=[1;0;1];
>> J = [-1+j*1 -1-j*1 -3];
>> K = acker(A,B,J)
K =  [3.0000   5.6667   5.0000]
```