Kaferelsheikh University

Department of Electrical Engineering

Subject: Electromagnetic waves

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Full Mark: 90 degree   Final Exam:Solution   Date: 10 /1/2018   Time allowed: 3 h

[1] Question One: (20 Mark)

A- Write down the conditions for the following mediums:

- Perfect dielectric - lossy dielectric -good conductor   [a.20.1 *(5 marks)*]

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$)
2. Lossless dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)
4. Good conductors ($\sigma \simeq \infty, \varepsilon = \varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \gg \omega\varepsilon$)

B- Starting with Maxwell equations, , **drive** the wave equation which describes the time and distance evolution for the electric field   [c.1.1*(5 marks)*]

From Maxwell equations in point form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

and

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times (\nabla \times \mathrm{E}) = -\mu\sigma\frac{\partial \mathrm{E}}{\partial t} - \epsilon\mu\frac{\partial^2 \mathrm{E}}{\partial t^2}, \quad \text{and} \quad \nabla \times (\nabla \times \mathrm{H}) = -\mu\sigma\frac{\partial \mathrm{H}}{\partial t} - \epsilon\mu\frac{\partial^2 \mathrm{H}}{\partial t^2}.$$

Using the vector identity $\nabla \times (\nabla \times\ ) \equiv \nabla(\nabla \bullet\ ) - \nabla^2(\ )$

$$\nabla^2\mathrm{E} - \epsilon\mu\frac{\partial^2 \mathrm{E}}{\partial t^2} - \mu\sigma\frac{\partial \mathrm{E}}{\partial t} = 0, \quad \nabla^2\mathrm{H} - \epsilon\mu\frac{\partial^2 \mathrm{H}}{\partial t^2} - \mu\sigma\frac{\partial \mathrm{H}}{\partial t} = 0.$$

If the field is time-harmonic and we use complex notation, we obtain

$$\nabla^2\underline{\mathrm{E}} + (\omega^2\epsilon\mu - j\omega\mu\sigma)\underline{\mathrm{E}} = 0,$$

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$$\nabla^2 \underline{\mathrm{E}} + (\omega^2 \epsilon \mu - \mathrm{j}\omega\mu\sigma)\underline{\mathrm{E}} = 0,$$

$$\nabla^2 \underline{\mathrm{H}} + (\omega^2 \epsilon \mu - \mathrm{j}\omega\mu\sigma)\underline{\mathrm{H}} = 0.$$

For the vector E in perfect dielectric medium (σ= 0) becomes

$$\frac{\partial^2 E_x}{\partial z^2} - \epsilon\mu \frac{\partial^2 E_x}{\partial t^2} = 0.$$

The solution to this equation is of the form

$$E_x(z, t) = E_1 f_1 \left(t - \frac{z}{c}\right) + E_2 f_2 \left(t + \frac{z}{c}\right).$$

E field consisting of incident and reflected plane waves. In this equation, Velocity of propagation of plane waves)

$$c = \frac{1}{\sqrt{\epsilon\mu}}$$

E1 and E2 are constants, andf_1 and f_2 are any functions of the arguments (t- z/c) and (t + z/c), respectively.

The electric and magnetic field vectors of the electromagnetic wave we considered are in planes perpendicular to the direction of propagation of the wave (the z direction).

C- <u>Discuss</u> the effect of the dispersive medium on the electromagnetic wave velocity

[a.21.1*(5 marks)*]

In many media, dispersion can be ignored. In quite a number of important cases, however, it must be taken into account, as it results in signal distortion. Namely, any signal is composed of time-harmonic components contained in a certain frequency band. Its shape is determined by relative amplitudes and phases of the time-harmonic components in this band. If the velocities of the time-harmonic components are not the same, their relative positions, which means their relative phases, change as the signal propagates, which means that the signal shape changes as well. For this reason, the frequency band of a signal is usually made small enough so that distortion can be ignored. The velocity c of plane waves described above is the velocity with which the phase of the wave propagates. To be more specific, it determines the progression of the z coordinates in the argument of the cosine function, which ensures that as time passes, the argument (i.e., the phase) of the cosine function remains unchanged. For this reason, the velocity C is termed the phase velocity.

the phase velocity, V_{ph} = c, can be expressed as:

$$v_{\mathrm{ph}}(\omega) = \frac{\omega}{\beta(\omega)} \qquad (\mathrm{m/s}).$$

There is another important concept connected to dispersion, known as the group velocity. It represents the velocity of the signal in a dispersive medium and can be defined only for cases where dispersion is small. To determine the group velocity, consider a simple signal in a weakly dispersive medium. Let the signal be obtained as a superposition of two plane waves propagating in the same direction and with slightly different angular frequencies, ω_1 and ω_2, and slightly different phase coefficients, β_1 and β_2 (due to dispersion). Without loss of generality, we can assume the amplitudes of both waves are the same, for example equal to 1, and consider a signal f(z, t) of the form

$$f(z,t) = \cos(\omega_1 t - \beta_1 z) + \cos(\omega_2 t - \beta_2 z).$$

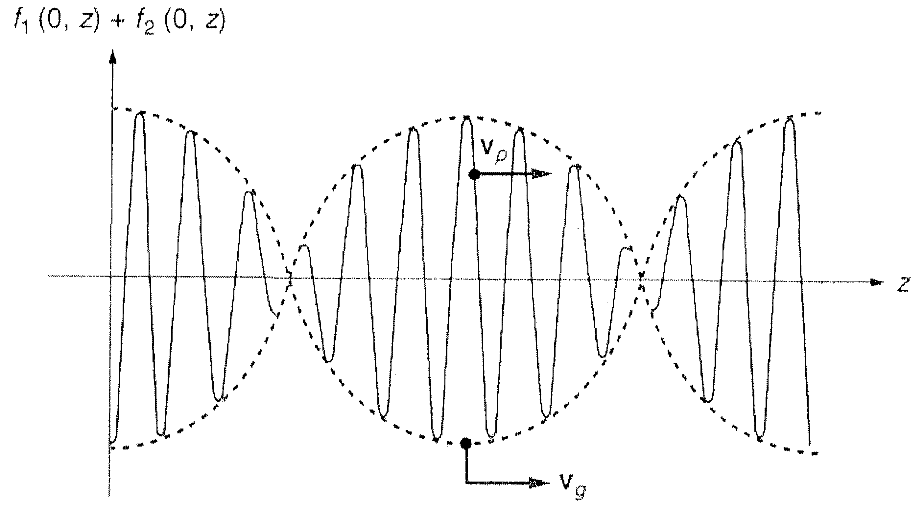
Now, $\cos a_1 + \cos a_2 = 2\cos[(a_2 - a_1)/2]\cos[(a_2 + a_1)/2]$, so that

$$f(z,t) = 2\cos(\Delta\omega t - \Delta\beta_1 z)\cos(\omega t - \beta z),$$

where

$$\Delta\beta = \frac{\beta_2 - \beta_1}{2}, \qquad \Delta\omega = \frac{\omega_2 - \omega_1}{2}, \qquad \omega = \frac{\omega_1 + \omega_2}{2}, \qquad \beta = \frac{\beta_1 + \beta_2}{2}.$$

Since $\Delta\omega \ll \omega$, the shape of this signal frozen in time is as sketched below. This is a rapidly varying wave (of frequency ω) modulated by a slowly varying wave (of frequency $\Delta\omega$). The velocity of propagation of the rapidly varying modulated wave (the solid line in Fig. ()) is simply ω/β.



Sum of two time-harmonic functions of equal amplitudes and slightly different frequencies

The velocity of the modulating wave (signal), however, is different-it is equal to the velocity of the envelope of the rapidly varying wave, indicated by the dashed lines. This means that the group velocity, V_g, is given by

$$v_g = \frac{\Delta\omega}{\Delta\beta} = \frac{1}{\Delta\beta/\Delta\omega}.$$

D- The electric field in free space is given by: $E = 50\cos(10^8 t + \beta x)a_y V/m$

i- **Find** the direction of wave propagation

−X direction

ii- **Calculate** β and the time it takes to travel a distance of λ/2 [c.1.1(*5 marks*)]

w= 10^8 Hz , β= w/C= 10^8 /(3x 10^8)=0.33 rad/m

time(t)=distance/velocity=(λ/2)/(λ. w/2π)=31.4 nano second

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[2] Question Two: (25 Mark)

A- A lossy dielectric has an intrinsic impedance of 200e$^{j\pi/6}$ at a particular frequency. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component $H = 10\,e^{-\alpha x}\cos(\omega t - 0.5x)a_y$ A/m. **find E** and **Determine** the skin depth and wave polarization [a.21.1, b.3.1(*7 marks*)]

Solution:

The given wave travels along $\mathbf{a}_x$ so that $\mathbf{a}_k = \mathbf{a}_x$; $\mathbf{a}_H = \mathbf{a}_y$, so

$$-\mathbf{a}_E = \mathbf{a}_k \times \mathbf{a}_H = \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

Also $H_o = 10$, so

$$\frac{E_o}{H_o} = \eta = 200\,\underline{/30^\circ} = 200\,e^{j\pi/6} \rightarrow E_o = 2000e^{j\pi/6}$$

Except for the amplitude and phase difference, **E** and **H** always have the same form. Hence

$$\mathbf{E} = \text{Re}\,(2000e^{j\pi/6}e^{-\gamma x}e^{j\omega t}\mathbf{a}_E)$$

$$\mathbf{E} = -2e^{-\alpha x}\cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right)\mathbf{a}_z \text{ kV/m}$$

Knowing that $\beta = 1/2$, we need to determine α. Since

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1}{\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1}\right]^{1/2}$$

But $\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60° = \sqrt{3}$. Hence,

$$\frac{\alpha}{\beta} = \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\beta}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2887 \text{ Np/m}$$

$$\delta = \frac{1}{\alpha} = 2\sqrt{3} = 3.464 \text{ m}$$

B- A 5-GHz uniform plane wave $E_{is} = 10e^{-j\beta z}a_x$ V/m in free space is incident normally on a large plane, lossless dielectric slab (z>0), having $\varepsilon_r = 4$ and $\mu_r = 1$. **Find** the electric field components of both the reflected and transmitted waves [a.21.1, b.3.1*(6 marks)*]

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}, \quad v = \frac{1}{\sqrt{\mu\varepsilon}} \quad \therefore \eta_2 = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \eta_1$$

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_1\left(\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1\right)}{\eta_1\left(\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1\right)} = \frac{-1/2}{3/2} = -0.33$$

$$\beta = \frac{\omega}{v} \Rightarrow \beta_2 = \frac{\omega}{v_2}, \quad v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\therefore \beta_2 = \frac{\omega}{1/\sqrt{\mu\varepsilon}} = 2\beta_1$$

Answer: $-3.333 \exp(j\beta_1 z)\, \mathbf{a}_x$ V/m, $6.667 \exp(-j\beta_2 z)\, \mathbf{a}_x$ V/m where $\beta_2 = 2\beta_1 = 200\pi/3$.

C- **Derive** an expression for the reflection coefficient in case of oblique incidence with parallel polarization. [a.21.1, c.1.1*(6 marks)*]

In this case uniform plane wave takes the general form of

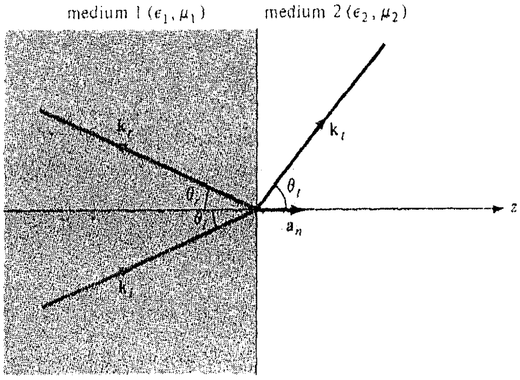
$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_o \cos(\mathbf{k}\cdot\mathbf{r} - \omega t) = \text{Re}\,[E_o e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}]$$

where $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the radius or position vector and $\mathbf{k} = k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z$ is the *wave number vector* or the *propagation vector;* **k** is always in the direction of wave propagation. The magnitude of **k** is related to ω according to the dispersion relation

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon \qquad \mathbf{k}\cdot\mathbf{r} = k_x x + k_y y + k_z z = \text{constant}$$

$$\mathbf{H} = \frac{1}{\omega\mu}\,\mathbf{k}\times\mathbf{E} = \frac{\mathbf{a}_k\times\mathbf{E}}{\eta}$$

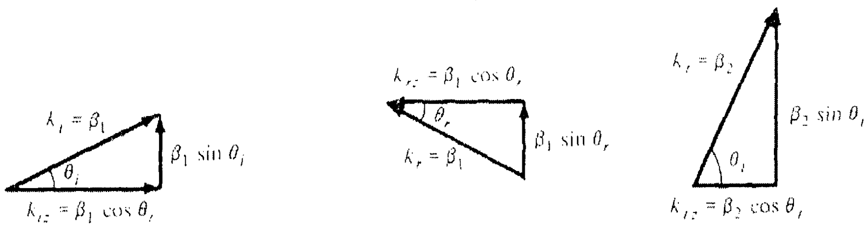
Again, both the incident and the reflected waves are in medium 1 while the transmitted (or refracted wave) is in medium 2. Let



$$\mathbf{E}_i = \mathbf{E}_{io}\cos(k_{ix}x + k_{iy}y + k_{iz}z - \omega_i t)$$

$$\mathbf{E}_r = \mathbf{E}_{ro}\cos(k_{rx}x + k_{ry}y + k_{rz}z - \omega_r t)$$

$$\mathbf{E}_t = \mathbf{E}_{to}\cos(k_{tx}x + k_{ty}y + k_{tz}z - \omega_t t)$$



Since the tangential component of **E** must be continuous at the boundary $z = 0$,

$$\mathbf{E}_i(z=0) + \mathbf{E}_r(z=0) = \mathbf{E}_t(z=0)$$

1. $\omega_i = \omega_r = \omega_t = \omega$
2. $k_{ix} = k_{rx} = k_{tx} = k_x$
3. $k_{iy} = k_{ry} = k_{ty} = k_y$

$$k_i\sin\theta_i = k_r\sin\theta_r$$

$$k_i\sin\theta_i = k_t\sin\theta_t$$

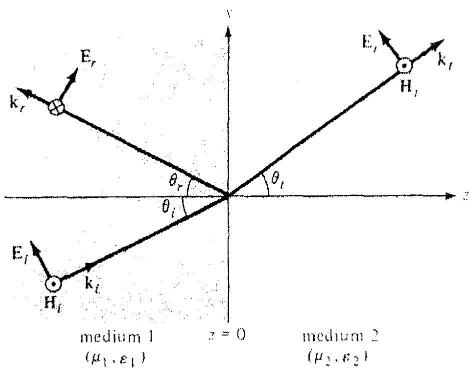
For Parallel Polarization

$$\mathbf{E}_{is} = E_{io}(\cos\theta_i\,\mathbf{a}_x - \sin\theta_i\,\mathbf{a}_z)\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\mathbf{H}_{is} = \frac{E_{io}}{\eta_1}\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\,\mathbf{a}_y$$

$$\mathbf{E}_{rs} = E_{ro}(\cos\theta_r\,\mathbf{a}_x + \sin\theta_r\,\mathbf{a}_z)\,e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\mathbf{H}_{rs} = -\frac{E_{ro}}{\eta_1}\,e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\,\mathbf{a}_y$$



The transmitted fields exist in medium 2 and are given by

$$\mathbf{E}_{ts} = E_{to}(\cos\theta_t\,\mathbf{a}_x - \sin\theta_t\,\mathbf{a}_z)\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\mathbf{H}_{ts} = \frac{E_{to}}{\eta_2}\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\,\mathbf{a}_y \qquad \text{where } \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}.$$

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2}E_{to}$$

Expressing E_{ro} and E_{to} in terms of E_{io}, we obtain

$$\Gamma_{\|} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

D- A polarized wave is incident from air to polystyrene with $\varepsilon_r = 3$ and $\mu_r = 1$ at Brewster angle. **Determine** the transmission angle [c.2.1*(6 marks)*]

$$\tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = 1.73 \Rightarrow \theta_B = 60^\circ$$

$$\eta_2\cos\theta_t = \eta_1\cos\theta_{B\|} \qquad , \eta_2 = \sqrt{\frac{\mu_r}{\varepsilon_r}}\,\eta_1$$

$$\therefore \cos\theta_t = 0.866$$

$$\therefore \theta_t = 30^\circ$$

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[3] Question three: (20 Mark)

A- Given a distortion less T.L having distributed parameters of R, L, G and C. **Deduce** the relation used to determine the attenuation constant [b.3.1*(6 marks)*]

Distortionless Line ($R/L = G/C$)

Thus, for a distortionless line,

$$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$
$$= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

$\alpha = \sqrt{RG}, \qquad \beta = \omega\sqrt{LC}$ showing that α does not depend on frequency

B- A 75 ohm lossless line is to be matched to a 100 - j80 (ohm) load with a shorted parallel stub. **Use smith chart to calculate** the stub length, its distance from the load, and the necessary stub admittance. [a.4.1, b.3.1*(8 marks)*]

$\tilde{Z}_L = Z_L / Z_o = 1.33 - 1.067$ Ω

Locate $\tilde{Z}_L$ on smith chart and then draw the constant SWR circle, by mirror obtain $\tilde{Y}_L$

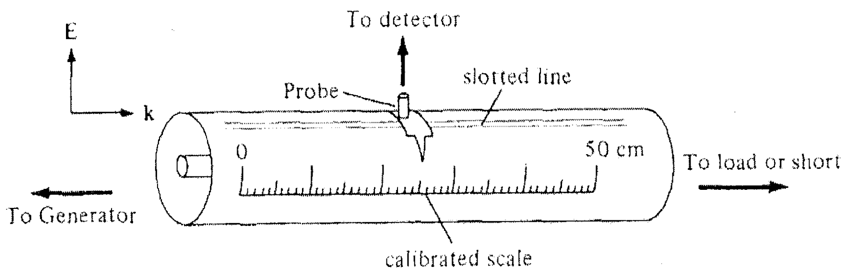
- The SWR circle intersects the unity circle into two points A and B at both of them the real part of $\bar{y}_{in} = 1$ $\bar{Y}_A = 1 + j\tilde{b}$, $\bar{Y}_B = 1 - j\tilde{b}$

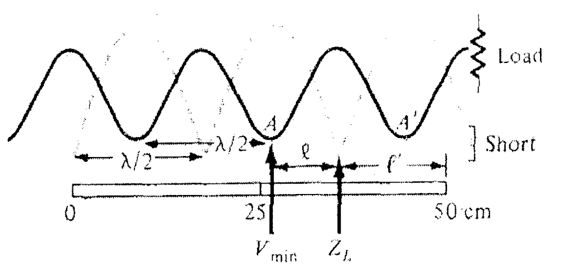
→ The stub will be connect at this distance

$$d = (\theta_{A or B} - \theta_L) \cdot \frac{\lambda}{720}$$

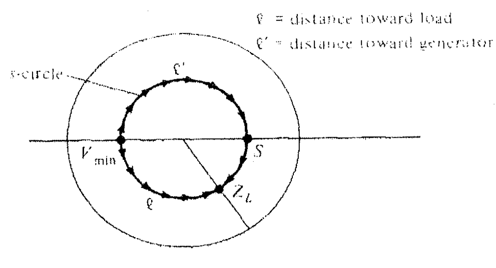
→ for stub length d move from the s.c point towards the generator until reaching $-j\tilde{b}$

C) Explain the slotted line technique used for impedance measurement. [a.4.1*(6 marks)*]





1. With the load connected, read s on the detection meter. With the value of s, draw the s-circle on the Smith chart.
2. With the load replaced by a short circuit, locate a reference position for z_L at a voltage minimum point.
3. With the load on the line, note the position of V_{min} and determine ℓ.
4. On the Smith chart, move toward the load a distance ℓ from the location of V_{min}. Find Z_L at that point.



$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{8.636}\right)^2+\left(\frac{0}{b}\right)^2} = 1.74\ GH$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{1.74\times10^9}{4\times10^9}\right)^2}} = 418.5\ \Omega$$

$= 0.9$

$$u_{ph} = \frac{u'}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} = 3.33 * 10^8\ m/s$$

$$u_g = \frac{u'\sqrt{1-\left(\frac{f}{f_c}\right)^2}}{\sqrt{1}} = 2.7 * 10^8\ m/s$$

C- Design a rectangular waveguide with an aspect ratio of 3 to 1 for use in the k band (18-26.5 GHz). Assume that the guide is air filled. [b.2.1, c.2.1 *(6 marks)*]

$f_c = 18$ GHz , $a/b = 3:1$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{u'}{2}\cdot\frac{1}{b}\sqrt{\left(\frac{m}{a/b}\right)^2+n^2}$$

for TE_{10}

$$18\ GHz = \frac{3\times10^8}{2}\cdot\frac{1}{b}\cdot\frac{1}{3} \Rightarrow b = 2.8\ mm$$

$a = 8.33$ mm

D- Design an air-filled cubical cavity to have its dominant resonant frequency at 3 GHz [b.2.2, c.2.1 *(6 marks)*]

$f_r = 3$ GHz

The dominant mode is TE_{101}

cubical $\Rightarrow a = b = c$

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$$

$$\therefore 3\,GHz = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{0}{a}\right)^2+\left(\frac{1}{a}\right)^2} = \frac{3\times10^8}{\sqrt{2}}\cdot\frac{1}{a}$$

$\therefore a = b = c = 7$ cm

[4] Question four: (25 Mark)

A- with the aid of equations, <u>prove</u> that the wave propagation through rectangular wave guides takes zigzag paths. [a.21.2, c.1.2*(6 marks)*]

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) \qquad \cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

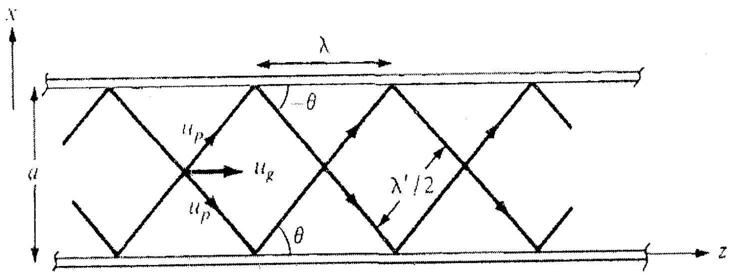
For the TE10 mode, for example,

$$E_{ys} = -\frac{j\omega\mu a}{\pi}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$
$$= -\frac{\omega\mu a}{2\pi}(e^{j\pi x/a} - e^{-j\pi x/a})\,e^{-j\beta z}$$
$$= \frac{\omega\mu a}{2\pi}[e^{-j\beta(z+\pi x/\beta a)} - e^{-j\beta(z-\pi x/\beta a)}]$$

The first term of the previous eq. represents a wave traveling in the positive z-direction at an angle

$$\theta = \tan^{-1}\left(\frac{\pi}{\beta a}\right)$$

The second term of represents a wave traveling in the positive z-direction at an angle -θ. The field may be depicted as a sum of two plane TEM waves propagating along zigzag paths between the guide walls at x = 0 and x = a as illustrated in figure



B- A standard air-filled rectangular waveguide with dimensions a = 8.636 cm, b = 4.318 cm is fed by a 4-GHz carrier from a coaxial cable. **<u>Determine</u>** the phase velocity and the group velocity if a TE_{10} mode will be propagated. [b.2.1, c.1.2*(7 marks)*]