



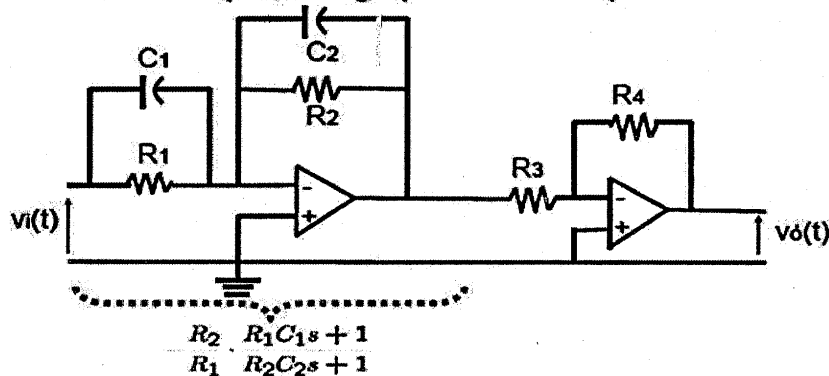
Answer all the following questions:

Intended learning outcomes (ILOs): [a1, a4, a12 , b2, b6, b11, b12, b14, c3,c14]

Problem 1: (35 Marks) - - (ILOs): [a1, a12, b2, b6, c3]

- a) Construct the configuration of the electronic lag compensator using operational amplifiers and drive the transfer function of the compensator. (10 Marks)

- One example, using operational amplifiers

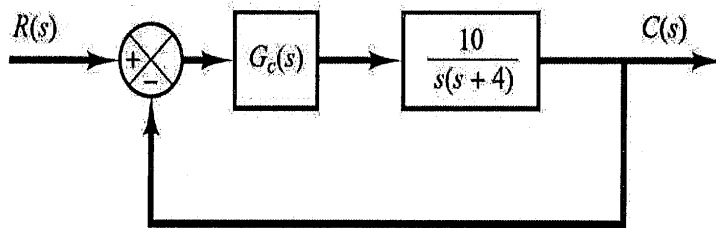


- Transfer function

$$C(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2}{C_2 s}}{R_2 + \frac{1}{C_2 s}} \cdot \frac{R_4}{R_3} = \frac{R_4 C_1}{R_3 C_2} \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}}$$

$\begin{matrix} K \\ \uparrow \\ \frac{R_4 C_1}{R_3 C_2} \\ \downarrow \end{matrix} \quad \begin{matrix} z \\ \uparrow \\ s + \frac{1}{R_1 C_1} \\ \downarrow \end{matrix} \quad \begin{matrix} p \\ \downarrow \\ s + \frac{1}{R_2 C_2} \\ \uparrow \end{matrix}$

- b) Design a lag compensator for following unity feedback system such that the static velocity error constant is 50 sec^{-1} without appreciably changing the closed loop poles, which are at $s = -2 \pm j\sqrt{6}$. (15 Marks)



Solution. Assume that the transfer function of the lag compensator is

$$G_c(s) = \hat{K}_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \quad (\beta > 1)$$

Since K_v is specified as 50 sec^{-1} , we have

$$K_v = \lim_{s \rightarrow 0} sG_c(s) \frac{10}{s(s+4)} = \hat{K}_c \beta 2.5 = 50$$

Thus

$$\hat{K}_c \beta = 20$$

Now choose $\hat{K}_c = 1$. Then

$$\beta = 20$$

Choose $T = 10$. Then the lag compensator can be given by

$$G_c(s) = \frac{s + 0.1}{s + 0.005}$$

The angle contribution of the lag compensator at the closed-loop pole $s = -2 + j\sqrt{6}$ is

$$\begin{aligned} \angle G_c(s) \Big|_{s = -2 + j\sqrt{6}} &= \tan^{-1} \frac{\sqrt{6}}{-1.9} - \tan^{-1} \frac{\sqrt{6}}{-1.995} \\ &= -1.3616^\circ \end{aligned}$$

which is small. Thus the change in the location of the dominant closed-loop poles is very small. The open-loop transfer function of the system becomes

$$G_c(s)G(s) = \frac{s + 0.1}{s + 0.005} \frac{10}{s(s+4)}$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10s + 1}{s^3 + 4.005s^2 + 10.02s + 1}$$

c) Draw the bode diagram of the following transfer function:

$$G(j\omega) = \frac{5(1 + j0.1\omega)}{j\omega(1 + j0.5\omega)(1 + j0.6\frac{\omega}{50} + (\frac{j\omega}{50})^2)}$$

After plotting Bode diagram evaluate the gain margin and the phase margin of the system and Comment on the stability of the system whose Bode diagram. (10 Marks)