



All questions measure ILOs: a1,a5,b1,b2 and b7.

**Question 1: Complete each of the following sentences:** (30 Marks)

- If the axis are turned through an angle  $(\tan^{-1}2)$ , then the equation  $4xy - 3x^2 = a^2$  becomes..... (a1,a5,b7)(6 Marks)
- The equation of the tangent to the circle  $x^2 + y^2 - 4y - 1 = 0$  at the point (2,1) is ..... (a5)(6 Marks)
- For the ellipse whose equation is  $4x^2 + 9y^2 - 48x + 72y + 144 = 0$ , the coordinates of the vertices are ....., and the length of the latus rectum is..... (a1,a5,b5)(6 Marks)
- The equation of the locus of a point that moves so that it always is equidistance from the line  $x = -2$  and the fixed point (2,0) is ..... (a1,a5,b5)(6 Marks)
- To represent the equation  $2x^2 + xy - y^2 - 11x - 5y + k = 0$  pair of lines, the value of  $k$  is ..... (a1,a5,b5)(6 Marks)

**Question 2: Put true (✓) or false (×) for each of the following sentences** (25 Marks)

- The area of the triangle, whose vertices are (2,1), (5, -3) and (-8, 0), is  $(43/4)$  units. ( ) (a1, b5)(5 Marks)
- The length of the latus rectum for any ellipse is  $(4b^4/a^2)^{1/2}$ . ( ) (a5,b5)(5 Marks)
- The center of the circle  $r^2 - 3r \cos \theta - 3\sqrt{3}r \sin \theta - 16 = 0$  is  $(\frac{3}{2}, \frac{3\sqrt{3}}{2})$ . ( ) (a1, b5)(5 Mark)
- The image of a point  $p(1, \sqrt{3})$  by rotation around the origin point measured by angle  $(\pi/3)$  is (0,2). ( ) (a5,b5)(5 Marks)
- The equation  $x^2 - y^2 = 4$  represents a circle. ( ) (a5,b5)(5 Marks)

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**Question 3: (25 Marks)**

1. Evaluate the following integrals: (a5,b7)(16 Marks)

a.  $\int \frac{x^4 - 4}{x^3 - 3x^2 - x + 3} dx$

b.  $\int e^{3x} \sqrt{1 - e^{2x}} dx$

c.  $\int \sqrt{\frac{x}{x-1}} dx$

d.  $\int \ln(1 + \sin x)^{\sin 2x} dx$

2. Derive the reduction formula for  $\int \sin^n x dx$  and hence find  $\int \sin^3 x dx$ .

(a5,b7)(5 Marks)

3. Find the length of the curve  $y = \ln(\sec x)$ , between the points  $x = 0$  and  $x = \pi/4$

(a5,b7) (4 Marks)

**Question 4: (30 Marks)**

1. Evaluate an approximate value for  $\int_1^2 \frac{1}{x+1} dx$  using numerical integration,

dividing the interval into 5 subintervals ( $n=5$ ) and deduce the value of the error.

(b1,b7)(6 Marks)

2. Evaluate the value of the integrals, if exist,

(a5,b1,b7)(12 Marks)

a.  $\int_{-1}^1 \frac{(\tan^{-1} x)^2 + \sin^3 x}{(1+x^2)} dx$

b.  $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

3. Find the area between the two curves  $y = x^2 - 3$  and  $y = |2x|$ .

(a5,b7) (6 Marks)

4. Find the volume of the solid generated by revolving about the Y-axis, the curve

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , in the first quadrant (الربع الاول) and then prove that the volume of

the sphere of radius ( $a$ ) is  $(\frac{4}{3} \pi a^3)$ .

(a5,b7) (6 Marks)

*With our best wishes*

A. A. Nassef      Samah  
Prof. A. A. Elgaber Nassef and Dr Samah El-Kholq