Kafrelsheikh University		Semester: 2nd Semester
Mechanical Engineering		Final Examination
Dept. Mechanical Engineering		Date: May 20th, 2018
Year: Fist Year		Time allowed: 3 hour
Instructor: Assoc. Prof. Maher		Full Mark: 60

Subject: Thermodynamics I (MEP1203)

Questions and Answers Booklet

Answer Model (نمــوذج الإجابـــــــــــــة)

(a) This exam measures ILOs no.: a.5 b.2 c.1 d7, and d9
(b) No. of questions: 6. No. of pages: 12 (only pages no [9/12] and [12/12] are empty)
(c) This is a close book exam. Only thermodynamics tables and calculator are permitted
(d) Clear, systematic answers and solutions are required. In general, marks will not be assigned for answers and solutions that require unreasonable (in the opinion of the instructor) effort to decipher.
(e) Retain all the significant figures of properties taken from tables. Final results should have at least 3 to 5 significant digits.
(f) Ask for clarification if any question statement is not clear to you.
(g) Solve all questions.
(h) The exam will be marked out of 60. There are 30 marks bonus.

Question #1 (27 Marks)

Choose the correct answer. Justify your answer with calculations or explanations or both whenever possible. If answer requires justification, marks will not be given to the correct answer without justification.

1. The latent heat of vaporization at critical point is **(1 Mark)**

(*a*) less than zero (*b*) greater than zero
(*c*) equal to zero (*d*) none of the above.

2. Select a correct statement of the first law if kinetic and potential energy changes are neglected. **(1 Marks)**

(A) Heat transfer equals the work done for a process.
(B) Net heat transfer equals the net work for a cycle.
(C) Net heat transfer minus net work equals internal energy change for a cycle.
(D) Heat transfer minus work equals internal energy for a process.

3. A definite area or space where some thermodynamic processes takes place is known as **(1 Mark)**

(a) thermodynamic system (b) thermodynamic cycle
(c) thermodynamic process (d) thermodynamic law.

4. An open system is one in which **(1 Mark)**

(a) heat and work cross the boundary of the system, but the mass of the working substance does not
(b) mass of working substance crosses the boundary of the system but the heat and work do not

(c) both the heat and work as well as mass of the working substances cross the boundary of the system
(d) neither the heat and work nor the mass of the working substances cross the boundary of the system.

5. An isolated system **(0.5 Mark)**
(a) is a specified region where transfer of energy and/or mass take place
(b) is a region of constant mass and only energy is allowed to cross the boundaries
(c) cannot transfer either energy or mass to or from the surroundings
(d) is one in which mass within the system is not necessarily constant

6. Which of the following is an intensive property of a thermodynamic system ? **(0.5 Mark)**
(a) Volume
(b) Temperature
(c) Mass
(d) Energy.

7. Which of the following is the extensive property of a thermodynamic system ? **(0.5 Mark)**
(a) Pressure
(b) Volume
(c) Temperature
(d) Density.

8. When two bodies are in thermal equilibrium with a third body they are also in thermal equilibrium witheach other. This statement is called **(0.5 Marks)**
(a) Zeroth law of thermodyamics
(b) First law of thermodynamics
(c) Second law of thermodynamics
(d) Kelvin Planck's law.

9. Select a correct statement of the first law if kinetic and potential energy changes are neglected. **(1 Marks)**

(A) Heat transfer equals the work done for a process.
(B) Net heat transfer equals the net work for a cycle.
(C) Net heat transfer minus net work equals internal energy change for a cycle.
(D) Heat transfer minus work equals internal energy for a process.

10. Absolute zero temperature is taken as **(0.5 Mark)**
(a) – 273°C
(b) 273°C
(c) 237°C
(d) – 373°C.

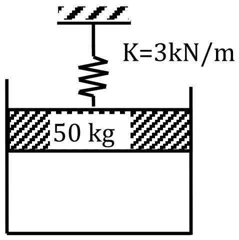
11. Which of the following is correct ? **(0.5 Mark)**
(a) Absolute pressure = gauge pressure + atmospheric pressure
(b) Gauge pressure = absolute pressure + atmospheric pressure
(c) Atmospheric pressure = absolute pressure + gauge pressure
(d) Absolute pressure = gauge pressure – atmospheric pressure

12. Calculate the pressure in the 140-mm-diameter cylinder shown. The spring is compressed 60 cm. Neglect friction. **(2 Marks)**
(A) 140 kPa
(B) 135 kPa
(C) 100 kPa
(D) 35 kPa

$$p = p_{atm} + (mg + kx)/A_p$$

$$p = 100 + \left(50\times 9.81\times 10^{-3} + 3\times 0.60\right)/\left(\frac{\pi}{4}\times(0.240)^2\right)$$

$$p = 140.20 \ \text{kPa}$$



13. The volume occupied by 4 kg of 200°C steam at a quality of 80 percent is nearest **(1 Marks)**
(A) 0.004 m³
(B) 0.104 m³
(C) 0.4 m³
(D) 4.1 m³

From saturated steam tables at 70 °C $v_g = 0.001157$ m³/kg $v_g = 0.12721$ m^3/kg

$$v = v_f + x(v_g - v_g) = 0.001157 + 0.8\times(0.12721 - 0.001157) = 0.102 \ \text{m}^3/\text{kg}$$

$$V = mv = 4\times 0.102 = 0.4080 \ \text{m}^3$$

14. Saturated steam is heated in a rigid tank from 70 to 800°C. P_2 is nearest **(2 Marks)**

(A) 100 kPa
(B) 200 kPa
(C) 300 kPa
(D) 400 kPa

From saturated steam tables at 70 °C

$$v_g = 5.0396 \text{ m}^3/\text{kg}$$

$$v_2 = v_1 = v_g = 5.0396 \text{ m}^3/\text{kg}$$

From superheated steam tables at 800 °C and 5.0396 m³/kg

$$p_2 = 0.099 \text{ MPa} \approx 0.1 \text{ MPa}$$

15. A vertical circular cylinder holds a height of 1 cm of liquid water and 100 cm of vapor. If P = 200 kPa, the quality is nearest **(1.5 Marks)**

(A) 0.01
(B) 0.1
(C) 0.4
(D) 0.8

From saturated steam tables at 200 kPa

$$v_f = 0.0.001061 \text{ m}^3/\text{kg}$$

$$v_g = 0.88578 \text{ m}^3/\text{kg}$$

$$x = \frac{m_g}{m_g + m_f} = \frac{V_g / v_g}{V_g / v_g + V_f / v_f} = \frac{h_g A / v_g}{h_g A / v_g + h_f A / v_f} = \frac{h_g / v_g}{h_g / v_g + h_f / v_f}$$

$$x = \frac{1.00/0.88578}{1.00/0.88578 + 0.01/0.001061} = 0.107$$

16. The point that connects the saturated-liquid line to the saturated-vapor line is called the **(0.5 Marks)**

(A) triple point
(B) critical point
(C) superheated point
(D) compressed liquid point

17. Air (R=0.287 kJ/kg.K) undergoes a three-process cycle. Find the net work done for 2 kg of air if the processes are **(4 Marks)**

$1 \rightarrow 2$: constant-pressure expansion
$2 \rightarrow 3$: constant volume
$3 \rightarrow 1$: constant-temperature compression

The necessary information is T_1 = 100°C, T_2 = 600°C, and P_1 = 200 kPa.

(A) 105 kJ
(B) 96 kJ
(C) 66 kJ
(D) 11.5 kJ

$$W_{1-2} = mp(v_2 - v_1) = mR(T_2 - T_1) = 2 \times 0.287 \times (600 - 100) = 287 \text{ kJ}$$

$$W_{2-3} = 0$$

$$W_{3-1} = mRT_1 \ln\left(\frac{v_1}{v_3}\right) = mRT_1 \ln\left(\frac{v_1}{v_2}\right) = mRT_1 \ln\left(\frac{RT_1/p_1}{RT_2/p_2}\right) = mRT_1 \ln\left(\frac{T_1}{T_2}\right)$$

$$W_{3-1} = 2 \times 0.287 \times 373 \times \ln\left(\frac{373}{873}\right) = -182.06 \text{ kJ}$$

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 287 + 0 - 182.06 = 104.94 \approx 105 \text{kJ}$$

18. Propane (C_3H_8) is an ideal gas is maintained at 6.39 MPa and 444 K. How much volume does 1 kg of this gas fill? **(1 Marks)**

(a) 8.78 liters
(b) 12.3 liters
(c) 13.1 liters
(d) 15.7 liters

$$pv = mRT \qquad V = mRT/p = 1 \times (8.314/44) \times 444/6390 = 0.0131 \text{m}^3 = 13.1 \text{ litre}$$

19. For each of the cases below, determine if the heat engine satisfies the first law (energy equation) and if it violates the second law. **(2 Marks)**

a. $\dot{Q}_H$ = 6 kW $\quad \dot{Q}_L$ = 4 kW $\quad \dot{W}$ = 2 kW

b. $\dot{Q}_H$ = 6 kW $\quad \dot{Q}_L$ = 0 kW $\quad \dot{W}$ = 6 kW

c. $\dot{Q}_H$ = 6 kW $\quad \dot{Q}_L$ = 2 kW $\quad \dot{W}$ = 5 kW

d. $\dot{Q}_H$ = 6 kW $\quad \dot{Q}_L$ = 6 kW $\quad \dot{W}$ = 0 kW

	1st Law	2nd Law
a.	√	√
b.	√	×
c.	×	×
d.	√	×

20. A heat pump is absorbing heat from the cold outdoors at 5 °C and supplying heat to a house at 25°C at a rate of 18,000 kJ/h. If the power consumed by the heat pump is 1.9 kW, the coefficient of performance of the heat pump is **(1 Marks)**

(*a*) 1.3
(*b*) 2.6
(*c*) 3.0
(*d*) 3.8
(*e*) 13.9

$$\text{COP}_{HP} = \frac{\dot{Q}_H}{\dot{W}} = \frac{18000/3600}{1.9} = 2.63$$

21. A heat engine cycle is executed with steam in the saturation dome. The pressure of steam is 1 MPa during heat addition and 0.4 MPa during heat rejection. The highest possible efficiency of this heat engine is **(2 Marks)**

(*a*) 8.0%
(*b*) 15.6%
(*c*) 20.2%
(*d*) 79.8%
(*e*) 100%

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_{S@0.4\,\text{MPa}}}{T_{S@1\,\text{MPS}}} = 1 - \frac{143.6+273}{179.9+274} = 0.08$$

22. A heat engine receives heat from a source at 1000°C and rejects the waste heat to a sink at 50°C. If heat is supplied to this engine at a rate of 100 kJ/s, the maximum power this heat engine can produce is **(2 Marks)**

(*a*) 25.4 kW
(*b*) 55.4 kW
(*c*) 74.6 kW
(*d*) 95.0 kW
(*e*) 100 kW

$$\eta_{HE,\max} = 1 - \frac{T_L}{T_H} = \frac{\dot{W}_{\max}}{\dot{Q}_H}$$

$$\dot{W}_{\max} = \dot{Q}_H\left(1 - \frac{T_L}{T_H}\right) = 100 \times \left(1 - \frac{50+273}{1000+273}\right) = 74.6\,\text{kW}$$

Question #2 (14 Marks)

A closed system, containing 1.5 kg of helium (He), is initially at a pressure of P_1=120 kPa and a temperature of T_1 =60°C, undergoes two quasi-equilibrium processes, one after the other. The first process (state 1 to state 2) is a polytropic compression until the pressure and temperature are P_2=500 kPa and T_2=150°C. The second process (state 2 to state 3) is an adiabatic expansion until the pressure and temperature are P_3=200 kPa and T_3=-10 °C

a. Calculate the value of the polytropic exponent, *n*, for the first process (state 1 to state 2). **(4 Marks)**
b. Calculate the work done by the system in the first process, W_{12} in kJ. **(2 Marks)**
c. Calculate the heat transfer by the system in the first process, Q_{12} in kJ. **(2 Marks)**
d. Calculate the work done by the system in the second process, W_{23} in kJ. **(2 Marks)**
e. Show the two processes on a P-V (pressure-volume) diagram. Clearly identify the states and show the processes paths with respect to constant temperature lies. **(4 Marks)**

(N.B. use the following constants for helium, R = 2.0785 kJ/kg.K, C_{vo}=3.1156 kJ/kg.K)

Solution

a. For a closed system, polytropic process

$$P_1\mathcal{V}_1^{\,n}=P_2\mathcal{V}_2^{\,n}\rightarrow\frac{P_2}{P_1}=\left(\frac{\mathcal{V}_1}{\mathcal{V}_2}\right)^n\rightarrow n=\frac{\ln(P_2/P_1)}{\ln(\mathcal{V}_1/\mathcal{V}_2)}$$

$$\mathcal{V}_1=mRT_1/P_1=1.5\times2.0785\times(60+273)/120=8.652\text{ m}^3$$

$$\mathcal{V}_2=mRT_2/P_2=1.5\times2.0785\times(150+273)/500=2.638\text{ m}^3$$

$$n=\frac{\ln(P_2/P_1)}{\ln(\mathcal{V}_1/\mathcal{V}_2)}=\frac{\ln(500/120)}{\ln(8.652/2.638)}=1.2$$

b. Work done by the system in the first process (process 1→2 is a polytropic process is expressed as

$$W_{12}=\frac{P_2\mathcal{V}_2-P_1\mathcal{V}_1}{1-n}=\frac{500\times2.638-120\times8.652}{1-1.2}=-1403.8\ \text{kJ}$$

c. Heat transfer by the system in the first process, Q_{12} is

$$Q_{12}-W_{1-2}=m(u_2-u_1)=$$

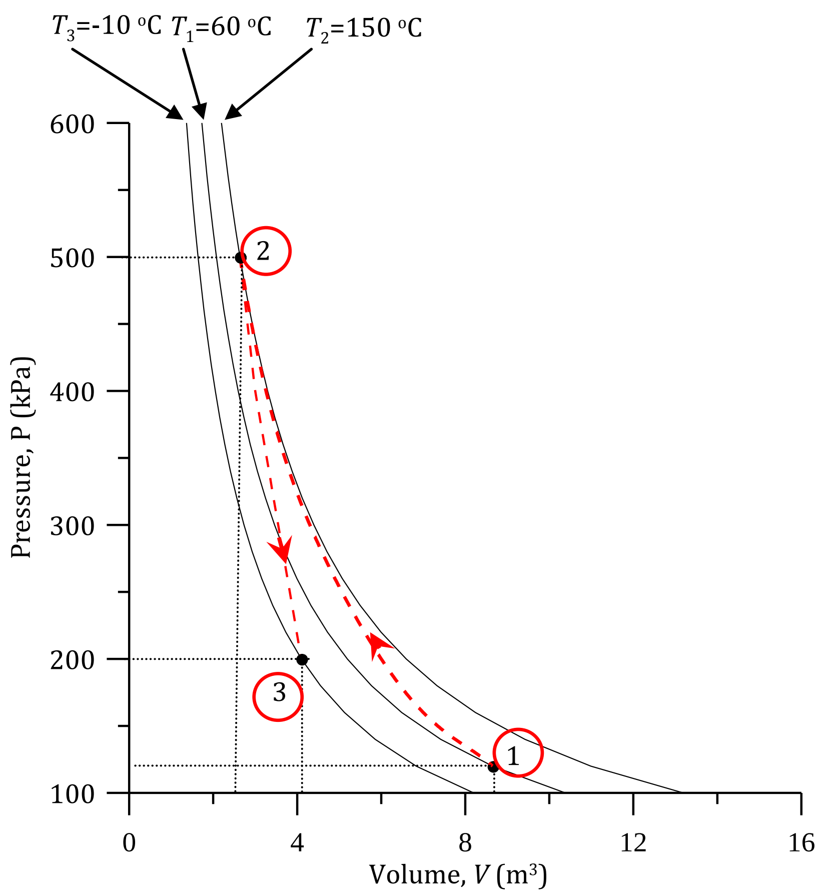
$$Q_{12}=+W_{1-2}+mC_v(T_2-T_1)=-1403.08+1.5\times3.1156\times(150-60)=-9832.15\ \text{kJ}$$

d. Work done by the system in the second process, W_{23}

$$Q_{2-3}-W_{2-3}=mC_v(T_3-T_2)$$

$$W_{2-3}=mC_{v0}(T_2-T_3)=1.5\times3.1156\times(150+10)=747.744\ \text{kJ}$$

e.



Question #3 (6 Marks)

A balloon behaves such that the pressure inside is proportional to its diameter squared. It contains 2kg of R-134a 5°C, 60% quality. The balloon and refrigerant R-143a are now heated so that a final pressure of 600 kPa is reached. Find the amount of work done in the process and also amount of heat transfer

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Solution

- Relation between p-and V can be obtained as follow

$$P \propto D^2$$
$$V \propto D^3$$
$$V^{2/3} \propto D^2$$
$$P \propto V^{2/3}$$
$$PV^{-2/3} = C$$

- From Tables of saturated R-134a at 5°C

$$p_1 = 349.7 \text{ kPa}$$
$$v_f = 0.0007824 \text{ m}^3/\text{kg}$$
$$v_g = 0.05837 \text{ m}^3/\text{kg}$$

$$v_1 = v_f + x(v_g - v_f) = 0.0007824 + 0.6 \times (0.05837 - 0.0007824) = 0.03533 \text{ m}^3/\text{kg}$$

$$\mathrm{V}_1 = mv_2 = 2 \times 0.03533 = 0.07066 \, \text{m}^3$$

$$\mathrm{V}_2 = V_1 \left(\frac{p_1}{p_2} \right)^{-3/2} = 0.03533 \times \left(\frac{349.7}{600} \right)^{-3/2} = 0.0794 \, \text{m}^3$$

$$\mathrm{W}_{1\text{-}2} = \frac{p_1 V_1 - p_2 V_2}{n-1} = \frac{349.7 \times 0.07066 - 600 \times 0.0794}{(-2/3) - 1} = 13.758 \text{ kJ}$$

- From Tables of saturated R-134a at 5°C

$$u_f = 206.5 \, \text{kJ/kg}$$
$$u_{fg} = 174.6 \text{ kJ/kg}$$

$$u_1 = u_f + x u_{fg} = 206.5 + 0.6 \times 174.6 = 311.26 \, \text{kJ/kg}$$

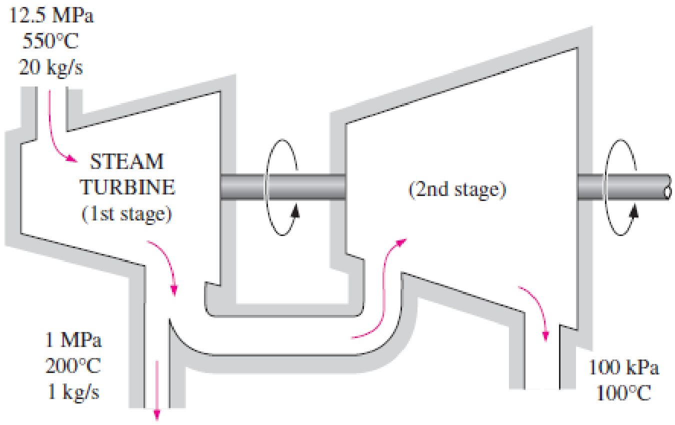
- From Tables of superheated R-134a at 600 kPa, and 0.0397 m^3/kg

$$u_2 = 414.8 \, \text{kJ/kg}$$

- Amount of heat transferred to the balloon is calculated as

$$\mathrm{Q}_{1\text{-}2} - \mathrm{W}_{1\text{-}2} = m(u_2 - u_1)$$
$$\mathrm{Q}_{1\text{-}2} = \mathrm{W}_{1\text{-}2} + m(u_2 - u_1) = 311.26 + 2 \times (414.8 - 311.26) = 518.34 \, \text{kJ}$$

Question #4 (10 Marks)



A portion of the steam passing through a steam turbine is sometimes removed for the purposes of feedwater heating as shown in figure . Consider an adiabatic steam turbine with 12.5 MPa and 550C steam entering at a rate of 20 kg/s. Steam is bled from this turbine at 1000 kPa and 200C with a mass flow rate of 1 kg/s. The remaining steam leaves the turbine at 100 kPa and 100C. Determine the power produced by this turbine.

Solution

Properties From the steam tables (Table A-6)

$$\left.\begin{array}{l} P_1 = 12.5\ \text{MPa} \\ T_1 = 550°\text{C} \end{array}\right\} h_1 = 3476.5\ \text{kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 1\ \text{MPa} \\ T_2 = 200°\text{C} \end{array}\right\} h_2 = 2828.3\ \text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 100\ \text{kPa} \\ T_3 = 100°\text{C} \end{array}\right\} h_3 = 2675.8\ \text{kJ/kg}$$

Analysis The mass flow rate through the second stage is

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 20 - 1 = 19\ \text{kg/s}$$

We take the entire turbine, including the connection part between the two stages, as the system, which is a control volume since mass crosses the boundary. Noting that one fluid stream enters the turbine and two fluid streams leave, the energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}_{\text{out}}$$

$$\dot{W}_{\text{out}} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Substituting, the power output of the turbine is

$$\dot{W}_{\text{out}} = (20\ \text{kg/s})(3476.5\ \text{kJ/kg}) - (1\ \text{kg/s})(2828.3\ \text{kJ/kg}) - (19\ \text{kg/s})(2675.8\ \text{kJ/kg})$$
$$= 15{,}860\ \text{kW}$$

Question #5 (22 Marks)

Two springs with different spring constants (K, 2K) are installed in a piton/cylinder arrangement with outside air at 100 kPa. The cylinder (shown in Figure 3) contains 1 kg of water initially at 110 ^oC and a quality of 15% (state 1). Heat is added to the cylinder until the pressure and temperature inside the cylinder are 1 MPa and 1300 C (state 4), respectively. If the piston comes in contact with the first spring with a constant of *K* when the volume of the cylinder equals =0.25 m^3 (state 2) and with the second spring of a constant of 2K when the volume of the cylinder is doubled (state 3). Calculate

a. Mass of piston if its cross sectional area is 500 cm^2. **(3 Marks)**
b. Springs constant. **(7 Marks)**
c. Pressure at which piston comes in contact with the second spring, P_3. **(1 Marks)**
d. Work done by water in each process and net work. **(4 Marks)**
e. Heat transfer to the cylinder. **(3 Marks)**
f. Draw a *P-V* diagram showing the state points and process path(s). label the values of P and V for each state point and clarify label the constant temperature lines that passes through the state points. **(4 Marks)**

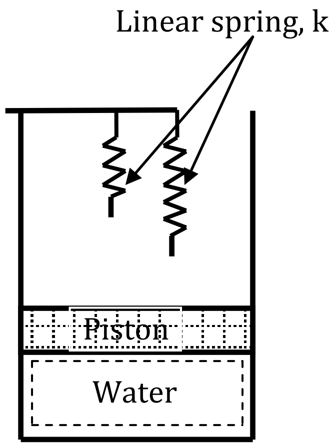
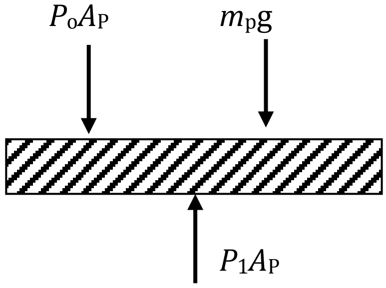


Figure 3 Sketch of problem in question #5

Solution

Summary of states

State 1 (T_1=110 ^oC, x_1=0.15, m_1=1 kg)
State 2 (P_2=P_1, V_2=0.25 m^3, m_2=1 kg)
State 3 (V_3=0.5 m^3, m_3=1 kg)
State 4 (P_4 = 1MPa,T_4=1300 ^oC, m_4=1 kg)



a. From saturated steam tables (Table A.4) at T_1=110 ^oC

$$P_1 = 143.38\,\text{kPa}$$

From foce balance of the piston

$$P_1 = P_o + \frac{m_p g}{A_p} \rightarrow 143.38 = 100 + 0.15 \times \frac{m_p \times 9.81}{0.05 \times 10^3}$$

$$m_p = \frac{(P_1 - P_o)A_p}{g} = \frac{(143.38 - 100) \times 10^3 \times 0.0500}{9.81} = 221.101 \text{ kg}$$

b. Calculating the spring constant
From saturated steam tables (Table A.4) at T_1=110 ^oC

$$v_{f1} = 0.001052 \text{ m}^3/\text{kg}$$
$$v_{g1} = 1.2094 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{V_2}{m_2} = \frac{0.25}{1} = 0.25 \text{ m}^3/\text{kg}$$

Since P_2=P_1, $v_{g2} = v_{g1} = 1.2094$, and $v_2 < v_{g2}$, state 2 is saturated mixture and P_2=P_1=143.38 kPa

From superheated team tables (Table A-6) at 1 MPa and 1300 °C

$$v_4 = 0.72610 \text{ m}^3/\text{kg}$$
$$V_4 = m_4 v_4 = 1 \times 0.72610 = 0.72610 \text{ m}^3$$

From spring equation

$$P_3 = P_2 + \frac{K}{A_p^2}(V_3 - V_2) \qquad (1)$$

$$P_4 = P_3 + \frac{3K}{A_p^2}(V_4 - V_3) \qquad (2)$$

Using Eq. (1) in Eq. (2)

$$P_4 = P_2 + \frac{K}{A_p^2}(V_3 - V_2) + \frac{3K}{A_p^2}(V_4 - V_3) = P_2 + \frac{K}{A_p^2}(3V_4 - V_2 - 2V_3)$$

$$K = \frac{A_p^2(P_4 - P_2)}{(3V_4 - V_2 - 2V_3)} = \frac{(0.0500)^2 \times (1000 - 143.38)}{(3 \times 0.7261 - 0.25 - 2 \times 0.50)} = 2.317 \text{ kN/m}$$

c. Calculating the Pressure P_3

Substitute value of K in Eq. (1)

$$P_3 = P_2 + \frac{K}{A_p^2}(V_3 - V_2) = 143.38 + \frac{2.317}{(0.0500)^2} \times (0.50 - 0.25) = 375.08 \text{ kPa}$$

d. Calculating the work done by water for each process

Process 1→2 is an isobaric expansion process

Process 2→3 is a spring expansion process with spring constant K

Process 3→4 is a spring expansion process with spring constant 2K

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.001052 + 0.15 \times (1.2095 - 0.001052) = 0.1823 \text{ m}^3/\text{kg}$$
$$V_1 = m_1 v_1 = 1 \times 0.1823 = 0.1823 \text{ m}^3$$
$$W_{12} = P_1(V_2 - V_1) = 143.38 \times (0.25 - 0.1823) = 9.707 \text{ kJ}$$
$$W_{23} = \frac{(P_2 + P_3)}{2}(V_3 - V_2) = \frac{(143.38 + 373.08)}{2} \times (0.50 - 0.25) = 64.558 \text{ kJ}$$
$$W_{34} = \frac{(P_3 + P_4)}{2}(V_4 - V_3) = \frac{(373.08 + 1000)}{2} \times (0.72610 - 0.50) = 155.227 \text{ kJ}$$
$$W_{net} = \sum W = W_{12} + W_{23} + W_{34} = 9.707 + 64.558 + 155.227 = 229.492 \text{ kJ}$$

e. Calculating the heat transfer to the cylinder

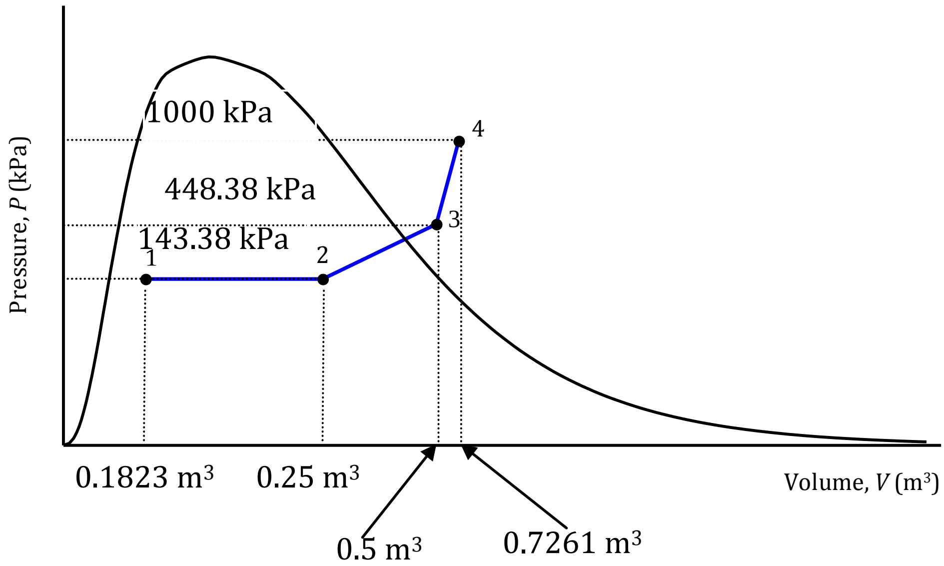
From saturated steam tables (Table A.4) at T_1=110 °C

$$u_{f1} = 461.27 \text{ kJ/kg}$$
$$u_{fg1} = 2056.4 \text{ kJ/kg}$$
$$u_1 = u_{f1} + x_1 u_{fg1} = 461.27 + 0.15 \times 2056.4 = 769.73 \text{ kJ/kg}$$

From superheated team tables (Table A-6) at 1 MPa and 1300 °C

$$u_4 = 4685.8 \text{ kJ/kg}$$
$$Q - W = m(u_4 - u_1)$$
$$Q - 229.492 = 1 \times (4685.8 - 769.73)$$
$$Q = 4145.56 \text{ kJ}$$

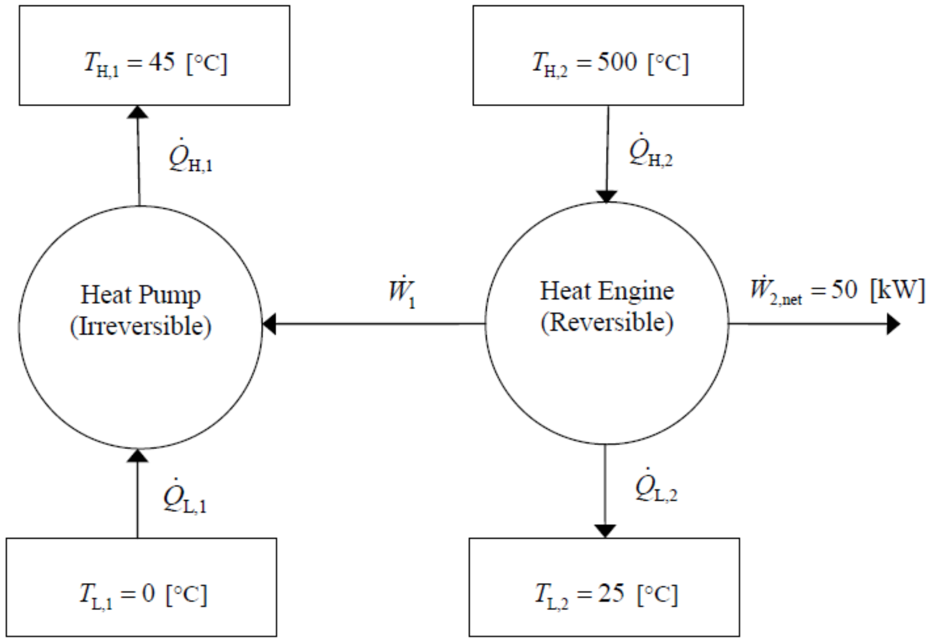
f. Drawing the *P-V* diagram showing the state points and process path



Question #6 (11 Marks)

A reversible (Carnot) heat engine operates between two thermal reservoirs at 500 °C and 25°C. The heat engine is used to drive an irreversible heat pump that removes heat from a low temperature reservoir at $T_{L,1}$=0°C and rejects heat to a high temperature reservoir at $T_{H,1}$ = 45°C. It is desired to provide input power, $\dot{W}_1$, to the heat pump such that the coefficient of performance of the irreversible heat pump is 60% of that for a reversible heat pump; *i.e.*, $\text{COP}_{\text{HP}} = 0.6(\text{COP}_{\text{HP,rev}})$.
The total power developed by the heat engine is divided into two parts: an amount $\dot{W}_1$ that is used to drive the heat pump, and $\dot{W}_{2,\text{net}}$ as the remaining power, where $\dot{W}_{2,\text{net}}$ =50 kW. Heat is transferred to the heat engine from a high temperature reservoir at the rate of $\dot{Q}_{H,2}$, and heat is "pumped" by the heat pump to a high temperature reservoir at the rate of $\dot{Q}_{H,1}$. It is known that the sum of these two rates of heat transfer is as follows: $\dot{Q}_{H,1} + \dot{Q}_{H,2} = 500$ kW

(a) Determine the thermal efficiency, η_{th}, for the Heat Engine and then coefficient of performance COP_{HP}, for the Heat Pump. **(3 Marks)**
(b) Determine the power input required for the Heat Pump, $\dot{W}_1$, in kW. **(4 Marks)**
(c) Determine the rates of heat transfer $\dot{Q}_{H,1}$ and $\dot{Q}_{L,1}$ for the Heat Pump, and determine the rates of heat transfer $\dot{Q}_{H,2}$ and $\dot{Q}_{L,2}$ for the Heat Engine. **(4 Marks)**



Solution

(a) thermal efficiency, η_{th}, for the Heat Engine and then coefficient of performance $\mathrm{COP_{HP}}$, for the Heat Pump

$$\eta_{\mathrm{th,rev}} = 1 - \frac{T_{L,2}}{T_{H,2}} = 1 - \frac{25+273}{500+273} = 0.614$$

$$\eta_{\mathrm{th}} = \eta_{\mathrm{th,rev}} 0.614$$

$$\mathrm{COP_{HP,rev}} = \frac{T_{\mathrm{H,1}}}{T_{\mathrm{H,1}} - T_{\mathrm{L,1}}} = \frac{45+273}{45-0} = 7.067$$

$$\mathrm{COP_{HP}} = 0.6 \times \mathrm{COP_{HP,rev}} = 0.6 \times 7.067 = 4.240$$

(b) Determine the power input required for the Heat Pump, $\dot{W}_1$, in kW. **(4 Marks)**

$$\eta_{\mathrm{th}} = \frac{\dot{W}_1 + \dot{W}_2}{\dot{Q}_{\mathrm{H,2}}} = 0.614$$

$$\dot{Q}_{\mathrm{H,2}} = \frac{\dot{W}_1 + \dot{W}_2}{0.614} = 1.627\left(\dot{W}_1 + \dot{W}_2\right) \qquad (1)$$

$$\mathrm{COP_{HP}} = \frac{\dot{Q}_{\mathrm{H,1}}}{\dot{W}_{\mathrm{L,1}}} = 4.240$$

$$\dot{Q}_{\mathrm{H,1}} == 4.240\dot{W}_{\mathrm{L,1}} \qquad (2)$$

By adding Eq. (1) to Eq. (2)

$$\dot{Q}_{\mathrm{H,1}} + \dot{Q}_{\mathrm{H,2}} = 4.240\dot{W}_{\mathrm{L,1}} + 1.627\left(\dot{W}_1 + \dot{W}_2\right)$$

$$\dot{Q}_{\mathrm{H,1}} + \dot{Q}_{\mathrm{H,2}} = (4.240 + 1.627)\dot{W}_{\mathrm{L,1}} + 1.627\dot{W}_2$$

$$500 = (4.240 + 1.627)\dot{W}_1 + 1.627 \times 150$$

$$\dot{W}_1 = \frac{500 - 1.627 \times 150}{(4.240 + 1.627)} = 43.625\,\text{kW}$$

(c) Determine the rates of heat transfer $\dot{Q}_{H,1}$ and $\dot{Q}_{L,1}$ for the Heat Pump, and determine the rates of heat transfer $\dot{Q}_{H,2}$ and $\dot{Q}_{L,2}$ for the Heat Engine.

From Eq. (2)

$$\dot{Q}_{H,1} = 4.240\dot{W}_{L,1} = 4.240 \times 43.625 = 184.971 \text{ kW}$$

$$\dot{Q}_{L,1} = \dot{Q}_{H,1} - \dot{W}_1 = 184.971 - 43.625 = 141.346 \text{ kW}$$

$$\dot{Q}_{H,2} = 500 - \dot{Q}_{H,1} == 500 - 184.971 = 315.029 \text{ kW}$$

$$\dot{Q}_{L,2} = \dot{Q}_{H,2} - \left(\dot{W}_{L,1} + \dot{W}_{L,2}\right) = 315.029 - (43.625 + 50) = 221.404 \text{ kW}$$