Kafrelsheikh University
Faculty of Engineering
Physics & Engineering Mathematics Dept.
Year: Prep.

Subject: Engineering Mathematics (1)

Date: 1/1/2019
Time allowed: 3 hours
Full Marks: 110
Final-Term Exam: 4 pages

[Q1] Choose the suitable answer: [15 marks][ILOs: a1,b1]

1) If $\frac{x^3-1}{(x+1)(x+2)}=(x+\alpha)+\frac{\beta}{x+1}+\frac{\gamma}{x+2}$, then the value of α,β,γ .

a) (-3,-2,9) b) (3,2,-9) c) (1,2,-1) d) (3,-2,-9)

2) If $Z=\left|\frac{2i}{3+i}\right|$, then $|Z|$ equal to

a) $\frac{2\sqrt{10}}{5}$ b) $\frac{\sqrt{10}}{3}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{10}}{5}$

3) For the system: $\begin{matrix}3i_1-2i_2+4i_3=2\\ i_1+3i_2-6i_3=8\\ 2i_1-i_2-2i_3=0\end{matrix}$,the value of i_3 is

a) $\frac{-1}{2}$ b) 2 c) $\sqrt{2}$ d) $\frac{1}{2}$

4) If $2\begin{bmatrix}a & -b\\ 3c & -2d\end{bmatrix}=\begin{bmatrix}3a & 1\\ c & 3+2d\end{bmatrix}-3\begin{bmatrix}-1 & a-b\\ \alpha+b & 2c\end{bmatrix}$, then a,b,c,d are

a) (-3,-2,3,5/2) b) (3,2,-3,-5/2) c) (-3,2,3,2/5) d) (2,3,-3,5)

[Q2] Put (√) or (X) in the following statements: [20 marks][ILOs: a1,b1]

(1) $\{-1,0,1,2\}\Leftrightarrow\{x:x\in Z,-1\le x\prec 3\}$ ( )

(2) $A\cap(A^C\cup B)=A\cap B$ ( )

(3)The equality of sets is not equivalence relation ( )

(4) $\{8\}\in\{2,4,6,.....\}$ ( )

(5) If $X=\{2,4,6,7\}, R=\{(x,y):x,y\in X \text{ and } x+y \text{ is an even integer}\}$, then R is equivalence relation ( )

(6)The matrix $A=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$ has an inverse ( )

(7) $\frac{(1+i)(2+i)}{(3+i)}=\frac{3}{5}-\frac{4}{5}i$ ( )

(8) $(\cos 15+i\ \sin 15)^6=-i^3$ ( )

(9) $A\leftrightarrow B\equiv(A\rightarrow B)\vee(B\rightarrow A)$ ( )

(10) $A\vee B\equiv\sim(\sim A\wedge\sim B)$ ( )

[Q3]Complete the following sentences: [20 marks][ILOs: a1,b1]

(1) $P=\{A_1,A_2,...,A_n\}$ *is a partation on A if* ..... , ... , ...

(2) $\{a,b\}\Delta\{b,c\}=$.....

(3) A is symmetric matrix if .....

(4) $(A\setminus B)\cap B=$.....

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(5) A *is orthogonal matrix if* .....

(6) $\left|e^{i\theta}\right| =$.....

(7) $\left(-2+\sqrt{-16}\right)\left(4-\sqrt{-9}\right) =$.....

(8) $\mathrm{Im}\left[\ \overline{(2+i)(3+i)}\ \right] =$.....

(9) $\left|2+\frac{3}{2}i\right| =$.....

(10) $A \vee (B \wedge C) =$.....

Q[4] [25 marks] [ILOs: a1,c1]

(*a*) Find y' *if* $x^{y} = 1+\ln(\sinh^{-1}x)^{x}$

(*b*) If $y = \frac{1}{x}+\sin(\frac{1}{x})+\cos(\frac{1}{x})$, show that $x^{5}y'' + 2x^{4}y' + xy = 1$

(*c*) Deduce Maclaurin expansion *of* $f(x) = \ln(1+x)$ then find Maclaurin expansion of $g(x) = e^{x}\ln(1+x)$ up to third degree

(d) Deduce $\frac{dy}{dx}$ *if* $y = \cosh^{-1}x$

(*e*) Deduce $y^{(n)}$ *of* $y = \cos(ax+b)$, then find $y^{(n)}$ if $y = \cos^{4}x$

Q[5] [30 marks] [ILOs: b7,c1]

Choose the correct answer from the following:

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(1) $\lim_{x\to\infty}(x-\ln x) =$ (*a*) 0 (*b*) 1 (*c*) *not exist* (*d*) 2

(2) $\lim_{x\to-\infty} x^{4}e^{x} =$ (*a*) 0 (*b*) 1 (*c*) ∞ (*d*) 4

(3) If $f(x) = \frac{1}{x^{2}+1}$ **and** $g(x) = \sqrt{x}$ **then the derivative of** $f(g(x))$ **is**

(*a*) $\frac{-\sqrt{x}}{(x^{2}+1)^{2}}$ (*b*) $\frac{1}{(x+1)^{2}}$ (*c*) $\frac{-2x}{(x^{2}+1)^{2}}$ (*d*) $-(x+1)^{-2}$

(4) Find y' **If** $3x^{2}-2xy+5y^{2} = 1$

(*a*) $\frac{3x+y}{x-5}$ (*b*) $\frac{y-3x}{5y-x}$ (*c*) $3x+5y$ (*d*) $\frac{3x+4y}{x}$

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(5) *Let* $y(x)=\frac{1}{9}(2x+1)^3$, Find an equation of the tangent line to the graph of y(x) at x=1

$(a)\ y=6 \qquad (b)\ y=6x-3 \qquad (c)\ y=3x \qquad (d)\ y=6x-6$

(6) $\lim_{x\to 0} x^2\cos(\frac{1}{x})=$ $(a)0 \qquad (b)\infty \qquad (c)$ *not exist* $\qquad (d)\ 2$

(7) *If the line* $3x-4y=0$ is tangent in the first quadrant to the curve $y=x^3+k$ *then* $K=$

$(a)0 \qquad (b)\frac{1}{4} \qquad (c)\frac{1}{2} \qquad (d)\frac{1}{3}$

(8) $\frac{d}{dx}\sin^2(\sin^{-1}\sqrt{x})=$ $(a)x^2 \qquad (b)2\sin(\sin^{-1}\sqrt{x}) \qquad (c)2x \qquad (d)1$

(9) $\lim_{x\to 0}\frac{\sec x-\cos x}{x^2}=$ $(a)\frac{1}{2} \qquad (b)1 \qquad (c)\frac{3}{2} \qquad (d)\frac{-1}{2}$

(10) If $\begin{cases} f(x)=\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & for\ x\neq 2 \\ f(2)=k \end{cases}$, is continuous at $x=2$, then $k=$

$(a)0 \qquad (b)\frac{1}{6} \qquad (c)\frac{1}{3} \qquad (d)1$

(11) If $y=\ln(\sec x)$, then $e^y-e^{-y}=$

$(a)\sin x\tan x \qquad (b)\frac{\sin x}{\cos x} \qquad (c)\cos x\sin^2 x \qquad (d)$ Non of these

(12) If $f(x)=\tanh^{-1}x$, Find D_f & R_f

$(a)\begin{cases} D_f=R \\ R_f=R \end{cases} \quad (b)\begin{cases} D_f=R \\ R_f=]-1,1[\end{cases} \quad (c)$ Non of these $\quad (d)\begin{cases} D_f=]-1,1[\\ R_f=R \end{cases}$

(13) Find D_f & R_f of the inverse of $f(x)=e^x$

$(a)\begin{cases} D_f=R \\ R_f=R \end{cases} \quad (a)\begin{cases} D_f=]0,\infty[\\ R_f=R \end{cases} \quad (a)\begin{cases} D_f=R \\ R_f=]0,\infty[\end{cases} \quad (a)$ Non of these

(14) *If* $y=a^{f(x)}$ *then* $y'=$

(a) *Non of these* $\quad (b)a^{f(x)}.\ln a f'(x) \quad (c)a^{f(x)}.\ln a \quad (d)a^{f(x)}$

(15) *If* $y=\ln(x)^x$, *then* $y'=$

$(a)\frac{1}{x}+\ln x \qquad (b)\frac{1}{x^x} \qquad (c)1+\ln x \qquad (d)\ln(x)^x$

(16) $\frac{d}{dx}\cot(\operatorname{cosec}^{-1}x)=$ $(a)\ \frac{x}{2\sqrt{x^2-1}} \quad (b)\ \frac{2x}{\sqrt{x^2-1}} \quad (c)\frac{x}{\sqrt{x^2-1}} \quad (d)\frac{x}{\sqrt{x^2+1}}$

(17) *If* $\tan(xy)=x$ *then* $\frac{dy}{dx}=$

$(a)\frac{\cos^2(xy)-y}{x} \quad (b)\frac{\sec^2(xy)-y}{x} \quad (c)\frac{\cos^2(xy)}{x} \quad (d)\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$

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(18) *If* $y=\tan u$ *and* $u=v-\frac{1}{v}$ *and* $v=\ln x$, *what is the value of* $\frac{dy}{dx}$ *at* $x=e$

$(a)\ x \qquad (b)\ \frac{1}{e} \qquad (c)\ \sec^2 e \qquad (d)\ \frac{2}{e}$

(19) *The domain and range of* $f(x)=\sec x$

$(a)\ \begin{cases} D_f = R-\{x=(n+\frac{1}{2})\pi \\ R_f = R-]-1,1[\end{cases} \quad (b)\ \begin{cases} D_f = R \\ R_f = R \end{cases} \quad (c)\ \begin{cases} D_f = R-]-1,1[\\ R_f = R-\{x=(n+\frac{1}{2})\pi \end{cases} \quad (d)$ *Non of these*

where $n=0,\pm1,\pm2,.....$

(20) *If* $y=\sqrt{x\sqrt{x}}$, *then* $y'=$

$(a)\ 2\sqrt{x\sqrt{x}} \qquad (b)\ \frac{1}{2\sqrt{x\sqrt{x}}} \qquad (c)\ \frac{3}{4}x^{(\frac{-1}{4})} \qquad (d)\ \frac{3}{4\sqrt{x\sqrt{x}}}$

(21) *If* $x=v^2-1$ *and* $y=2e^v$, *then* $\frac{dy}{dx}=$

$(a)\ \frac{4e^v}{2v-1} \qquad (b)\ \frac{2e^v}{v} \qquad (c)\ \frac{e^v}{v} \qquad (d)\ e^v$

(22) *If* $\sin x-\cos y-2=0$, *then* $y'=$

$(a)\ -\operatorname{cosec} y\cos x \qquad (b)\ -\cot x \qquad (c)\ -\cot y \qquad (d)\ \operatorname{cosec} y\cos x$

(23) *If* $y=e^{2\ln\sqrt{\cos x}}$, *then* $y'=$

$(a)\ \frac{\sin x}{2\sqrt{\cos x}} \qquad (b)\ \sin x \qquad (c)\ \cos x \qquad (d)\ -\sin x$

(24) Let $f(x)=(2x+1)^3$ and $g(x)$ be the inverse function of $f(x)$, what is the value of $g'(1)$? $(a)\ \frac{1}{27} \qquad (b)\ \frac{-2}{27} \qquad (c)\ \frac{1}{6} \qquad (d)\ \frac{1}{54}$

(25) *If* $y=\cos^2(10^x)$, *then* $y'=$

$(a)\ -2\cos(10^x)\sin(10^x)10^x\ln 10 \qquad (b)\ -\cos(10^x)\sin(10^x) \qquad (c)$ Non of these $\qquad (d)\ \sin^2(10^x)10^x\ln 10$

(26) *If* $y=\sin^2 x$, *then* $y^{(n)}=$

$(a)\ \frac{1}{2}2^n\sin(2x+\frac{n\pi}{2}) \qquad (b)\ \frac{-1}{2}2^n\cos(2x+\frac{n\pi}{2}) \qquad (c)\ \frac{1}{2}2^n\cos(2x+\frac{n\pi}{2}) \qquad (d)$ Non of these

(27) *If* $y=(\cos x)^x$, *then* $y'=$

$(a)\ -x(\cos x)^{x-1}\sin x \qquad (b)(\cos x)^x[-x\tan x+\ln\cos x] \qquad (c)\ (\sin x)^x \qquad (d)$ Non of these

(28) *The points of discontinuity of* $f(x)=\cot x$ *are* :

(a) Non of these $\qquad (b)\{\pm1\} \qquad (c)\ \{0\} \qquad (d)\{n\pi\},\ n=0,\pm1,\pm2,...$

(29) *If* $f(x)=\tan x$ *is periodic function, then* $p=$

$(a)\ 2\pi \qquad (b)\ \pi \qquad (c)$ *Non of these* $\qquad (d)\ \pi/2$

(30) *If* $f(x)=\sec x$ *is periodic function, then* $p=$

$(a)\ 2\pi \qquad (b)\ \pi \qquad (c)\ \pi/2 \qquad (d)$ *Non of these*

انتهت الأسئلة

With my best wishes ➔ *Prof. Dr. Arafa Nasef & Dr. Manal El-said Ali*