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المادة : An introduction to linear algebra
الزمن : ١٠-١٢
الدرجة : ٦٠ درجة

Answer the following questions:

Part (1): Choose the correct answer between a, b, c and d of the following questions:

1- A matrix X for solving the equation $3X + A = B$, where $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$ is.....

A) $\begin{bmatrix} \frac{-4}{3} & \frac{2}{3} \\ 2 & \frac{-2}{3} \end{bmatrix}$ B) $\begin{bmatrix} \frac{-4}{3} & 2 \\ \frac{2}{3} & \frac{-2}{3} \end{bmatrix}$ C) $\begin{bmatrix} \frac{4}{3} & 2 \\ \frac{2}{3} & \frac{-2}{3} \end{bmatrix}$ D) $\begin{bmatrix} \frac{4}{3} & 2 \\ -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$

2-Which of the vectors below are scalar multiples of $z = (\frac{1}{2}, \frac{-2}{3}, \frac{3}{4})$

A) $u = (-1, \frac{4}{3}, \frac{-3}{2})$ B) $v = (6, -4, 9)$ C) $w = (12, 0, 9)$ D) Non of these

3- The value of the determinant $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$ equal to.....

A) $x^2(x+3)$ B) x^3 C) $3x^3$ D) 0

4-The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ are .....

A) $-1, 5$ B) $1, 5$ C) $5, -1$ D) $5, 1$

5- The inverse matrix of A, where $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ is.....

A) $\begin{bmatrix} 1 & 0 \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$ C) $\begin{bmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ D) $\begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$

6-The set of vectors $S = \{(1,2), (2,4)\}$ is .....in R^2

A) Linearly dependent B) Linearly independent

C) Neither linearly dependent nor linearly independent D) Non of these

7-If A, B are matrices, where $A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$, then $(AB)^T$ equal to .....

A) $\begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 6 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 6 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & -2 \end{bmatrix}$

8-A system of linear equations $x-2y+z=9,\ y++3z=5,\ z=2$ has a solution $(x,y,z)=$ .....

A) $(1,-1,2)$ B) $(-1,1,2)$ C) $(1,1,2)$ D) $(1,1,-2)$

9- If A,B are matrices, where $A=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then $A+B$ is called.....

A) singular B) nonsingular C) neither singular nor nonsingular D) Non of these

10- If $A=\begin{bmatrix} \beta & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ such that $A^2=B$, then β equal to .....

A) 1 B) -1 C) 4 D) Non of these

11- The eigenvector corresponding the eigenvalue $\lambda=5$ of the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ is .....

A) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ D) Non of these

12-Which of the vectors below are scalar multiples of $z=(3,2,-5)$

A) $u=(2,\frac{4}{3},\frac{-10}{3})$ B) $v=(6,4,10)$ C) $w=(-6,-4,10)$ D) Non of these

13- If A and B are matrices (with sizes such that the given matrix operations are defined), then $\left(A^T B^T\right)^T$ = ...

A) $B^T A^T$ B) $A\ B$ C) $(BA)^T$ D) BA

14- The set of vectors $M=\{(1,2,3),(0,1,2),(-2,0,1)\}$ is .....in R^3

A) Linearly dependent B) Linearly independent

C) Neither linearly dependent nor linearly independent D) Non of these

15-A value of c for which the system of equations:

$x+y=1,(c-2)x+(c+4)y=6,\ (c+2)^2x+(c+4)^2y=36$ is .....

A) 1 B) 2 C) -2 D) Non of these

16-The eigenvalues of the matrix $A=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ are .....

A) $-1,1,3$ B) $1,1,3$ C) $1,1,-3$ D) Non of these

17- The value of the matrix $A=\begin{bmatrix}2 & 3 & -1\\ 0 & -1 & 2\\ 0 & 0 & 3\end{bmatrix}$ equal to.....

A) -6 B) 6 C) $\frac{1}{6}$ D) Non of these

18-If $\begin{vmatrix}a+x & a & x\\ a-x & a & x\\ a-x & a & -x\end{vmatrix}=0$, then x equal to.....

A) 0 B) $2a$ C) a D) 3

19- If A and B are matrices (with sizes such that the given matrix operations are defined), then $(A^T+B)^T$ = ...

A) $A+B^T$ B) $B+A^T$ C) A^T+B^T D) Non of these

20- The eigenvector corresponding the eigenvalue $\lambda=1$ of the matrix $A=\begin{bmatrix}1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ is .....

A) $\begin{bmatrix}-2\\ -1\\ 1\end{bmatrix}$ B) $\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$ C) $\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$ D) Non of these

21- If $\begin{vmatrix}a+x & a-x & a-x\\ a-x & a+x & a-x\\ a-x & a-x & a+x\end{vmatrix}=0$, then x equal to.....

A) 0 B) $2a$ C) a D) $3a$

22- If the matrix $A=\begin{bmatrix}1 & 3\\ 0 & -2\\ -2 & -1\end{bmatrix}$, then AA^T equal to .....

A) $\begin{bmatrix}10 & -6 & -5\\ -6 & 4 & 2\\ -5 & 2 & 5\end{bmatrix}$ B) $\begin{bmatrix}10 & -6 & -5\\ 6 & 4 & 2\\ 5 & 2 & 5\end{bmatrix}$ C) $\begin{bmatrix}10 & 6 & -5\\ -6 & 4 & 2\\ -5 & 2 & 5\end{bmatrix}$ D) $\begin{bmatrix}10 & -6 & 5\\ -6 & 4 & 2\\ -5 & 2 & 5\end{bmatrix}$

23- The eigenvector corresponding the eigenvalue $\lambda=-1$ of the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ is .....

A) $\begin{bmatrix}2\\ 1\end{bmatrix}$ B) $\begin{bmatrix}-2\\ 1\end{bmatrix}$ C) $\begin{bmatrix}2\\ -1\end{bmatrix}$ D) Non of these

24- The eigenvector corresponding the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is .....

A) $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ D) Non of these

25- The eigenvector corresponding the eigenvalue $\lambda = 3$ of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is .....

A) $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ D) Non of these

Part (2) : Put the sign (T) or (F) beside the following statements :

1- The product of the lower triangular matrices is not lower triangular matrix. (T) (F)

2- The inverse of the matrix exists, it is not unique. (T) (F)

3- The product of matrices satisfies the commutative law. (T) (F)

4- A square matrix of size $n \times n$ can be written as the product of a lower triangular matrix and an upper triangular matrix . (T) (F)

5- The intersection of two subspaces of vector space is not subspace. (T) (F)

6- The sum of matrices satisfies the commutative law, if the matrices have the same size. (T) (F)

7- The homogeneous system of equations has only trivial solution, if the matrix coefficient has invertible . (T) (F)

8- For a vector space R^n , the additive identity is unique. (T) (F)

9- The set of all integers with standard operations forms a vector space. (T) (F)

10- The union of two subspaces of vector space is a subspace. (T) (F)

11- The non-homogeneous system of equations has an infinite solution, if the matrix coefficient has invertible. (T) (F)

12- The set of all linear combinations of a vector space forms a subspace. (T) (F)

13- The product of the upper triangular matrices is upper triangular matrix. (T) (F)

14- The sum of two square singular matrices is a non singular matrix. (T) (F)

15- A square matrix of size $n \times n$ cannot be written as the product of elementary matrices, if it invertible. (T) (F)

End the questions

Best regards

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