(4) (a) If $L$ is lie algebra with ideals $L = I_0 \supseteq I_1 \supseteq I_2 \cdots \supseteq I_{m-1} \supseteq I_m = 0$ such

That $I_{k-1}/I_k$ is abelian for $1 \leq k \leq m$ proof that $L$ is solvable.

(b) If $L$ is lie algebra has an ideal $I$ such that $I$ and $L/I$ is solvable proof that $L$ is solvable.

(c) Let $L$ be a lie sub algebra of $gL(V)$ If $x \in L$ and the linear map $x : V \to V$ is nilpotent Proof that $adx : L \to L$ is nilpotent

(d) Proof that any nilpotent Lie algebra is solvable.

(e) Let $L$ be a lie sub algebra of $gL(V)$ where $V$ is non-zero such that every element of $L$ is nilpotent linear transformation proof that there is some non-zero $v \in V$ such that $xv = 0..x \in L$

Kafr El-Sheikh University
Faculty of Science
Department of Math.

Forth Mathematical year
Lie Algebra (437)ر
Time: 2 hours

Date : 1/1/2018

**Answer the following questions:**

**(1) (a)Let** $\Im(R) = \{f \mid f:[a,b] \to R\}$ **Proof that (** $\Im(R)$ **,+,.) is vector space where + , .**

**defined as** $\forall f, g \in \Im(R)....,\ \forall s \in [a,b]..(f+g)(s) = f(s)+g(s).....(fg)(s) = f(s)g(s)$

**(b) Proof the Cayley theorem and explain it by example**

**(2) (a) Let** $L(V,V)$ **be the set of all linear transformations from the Vector space**

$V$ **over the field F into itself, Proof that** $(L(V,V),+,\circ)$ **is Algebra where the**

**operation +,**$\circ$ **defined by**

$s \in V...,\ \forall f, g \in L(V,V)...(f+g)(s) = f(s)+g(s).........(f \circ g)(s) = f(g(s)$

**(b) Let** $L(V,V)$ **be the vector .Proof that** $L(V,V)$ **together with the map**

$f : L(V,V) \times L(V,V) \to L(V,V)$ **Defined by** $f(f,g) = [f,g] = f \circ g - g \circ f$

**Is lie algebra.**

**(3) (a) Let** $sl(n,f)$ **be the set of all** $n \times n$ **matrices of trace zero Proof**

**That** $sl(n,f)$ **is lie sub algebra of** $gl(n,f)$ **.**

**(b) Let L be lie algebra for** $x \in L$ **Proof that** $(adx): L \to L$ **which defined**

**By** $(adx)(y) = [x,y]......\forall y \in L$ **is morphism.**

**(c) Let** $I, J$ **be an ideals of a lie algebra L proof that** $[I,J]$ **is an ideal of L.**

انظر بقية الاسئلة في الخلف

## The third question

**(a) Use the variation of parameters technique to find a particular**

**Solution,** $X_p$ **of** $X' = AX + b(t)$ **for**

$$\mathbf{A}=\begin{bmatrix} -1 & -2 & 2 \\ 2 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} \qquad b=\begin{bmatrix} -e^{3t} \\ 4e^{3t} \\ 3e^{3t} \end{bmatrix}$$

**(b) Let A be non-defective** $n \times n$ **matrix with linearly indepent**

**eigenvectors** $v_1, v_2, v_3 \ldots, v_n$ **and corresponding eigenvalues** $\lambda_1, \lambda_2, \lambda_3 \ldots, \lambda_n$

**then** $e^{At} = Se^{Dt}S^{-1}$ **where** $S = [v_1, v_2, v_3 \ldots, v_n]$ **and** $D = diag(\lambda_1, \lambda_2, \lambda_3 \ldots, \lambda_n)$

**(c) Use the above theorem to find** $e^{At}$

**where** $\mathbf{A}=\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

Kafr El-Sheikh University
Faculty of Science
Department of Math.

Forth Mathematical year
Theorem of differential equations
Time: 2 hours

**Solve the following questions:**

**The first question**

**(a) Proof that the change of variable** $x = e^Z$ **Transform the Cauchy-Euler equation** $x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + a_2 x^{n-2} y^{(n-2)} + .... + a_{n-1} x y^{(1)} + a_n y = 0......, x \succ 0$

**Into the constant coefficient equation**

$$[D(D-1)(D-2).......(D-n+1) + a_1 D(D-1)(D-2)...(D-n+2) + .... + a_{n-1} D + a_n]y = 0$$

**(b) Determine the general solution of the given differential Equation**

$$x^4 y^{(4)} + 6x^3 y^{(3)} + 9x^2 y^{(2)} + 3xy^{(1)} + y = 0$$

**(c) Use Variation of parameters technique to determine a particular solution for** $y''' - 12y'' + 48y' - 64y = 36e^{4x} linx$

**The second question**

**(a) Find the general solution of** $x^2 y'' - 5xy' + 9y = x^3 linx$

**(b) Determine the general solution of the system X'= AX for**

$$\mathbf{A} = \begin{bmatrix} 5 & -3 & -2 \\ 8 & -5 & -4 \\ -4 & 3 & 3 \end{bmatrix}$$

أنظر بقية الأسئلة بالخلف

| Kafrelsheikh University<br>Faculty of Science<br>Mathematics Department<br>Final Exam of First Term<br>2017-2018 | Numerical Analysis (2)<br>M436 | Level : $4^{th}$ year mathematics<br>Date : 28 \12 \2017<br>Time Allowed : 2H<br>Total Marks :100 (70 Written, 10 Oral,20 Exe...<br>Exam in one Page |
|---|---|---|

## Answer the following questions

**Question1** (10 Ma...

Prove that the error in second order Runge- Kutta method is of $O(h^3)$ ?

**Question2** (25 M...

Find the solution of the following linear algebraic system by numerical method (n=2) :-

$$-2x_2 + 3x_3 - x_4 = 5$$
$$x_1 - x_4 = 2$$
$$x_1 - 2x_2 + 2x_4 = 4$$
$$x_1 + x_3 - 2x_4 = 3$$

$$x^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

And find the error?

**Question3** (20 Marks)

Apply Newton – Raphson 's method to solve the Equation (n=2) :-

$$x^3 + x^2 - 2 = 0 \quad , x_0 = 1$$

And find the error?

**Question4** (15 Marks)

Given the data

| x | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | -1.5 | -3 | -1.5 | 3 | -10.5 |

Find the optimal curve of degree 2 fitting data? , $x_0 = 1$

With Best Wishes Dr. Amin Elfeky

| | | |
|---|---|---|
| Kafrelsheikh University<br>Faculty of Science<br>Math. Department<br>Forth level physics.<br>First Term 2017/2018 | | Time: 2h<br>Subject : (2) فزياء رياضية<br>Total Marks: 100 (70 Written, 10 Oral, 20 Exercises)<br>Date : |

## Answer the following questions

Q.1.

Solve the P.D.E. $x\dfrac{\partial^2 u}{\partial x \partial y}+\dfrac{\partial u}{\partial y}=0$ ; and find a particular solution, for which

$u(x,0)=x^5, \qquad u(1,y)=y^3$ (10)

Q.2.

Show that $u(x,t)=e^{-8t}\sin 2x$ is a solution of the following B.V.P.

$$y\frac{\partial^2 u}{\partial x \partial y}+2\frac{\partial u}{\partial x}=0 \ ;$$

$u(x,1)=x^2$ and $u(0,y)=y$ (10)

Q.3.

(i)Solve the B.V.P

$$\frac{\partial^2 u}{\partial x^2}=\frac{y^3}{1+x^2}; \quad u(0,y)=e^y \qquad u(1,y)=y^3\left(\frac{\pi}{4}-\frac{1}{2}\log 2\right)$$

(ii) Solve the B.V.P. by the M.S. Vars.

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}+u; \qquad u(0,t)=0, \qquad u(a,t)=\sin t$$ (30)

Q.4.

By using L.T. solve the problem

$$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=x; \qquad u(x,0)=0, \qquad u(0,t)=0$$

(20)

| | | |
|---|---|---|
| جامعة كفر الشيخ - كلية العلوم<br>قسم الرياضيات<br>الفرقة : الرابعه – رياضيات<br>المادة : توبولوجي ٢<br>رمز المقرر: MATH 871 | | الاختبار النهائي الفصل الاول ٢٠١٧/ ٢٠١٨م<br>التاريخ: ٨/ ١/ ٢٠١٨ م<br>الزمن : ١٠-١٢<br>الدرجه : ٧٠ درجه |

**Solve the following questions:**

**Question (1)** ( 18 Marks )

a- Give the implication between $T_i$ −spaces, where $i = 0,1,2,3,4$; show that the converse of $T_i$ − spaces is not true, where $i = 0,1,2$..

b- Prove that a co finite space is compact .

c- If $A, B$ are non-empty separated sets, show that $A \bigcup B$ is disconnected.

**Question (2)** ( 17 Marks)

a- State and prove a Heni Borel theorem for compact sets.

b- Show that the property of being a compact space is a weak hereditary property.

**Question (3)** ( 17 Marks )

a- Show that a space $(X,\tau)$ is compact if and only if every collection of closed sets $\{F_i : i \in I\}$ such that $\bigcap F_i = \varphi$ , there exists a finite sub collection of closed sets $\{F_i : i = 1,2,3,\ldots m\}$ such that $\bigcap_{i=1}^{m} F_i = \varphi$ .

b- If $f:(X,\tau) \to (Y,\sigma)$ is a bijective continuous mapping, $(X,\tau)$ is a compact space and $(Y,\sigma)$ is a $T_2$ space, show that $f:(X,\tau) \to (Y,\sigma)$ is a homeomorphism.

**Question (4)** ( 18 Marks)

a- Prove that compactness is a topological property.

b- If $(X,\tau)$ is a topological space , prove that a subset $A$ of $X$ is connected if and only if $A$ can not be expressed as the union of two non-empty separated sets.

*End Questions*

*Best regards*

*Prof. Dr. Ahmed EL-Maghrabi*

بحوث عمليات (٤) رياضيات

| | | |
|---|---|---|
| Kafrelsheikh University<br>Faculty of Science<br>Mathematics Department<br>Final Exam of First Term<br>2017-2018 | Operation Research<br>M439 | Level : $4^{th}$ year mathematics<br>Date : 15 \1 \2018<br>Time Allowed : 2H<br>Total Marks :100 (70 Written, 10 Oral,20 Exercises)<br>Exam in one Page |

## Answer the following questions

### Question1 (25 Marks)

**a)** Let $f(x)$ be a numerical function defined on an open set $S \subseteq R^n$. A necessary and sufficient condition that $f(x)$ be a convex on $S$ ,is that for each $x^1, x^2 \in S$ then $\left[\nabla f(x^2) - \nabla f(x^1)\right]'(x^2 - x^1) \geq 0$ *for each* $x \in S.$

**b)** Show that $f(x)$ is twice differentiable where,

$$f(x) = 2x_1 + 6x_2 - 2x_1^2 + 4x_1x_2 - 3x_2^2 \quad , \quad where\ \bar{x} = (0,0)'$$

### Question2 (25 Marks)

**a)** Let $S$ be a nonempty open convex set in $R^n$ and let $f(x): S \to R$ be twice differentiable on $S$. Prove that $f(x)$ is convex if and only if $H(\bar{x})$ is positive semidefinite at each point in $S$ .

**b)** Solve by using KTs-P of the following :-

$$\begin{aligned} &Min\ f(x) = 2x_1 - x_2 \\ &s.t. \quad x_1 + x_2 \leq 1 \\ &\qquad\ \ x_1^2 + x_2^2 \leq 1 \end{aligned}$$

### Question3 (20 Marks)

By the Quadratic Programming method ,solve the following :-

$$\begin{aligned} &Min \quad f(x) = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2 \\ &s.t. \quad 2x_1 + x_2 \leq 6 , \quad x_1 - 4x_2 \leq 0 , \\ &\qquad and \quad x_1, x_2 \geq 0 \end{aligned}$$

***With Best Wishes Dr. Amin Elfeky***